Name: ________________________________ Block/Period: ___

Version 3

Exp-Log Packet

Exponential and Logarithmic Functions

LESSON
8.1

NAME ______________________________ DATE __________

Practice with Examples

For use with pages 465–472

GOAL **Graph exponential growth functions and use exponential growth functions to model real-life situations**

VOCABULARY

An **exponential function** involves the expression b^x where the **base** b is a positive number other than 1. If $a > 0$ and $b > 1$, the function $y = ab^x$ is an exponential growth function.

An **asymptote** is a line that a graph approaches as you move away from the origin. In the exponential growth model $y = a(1 + r)^t$, y is the quantity after t years, a is the initial amount, r is the percent increase expressed as a decimal, and the quantity $1 + r$ is called the **growth factor.**

Compound Interest Consider an initial principal P deposited in an account that pays interest at an annual rate r (expressed as a decimal), compounded n times per year. The amount A in the account after t years can be modeled by this equation: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

EXAMPLE 1

Graphing Exponential Functions

Graph the function (a) $y = -2 \cdot 3^x$ and (b) $y = 2 \cdot 3^x$.

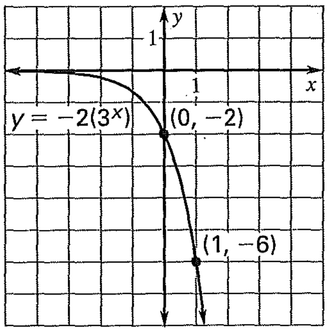
SOLUTION

Begin by plotting two points on the graph. To find these two points, evaluate the function when $x = 0$ and $x = 1$.

a. $y = -2 \cdot 3^0 = -2 \cdot 1 = -2$

$y = -2 \cdot 3^1 = -2 \cdot 3 = -6$

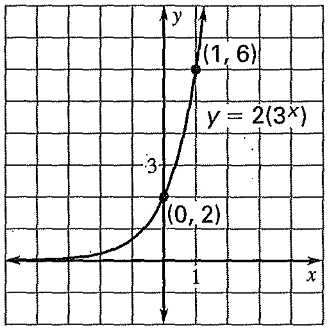
Plot $(0, -2)$ and $(1, -6)$. Then, from left to right, draw a curve that begins just below the x-axis, passes through the two points, and moves down to the right.



b. $y = 2 \cdot 3^0 = 2 \cdot 1 = 2$

$y = 2 \cdot 3^1 = 2 \cdot 3 = 6$

Plot $(0, 2)$ and $(1, 6)$. Then, from left to right, draw a curve that begins just above the x-axis, passes through the two points, and moves up to the right.



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NAME ______________________________ DATE __________

Practice with Examples

For use with pages 465–472

Exercises for Example 1

Graph the function.

1. $y = 2^x$

2. $y = -4^x$

3. $y = -3 \cdot 2^x$

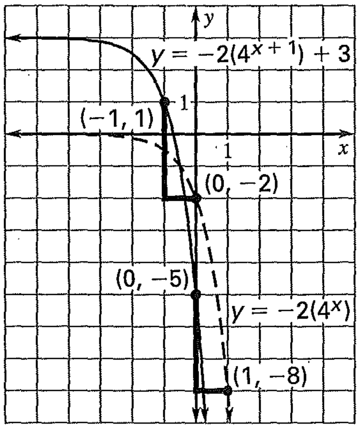
4. $y = 4 \cdot 2^x$

EXAMPLE 2 Graphing a General Exponential Function

Graph $y = -2 \cdot 4^{x+1} + 3$. State the domain and range.

SOLUTION

Begin by lightly sketching the graph of $y = -2 \cdot 4^x$, which passes through $(0, -2)$ and $(1, -8)$. Then because $h = -1$ and $k = 3$, translate the graph 1 unit to the left and 3 units up. Notice that the graph passes through $(-1, 1)$ and $(0, -5)$. The graph's asymptote is $y = 3$. The domain is all real numbers and the range is $y < 3$.



Exercises for Example 2

Graph the function. State the domain and range.

5. $y = -3 \cdot 2^{x+4}$

6. $y = 5 \cdot 2^{x-1}$

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Practice with Examples

For use with pages 465–472

7. $y = 3^{x-2} + 4$

8. $y = 4^{x+2} - 3$

EXAMPLE 3 Modeling Exponential Growth

A diamond ring was purchased twenty years ago for $500. The value of the ring increased by 8% each year. What is the value of the ring today?

