6–4 Inequalities for One Triangle

T28

Theorem 6–2 ***Triangle Angle-Side Size-Ordering Theorem***
If two sides of a triangle are unequal, the angles opposite them are unequal in the same order.

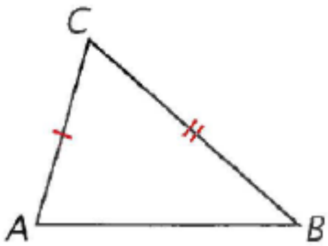
Theorem 6–3 ***Triangle Angle-Side Size-Ordering Converse***
If two angles of a triangle are unequal, the sides opposite them are unequal in the same order.

> **Corollary 1**: The perpendicular segment from a point to a line is the shortest segment from the point to the line.

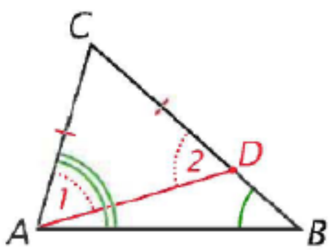
Example of 6–2

Example of 6–3

Theorem 6–2



Given: $\triangle ABC$ with $BC > AC$.
Prove: $\angle A > \angle B$.

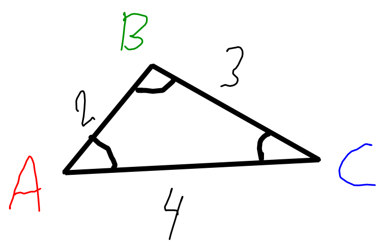


Plan for proof of Theorem 6–2:
Euclid observed the following. The shorter side, AC, is copied along the longer side CB as CD. AD is then drawn to form an isosceles triangle. Euclid's proof is based on observing that $m\angle CAB > m\angle 1$ and $m\angle 1 = m\angle 2$; so $m\angle CAB > m\angle 2$. But $m\angle 2 > m\angle B$; so $m\angle CAB > m\angle B$.

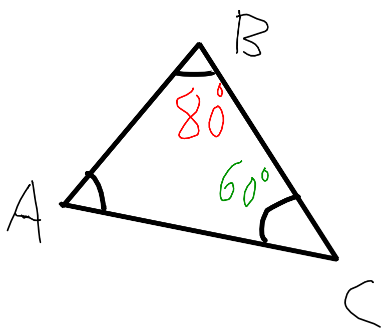
Triangle Side-Angle Order (not on T28)

Do now

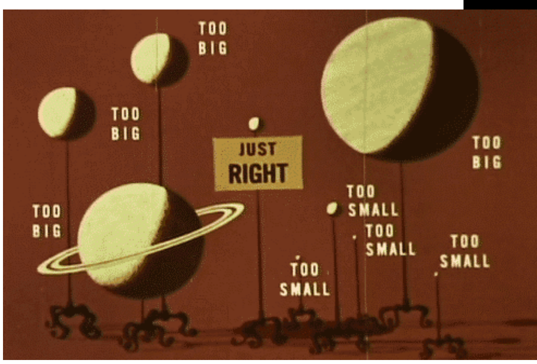
1. Order the lettered angles from largest to smallest

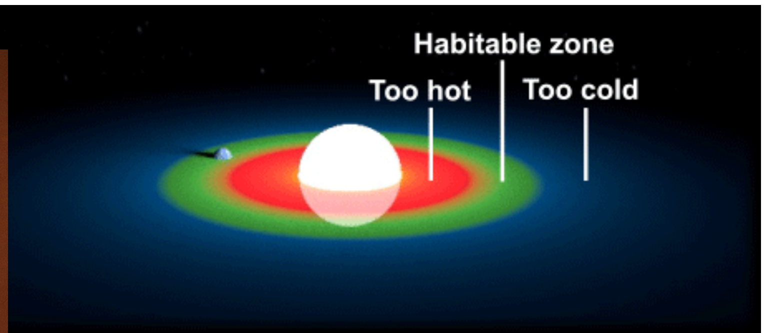


2. Order the lettered sides from largest to smallest



"Goldilocks Principle"





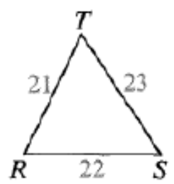
Venus is too hot, Mars is too cold, Earth is just right

Neptune is too big, Mars is too small, Earth is just right.

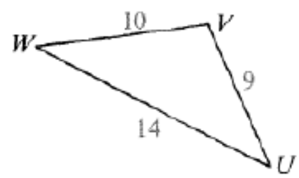
The Goldilocks Principle is used in Astrobiology, Psychology, Medicine, and Economics.