SOLUTION

The initial amount is $a = 500$, the percent increase expressed in decimal form is $r = 0.08$, and the time in years is $t = 20$.

$y = a(1 + r)^t$	Write exponential growth model.
$= 500(1 + 0.08)^{20}$	Substitute $a = 500$, $r = 0.08$, and $t = 20$.
$= 500 \cdot 1.08^{20}$	Simplify.
≈ 2330.48	Use a calculator.

The value of the ring today is about $2330.48.

Exercises for Example 3

9. A customer purchases a television set for $800 using a credit card. The interest is charged on any unpaid balance at the rate of 18% per year compounded monthly. If the customer makes no payment for one year, how much is owed at the end of the year?

10. A house was purchased for $90,000 in 1995. If the value of the home increases by 5% per year, what is it worth in the year 2020?

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LESSON 8.2

NAME ______________________________ DATE __________

Practice with Examples

For use with pages 474–479

GOAL **Graph exponential decay functions and use exponential decay functions to model real-life situations**

VOCABULARY

An **exponential decay function** has the form $f(x) = ab^x$, where $a > 0$ and $0 < b < 1$.

An exponential decay model has the form $y = a(1 - r)^t$, where y is the quantity after t years, a is the initial amount, r is the percent decrease expressed as a decimal, and the quantity $1 - r$ is called the **decay factor.**

EXAMPLE 1 *Recognizing Exponential Growth and Decay*

State whether $f(x)$ is an exponential growth or exponential decay function.

a. $f(x) = 4\left(\frac{1}{3}\right)^x$ **b.** $f(x) = 5\left(\frac{3}{4}\right)^{-x}$ **c.** $f(x) = 2(0.15)^x$

SOLUTION

a. Because $b = \frac{1}{3}$, and $0 < b < 1$, f is an exponential decay function.

b. Rewrite the function without negative exponents as $f(x) = 5 \cdot \left(\frac{4}{3}\right)^x$. Because $b = \frac{4}{3}$, and $b > 1$, f is an exponential growth function.

c. Because $b = 0.15$, and $0 < b < 1$, f is an exponential decay function.

Exercises for Example 1

State whether the function represents *exponential growth* or *exponential decay*.

1. $f(x) = 3 \cdot 4^x$

2. $f(x) = 2 \cdot (0.75)^x$

3. $f(x) = 4\left(\frac{1}{3}\right)^x$

4. $f(x) = 4\left(\frac{6}{5}\right)^x$

5. $f(x) = 3\left(\frac{1}{4}\right)^{-x}$

6. $f(x) = 7\left(\frac{5}{2}\right)^{-x}$

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LESSON
8.2
CONTINUED

NAME ______________________________ DATE __________

Practice with Examples

For use with pages 474–479

EXAMPLE 2 Graphing Exponential Functions

Graph the function (a) $y = -2\left(\frac{1}{3}\right)^x$ and (b) $y = 3\left(\frac{2}{3}\right)^x$.

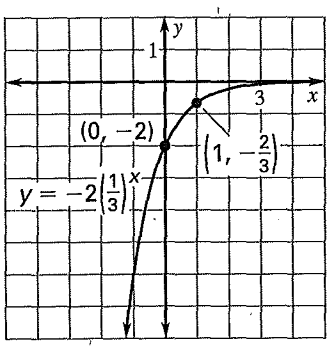
SOLUTION

Begin by plotting two points on the graph. To find these two points, evaluate the function when $x = 0$ and $x = 1$.

a. $y = -2\left(\frac{1}{3}\right)^0 = -2$

$y = -2\left(\frac{1}{3}\right)^1 = -\frac{2}{3}$

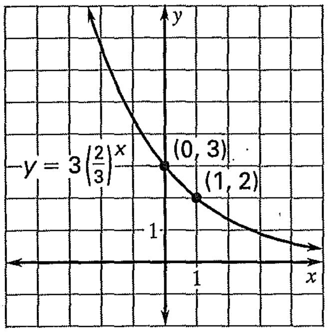
Plot $(0, -2)$ and $\left(1, -\frac{2}{3}\right)$. Then, from *right* to *left,* draw a curve that begins just below the x-axis, passes through the two points, and moves down to the left.



b. $y = 3\left(\frac{2}{3}\right)^0 = 3$

$y = 3\left(\frac{2}{3}\right)^1 = 2$

Plot $(0, 3)$ and $(1, 2)$. Then, from *right* to *left* draw a curve that begins just above the x-axis, passes through the two points, and moves up to the left.



Exercises for Example 2

Graph the function.

7. $y = 2\left(\frac{1}{4}\right)^x$

8. $y = -3\left(\frac{1}{2}\right)^x$

9. $y = 4\left(\frac{3}{4}\right)^x$

10. $y = -5\left(\frac{2}{3}\right)^x$

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Practice with Examples

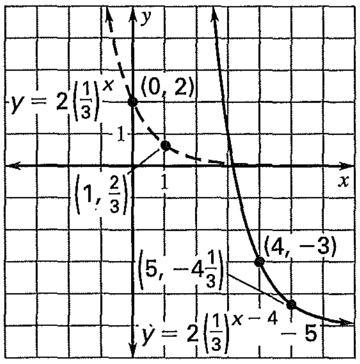
For use with pages 474–479

EXAMPLE 3 Graphing a General Exponential Function

Graph $y = 2\left(\frac{1}{3}\right)^{x-4} - 5$. State the domain and range.

SOLUTION

Begin by lightly sketching the graph of $y = 2\left(\frac{1}{3}\right)^{x}$, which passes through (0, 2) and $\left(1, \frac{2}{3}\right)$. Then, because $h = 4$ and $k = -5$, translate the graph 4 units to the right and 5 units down. Notice that the graph passes through (4, −3) and $\left(5, -4\frac{1}{3}\right)$. The graph's asymptote is the line $y = -5$. The domain is all real numbers and the range is $y > -5$.



Exercises for Example 3

Graph the function. State the domain and range.

11. $y = 2\left(\frac{1}{2}\right)^{x+3}$

12. $y = -3\left(\frac{2}{3}\right)^{x-4}$

13. $y = -\left(\frac{1}{4}\right)^{x} + 2$

14. $y = 4\left(\frac{1}{2}\right)^{x+4} - 3$

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LESSON

8.3

NAME ______________________________ DATE ____________

Practice with Examples

For use with pages 480–485

GOAL **Use the number e as the base of exponential functions**

VOCABULARY

The **natural base e** is irrational. It is defined as follows:
As n approaches $+\infty$, $\left(1 + \frac{1}{n}\right)^n$ approaches $e \approx 2.718281828459$.

EXAMPLE 1

Simplifying Natural Base Expressions

Simplify the expression.

a. $2e \cdot e^{-4}$ **b.** $\frac{6e^{5x}}{2e^{3x}}$ **c.** $(-5e^2)^3$

SOLUTION

a. $2e \cdot e^{-4} = 2e^{1+(-4)}$
$= 2e^{-3}$
$= \frac{2}{e^3}$

b. $\frac{6e^{5x}}{2e^{3x}} = 3e^{5x-3x}$
$= 3e^{2x}$

c. $(-5e^2)^3 = (-5)^3 e^{(2)(3)}$
$= -125e^6$

Exercises for Example 1

Simplify the expression.

1. $e^{-2} \cdot e^6$ **2.** $5e^3 \cdot 4e^2$ **3.** $e^{2x} \cdot e^{4x}$

4. $(2e^3)^3$ **5.** $\frac{e^5}{e^2}$ **6.** $\frac{10e^2}{2e^4}$

Chapter 8

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Practice with Examples

For use with pages 480–485

EXAMPLE 2 Evaluating Natural Base Expressions

Use a calculator to evaluate the expression (a) $e^{2/3}$ and (b) e^{-2}.

SOLUTION

	Expression	*Keystrokes*	*Display*
a.	$e^{2/3}$	2nd [e^x]2 ÷ 3) ENTER	1.947734041
b.	e^{-2}	2nd [e^x] (-) 2) ENTER	0.1353352832

Exercises for Example 2

Use a calculator to evaluate the expression. Round the result to three decimal places.