Name the largest angle and the smallest angle of the triangle.

1.



2.



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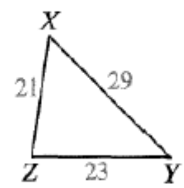

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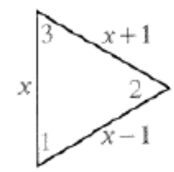
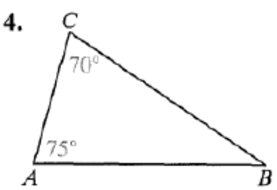


Name the longest side and the shortest side of the triangle.

4.



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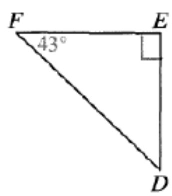

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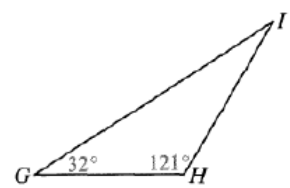

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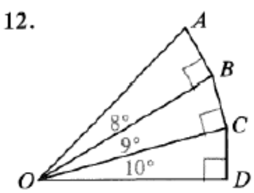


T29

The Triangle Inequality (Theorem 6–4)

The sum of any two sides of a triangle is greater than the third side.

From this and the triangle inequality,

$$\left|\frac{f(x)}{g(x)} - L\right| \le \left|\frac{f(x)}{g(x)} - Lu(x)\right| + |Lu(x) - L| \le \epsilon|u(x)| + |L|\,|u(x) - 1|.$$

Above is from a proof in a college course in Real Analysis.

Given three sides, are these triangles possible?

1. 1,1,1
2. 1,2,1
3. 1,3,1
4. 1,3,2
5. 2,3,2
6. 10,4,5
7. 2,4,7
8. 3,4,5

Two sides of a triangle have lengths of 4 and 7. The length of the third side can be any number between ________ and ________.

The Triangle Inequality (Theorem 6–4)

T29

The sum of any two sides of a triangle is greater than the third side.

Is it possible for a triangle to have sides with the lengths indicated?

7. 10, 9, 8

8. 6, 6, 20

9. 7, 7, 14.1

10. 16, 11, 5

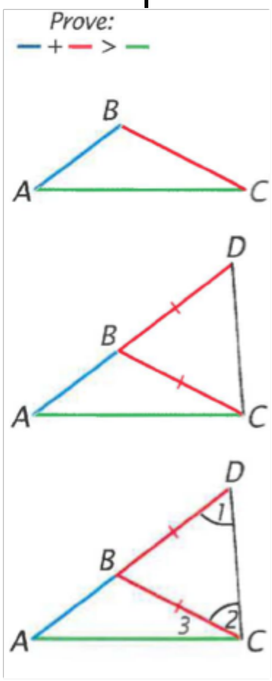
11. 0.6, 0.5, 1

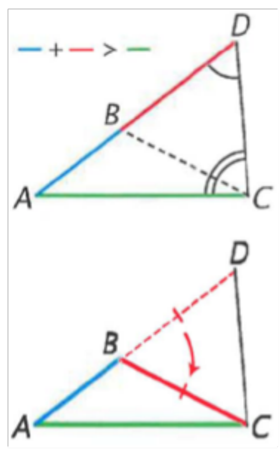
12. 18, 18, 0.06

16. Two sides of a triangle have lengths 15 and 20. The length of the third side can be any number between __?__ and __?__.

Visual proof of The Triangle Inequality (Euclid's method)

T29

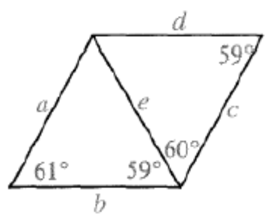




T29

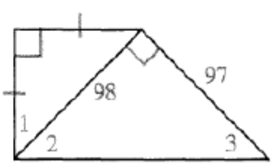
15. Use the lengths a, b, c, d, and e to complete:

$\underline{\ ?\ } > \underline{\ ?\ } > \underline{\ ?\ } > \underline{\ ?\ } > \underline{\ ?\ }$



16. Use $m\angle 1$, $m\angle 2$, and $m\angle 3$ to complete:

$\underline{\ ?\ } > \underline{\ ?\ } > \underline{\ ?\ }$



19. Given: ▱$EFGH$; $EF > FG$

Prove: $m\angle 1 > m\angle 2$