7. e^4

8. $e^{1/3}$

9. $e^{1.2}$

10. $2e^{-1/5}$

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Practice with Examples

For use with pages 480–485

EXAMPLE 3 Graphing Natural Base Functions

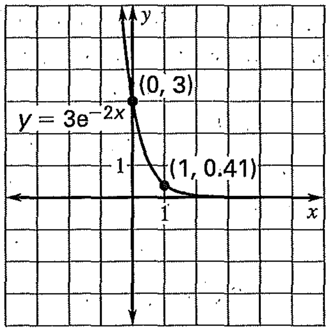
Graph the function. State the domain and range.

a. $y = 3e^{-2x}$

b. $y = \frac{1}{2}e^{x} - 5$

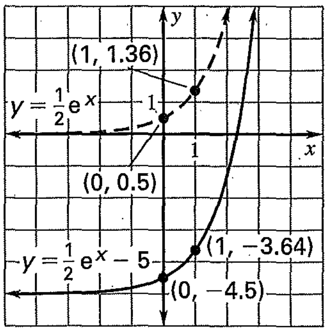
SOLUTION

a. Because $a = 3$ is positive and $r = -2$ is negative, the function is an exponential decay function. Plot points (0, 3) and (1, 0.41) and draw the curve.



The domain is all real numbers, and the range is $y > 0$.

b. Because $a = \frac{1}{2}$ is positive and $r = 1$ is positive, the function is an exponential growth function. Translate the graph of $y = \frac{1}{2}e^{x}$ down 5 units.



The domain is all real numbers, and the range is $y > -5$.

Exercises for Example 3

Graph the function. State the domain and range.

11. $y = 2e^{-x}$

12. $y = e^{x-3}$

13. $y = 4e^{x} - 3$

14. $y = e^{-2x} + 1$

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LESSON

8.4

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Practice with Examples

For use with pages 486–492

GOAL **Evaluate logarithmic functions, and graph logarithmic functions**

> **VOCABULARY**
>
> Let b and y be positive numbers, $b \neq 1$. The **logarithm of y with base b** is denoted by $\log_b y$ and is defined as follows: $\log_b y = x$ if and only if $b^x = y$. The expression $\log_b y$ is read as "log base b of y."
>
> The logarithm with base 10 is called the **common logarithm,** denoted by $\log_{10}$ or simply by log.
>
> The logarithm with base e is called the **natural logarithm,** denoted by $\log_e$ or more often by ln.
>
> If b is a positive real number such that $b \neq 1$, then $\log_b 1 = 0$ because $b^0 = 1$ and $\log_b b = 1$ because $b^1 = b$.

EXAMPLE 1 *Rewriting Logarithmic Equations*

Logarithmic Form	*Exponential Form*
a. $\log_{10} 1000 = 3$	$10^3 = 1000$
b. $\log_4 1 = 0$	$4^0 = 1$
c. $\log_9 \frac{1}{81} = -2$	$9^{-2} = \frac{1}{81}$

Exercises for Example 1

Rewrite the equation in exponential form.

1. $\log_4 64 = \cancel{4}\ 3$

2. $\log_5 125 = 3$

3. $\log_7 1 = 0$

4. $\log_2 \frac{1}{8} = -3$

5. $\log_8 8 = 1$

6. $\log_{1/3} 3 = -1$

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NAME ____________________ DATE __________

Practice with Examples

For use with pages 486–492

EXAMPLE 2 Evaluating Logarithmic Expressions

Evaluate the expressions (a) $\log_{27} 3$ and (b) $\log_6 216$.

SOLUTION

To evaluate a logarithm, you are finding an exponent. To help you evaluate $\log_b y$, ask yourself what power of b gives you y.

a. 27 to what power gives 3?
$27^{1/3} = 3$, so $\log_{27} 3 = \frac{1}{3}$.

b. 6 to what power gives 216?
$6^3 = 216$, so $\log_6 216 = 3$.

Exercises for Example 2

Evaluate the expression without using a calculator.

7. $\log_3 243$

8. $\log_2 2$

9. $\log_5 1$

10. $\log_{16} 4$

11. $\log_{1/3} 9$

12. $\log_{1/2} \frac{1}{32}$

EXAMPLE 3 Using Inverse Properties

Simplify the expressions (a) $5^{\log_5 4}$ and (b) $\log_2 8^x$.

SOLUTION

a. $5^{\log_5 4} = 4$ Use the inverse property $b^{\log_b x} = x$.

b. $\log_2 8^x = \log_2 (2^3)^x$ Rewrite 8 as a power of the base 2.

$= \log_2 2^{3x}$ Use power rule of exponents.

$= 3x$ Use the inverse property $\log_b b^x = x$.

Exercises for Example 3

Simplify the expression.

13. $4^{\log_4 x}$

14. $8^{\log_8 10}$

15. $\log_6 6^x$

16. $\log_3 81^x$

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Practice with Examples

For use with pages 486–492

EXAMPLE 4 Graphing Logarithmic Functions

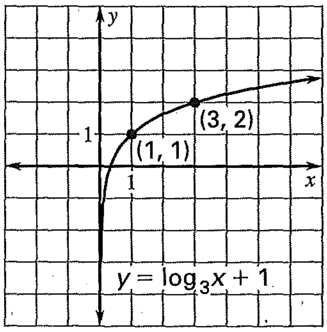
Graph the function. State the domain and range.

a. $y = \log_3 x + 1$

b. $y = \ln(x - 2)$

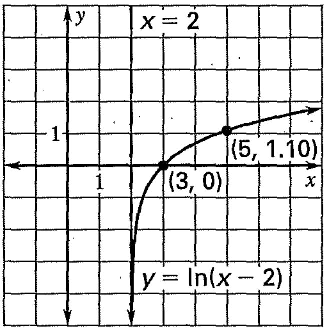
SOLUTION

a. Because $h = 0$, the vertical line $x = 0$ is an asymptote. Plot the points (1, 1) and (3, 2). Because $b > 1$, from left to right, draw a curve that starts just to the right of the line $x = 0$ and moves up.



The domain is $x > 0$, and the range is all real numbers.

b. Because $h = 2$, the vertical line $x = 2$ is an asymptote. Plot the points (3, 0) and (5, 1.10). Because $b > 1$, from left to right, draw a curve that starts just to the right of the line $x = 2$ and moves up.



The domain is $x > 2$, and the range is all real numbers.

Exercises for Example 4

Graph the function. State the domain and range.

17. $y = \log_2 x$

18. $y = \log_{1/2} x$

19. $\ln(x + 2)$

20. $\ln x - 3$

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LESSON **8.5**

NAME ______________________________ DATE __________

Practice with Examples

For use with pages 493–499

GOAL Use properties of logarithms

VOCABULARY

Properties of Logarithms
Let b, u, and v be positive numbers such that $b \neq 1$.

Product Property $\log_b uv = \log_b u + \log_b v$

Quotient Property $\log_b \frac{u}{v} = \log_b u - \log_b v$

Power Property $\log_b u^n = n \log_b u$

Change-of-Base Formula Let u, b, and c be positive numbers with $b \neq 1$ and $c \neq 1$. Then: $\log_c u = \frac{\log_b u}{\log_b c}$.

In particular, $\log_c u = \frac{\log u}{\log c}$ and $\log_c u = \frac{\ln u}{\ln c}$.

EXAMPLE 1 Using Properties of Logarithms

Use $\log_3 2 \approx 0.631$ and $\log_3 5 \approx 1.465$ to approximate the following.

a. $\log_3 \frac{2}{5}$ **b.** $\log_3 10$ **c.** $\log_3 125$

SOLUTION

a. $\log_3 \frac{2}{5} = \log_3 2 - \log_3 5 \approx 0.631 - 1.465 = -0.834$

b. $\log_3 10 = \log_3 (2 \cdot 5) = \log_3 2 + \log_3 5 \approx 0.631 + 1.465 = 2.096$

c. $\log_3 125 = \log_3 5^3 = 3 \log_3 5 \approx 3(1.465) = 4.395$

Exercises for Example 1

Use $\log_6 4 \approx 0.774$ and $\log_6 10 \approx 1.285$ to approximate the value of the expression.

1. $\log_6 40$

2. $\log_6 100$

3. $\log_6 \frac{10}{4}$

4. $\log_6 64$

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Practice with Examples

For use with pages 493–499

EXAMPLE 2 Expanding a Logarithmic Expression

Expand $\ln 6x^5$. Assume x is positive.

SOLUTION

$\ln 6x^5 = \ln 6 + \ln x^5$	Product Property
$= \ln 6 + 5 \ln x$	Power Property

Exercises for Example 2

Expand the expression.

5. $\log 9x$

6. $\log_2 6x^3$

7. $\log_6 \frac{2}{3}$

8. $\log_3 \dfrac{4x}{5}$

9. $\ln 2xy$

10. $\ln \dfrac{2x^2}{y}$

EXAMPLE 3 Condensing a Logarithmic Expression

Condense $3 \ln x + \ln 4 - \ln 7x$.

SOLUTION

$3 \ln x + \ln 4 - \ln 7x = \ln x^3 + \ln 4 - \ln 7x$	Power Property
$= \ln (x^3 \cdot 4) - \ln 7x$	Product Property
$= \ln \dfrac{4x^3}{7x}$	Quotient Property
$= \ln \dfrac{4x^2}{7}$	Simplify.

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Practice with Examples

For use with pages 493–499

Exercises for Example 3

Condense the expression.

11. $\log_4 12 + \log_4 5$

12. $\log x - \log y$

13. $\ln 3 + \ln 6 - \ln 9$

14. $3 \log_2 3$

15. $6 \log_2 x + 3 \log_2 x$

16. $\ln 24 - 3 \ln 2$

EXAMPLE 4 Using the Change-of-Base Formula

Evaluate the expression $\log_2 9$ using common and natural logarithms.

SOLUTION

Notice that the base of the logarithm is two. Most scientific calculators can only evaluate common logarithms of base ten and natural logarithms of base *e*. You must use the change-of-base formula.

Using common logarithms: $\log_2 9 = \frac{\log 9}{\log 2} \approx \frac{0.9542}{0.3010} \approx 3.170$

Using natural logarithms: $\log_2 9 = \frac{\ln 9}{\ln 2} \approx \frac{2.1972}{0.6931} \approx 3.170$

Notice that you obtain the same result using either common or natural logarithms.

Exercises for Example 4

Use the change-of-base formula to evaluate the expression.

17. $\log_3 30$

18. $\log_4 13$

19. $\log_2 17$

20. $\log_5 10$

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NAME ______________________________ DATE __________

Practice with Examples

For use with pages 501–508

GOAL Solve exponential equations and logarithmic equations

VOCABULARY

For $b > 0$ and $b \neq 1$, if $b^x = b^y$, then $x = y$.

For positive numbers b, x, and y where $b \neq 1$, $\log_b x = \log_b y$ if and only if $x = y$.

EXAMPLE 1 *Solving by Equating Exponents*

Solve $9^{x+1} = 27^{x-1}$.

SOLUTION

$9^{x+1} = 27^{x-1}$	Write original equation.
$(3^2)^{x+1} = (3^3)^{x-1}$	Rewrite each power with base 3.
$3^{2x+2} = 3^{3x-3}$	Power of a power property
$2x + 2 = 3x - 3$	Equate exponents
$x = 5$	Solve for x.

The solution is 5.

Exercises for Example 1

Solve the equation.

1. $5^{3x} = 5^{x+8}$

2. $10^{2x+3} = 10^{4x-1}$

3. $25^{2x+(1/2)} = 125^x$

4. $16 = 4^{x+1}$

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NAME ______________________ DATE __________

Practice with Examples

For use with pages 501–508

EXAMPLE 2 Taking a Logarithm of Each Side

Solve $e^{-x} - 6 = 9$.

SOLUTION

Notice that you cannot rewrite each number with the same base. You can solve the equation by taking a logarithm of each side.

$e^{-x} - 6 = 9$	Write original equation.
$e^{-x} = 15$	Add 6 to each side.
$\ln e^{-x} = \ln 15$	Take natural log of each side.
$-x = \ln 15$	$\ln e^x = x$
$x \approx -2.708$	Divide each side by -1 and use a calculator.

The solution is about -2.708.

Exercises for Example 2

Solve the equation.

5. $5^x = 8$

6. $e^{-x} = 5$

7. $2^x + 1 = 5$

8. $10^{2x} - 6 = 146$

9. $9 - 4e^x = 5$

10. $\frac{1}{2}e^{-2x} = 6$

EXAMPLE 3 Solving a Logarithmic Equation

Solve $\ln(2x + 3) = \ln(5x - 6)$.

SOLUTION

$\ln(2x + 3) = \ln(5x - 6)$	Write original equation.
$2x + 3 = 5x - 6$	$\log_b x = \log_b y$ implies $x = y$.
$9 = 3x$	Subtract $2x$ and add 6 to each side.
$3 = x$	Divide each side by 3.

The solution is 3.

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NAME ______________________________ DATE __________

Practice with Examples

For use with pages 501–508

Exercises for Example 3

Solve the equation.

11. $\log(x + 3) = \log(3x + 1)$

12. $\log_2(x - 1) = \log_2(2x + 1)$

13. $\ln(4 - x) = \ln(4x - 11)$

EXAMPLE 4

Exponentiating Each Side

Solve $4\log_3 3x = 20$.

SOLUTION

$4\log_3 3x = 20$	Write original equation.
$\log_3 3x = 5$	Divide each side by 4.
$3^{\log_3 3x} = 3^5$	Exponentiate each side using base 3.
$3x = 243$	$b^{\log_b x} = x$
$x = 81$	Solve for x.

The solution is 81.

Exercises for Example 4

Solve the equation.

14. $\log_8(x - 5) = \frac{2}{3}$

15. $3\log_5(x + 2) = 6$

16. $4\ln 2x = 5$

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LESSON 8.7

NAME ______________________________ DATE __________

Practice with Examples

For use with pages 509–516

GOAL **Model data with exponential functions and power functions**

EXAMPLE 1

Writing an Exponential Function

Write an exponential function $y = ab^x$ whose graph passes through $(2, -36)$ and $(0, -4)$.

SOLUTION

Begin by substituting the coordinates of the two given points to obtain two equations in a and b.

$-36 = ab^2$ Substitute -36 for y and 2 for x.

$-4 = ab^0$ Substitute -4 for y and 0 for x.

Notice that the second equation becomes $-4 = a$ because $b^0 = 1$. Substitute $a = -4$ in the first equation and solve for b:

$-36 = (-4)b^2$ Substitute -4 for a.

$9 = b^2$ Divide each side by -4.

$3 = b$ Take the positive square root.

So, $y = -4 \cdot 3^x$.

Exercises for Example 1

Write an exponential function $y = ab^x$ whose graph passes through the given points.

1. $(0, 7), (1, 14)$

2. $(1, -12), (-1, -3)$

3. $(1, 9), (-1, 1)$

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NAME ______________________________ DATE __________

Practice with Examples

For use with pages 509–516

EXAMPLE 2 Writing a Power Function

Write a power function $y = ax^b$ whose graph passes through (2, 4) and (4, 32).

SOLUTION

Begin by substituting the coordinates of the two points to obtain two equations in a and b.

$4 = a \cdot 2^b$ Substitute 4 for y and 2 for x.

$32 = a \cdot 4^b$ Substitute 32 for y and 4 for x.

To solve the system, solve for a in the first equation to get $a = \frac{4}{2^b}$, then substitute into the second equation.

$$32 = \left(\frac{4}{2^b}\right)4^b$$

$$32 = 4 \cdot 2^b$$

$$8 = 2^b$$

By inspection, $b = 3$, so $a = \frac{4}{2^b} = \frac{4}{2^3} = \frac{4}{8} = 0.5$ and $y = 0.5x^3$.

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LESSON
8.7
CONTINUED

NAME ______________________________ DATE __________

Practice with Examples

For use with pages 509–516

Exercises for Example 2

Write a power function of the form $y = ax^b$ whose graph passes through the given points.

4. (2, 1), (6, 9)

5. (4, 48), (2, 6)

6. (9, 6), (4, 4)

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Practice with Examples

For use with pages 517–522

GOAL Evaluate and graph logistic growth functions

VOCABULARY

Logistic growth functions are written as $y = \frac{c}{1 + ae^{-rx}}$, where c, a, and r are positive constants.

The graph of $y = \frac{c}{1 + ae^{-rx}}$ has the following characteristics:

- The horizontal lines $y = 0$ and $y = c$ are asymptotes.
- The y-intercept is $\frac{c}{1 + a}$.
- The domain is all real numbers, and the range is $0 < y < c$.
- The graph is increasing from left to right. To the left of its point of maximum growth, $\left(\frac{\ln a}{r}, \frac{c}{2}\right)$, the rate of increase is increasing. To the right of its point of maximum growth, the rate of increase is decreasing.

EXAMPLE 1 *Evaluating a Logistic Growth Function*

Evaluate $f(x) = \frac{300}{1 + e^{-2x}}$ for (a) $f(-2)$, (b) $f(0)$, and (c) $f(3)$.

SOLUTION

a. $f(-2) = \frac{300}{1 + e^{-2(-2)}} = \frac{300}{1 + e^4} \approx 5.4$

b. $f(0) = \frac{300}{1 + e^{-2(0)}} = \frac{300}{1 + e^0} = \frac{300}{1 + 1} = 150$

c. $f(3) = \frac{300}{1 + e^{-2(3)}} = \frac{300}{1 + e^{-6}} \approx 299.3$

Exercises for Example 1

Evaluate the function $f(x) = \frac{5}{1 + e^{-0.3x}}$ for the given value of x.

1. $f(0)$

2. $f(1)$

3. $f(-1)$

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NAME ______________________________ DATE __________

Practice with Examples

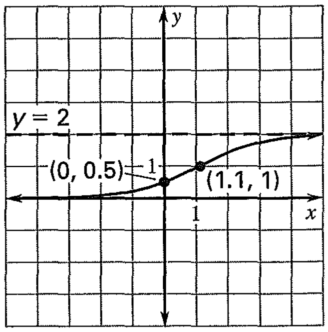
For use with pages 517–522

4. $f(4)$ **5.** $f(-3)$ **6.** $f(0.6)$

EXAMPLE 2 Graphing a Logistic Growth Function

Graph $y = \frac{2}{1 + 3e^{-x}}$.

SOLUTION



Begin by sketching the horizontal asymptote, $y = 2$.

Then find the y-intercept at $y = \frac{2}{1 + 3} = 0.5$. The point of maximum growth is $\left(\frac{\ln 3}{1}, \frac{2}{2}\right) \approx (1.1, 1)$. Plot these points. Finally, from left to right, draw a curve that starts just above the x-axis, curves up to the point of maximum growth, and then levels off as it approaches the upper horizontal asymptote, $y = 2$.

Exercises for Example 2

Graph the function. Identify the asymptotes, *y*-intercept, and point of maximum growth.

7. $y = \frac{4}{1 + 3e^{-x}}$ **8.** $y = \frac{3}{1 + e^{-0.02x}}$ **9.** $y = \frac{2}{1 + 2e^{-3x}}$

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NAME ______________________________ DATE __________

Practice with Examples

For use with pages 517–522

EXAMPLE 3 Solving a Logistic Growth Equation

Solve $\frac{12}{1 + 3e^{-2x}} = 10$.

SOLUTION

$\frac{12}{1 + 3e^{-2x}} = 10$	Write original equation.
$12 = 10(1 + 3e^{-2x})$	Multiply each side by $1 + 3e^{-2x}$.
$12 = 10 + 30e^{-2x}$	Use distributive property.
$2 = 30e^{-2x}$	Subtract 10 from each side.
$0.067 = e^{-2x}$	Divide each side by 30.
$\ln 0.067 = \ln e^{-2x}$	Take natural log of each side.
$\ln 0.067 = -2x$	$\ln e^x = x$
$-\frac{1}{2} \ln 0.067 = x$	Multiply each side by $-\frac{1}{2}$.
$1.35 \approx x$	Use a calculator.

The solution is about 1.35.

Exercises for Example 3

Solve the equation.

10. $\frac{25}{1 + 2e^{-x}} = 20$

11. $\frac{4}{1 + e^{-4x}} = 1$

12. $\frac{100}{1 + 5e^{-3x}} = 50$

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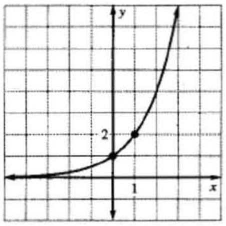
Chapter 8

PW Answers

Lesson 8.1

1.



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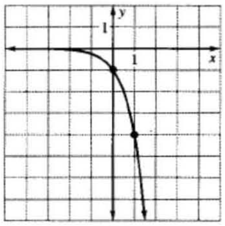

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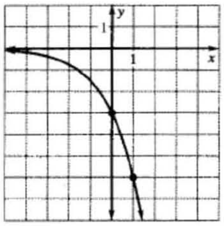

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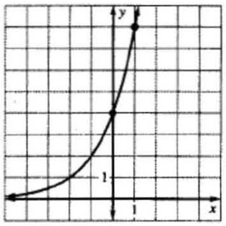
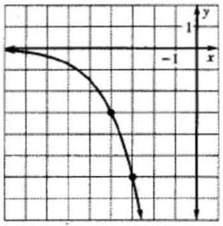
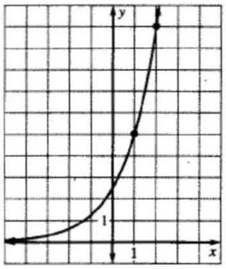


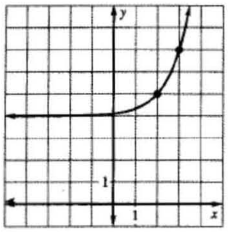
5.
domain: all real numbers
range: $y < 0$



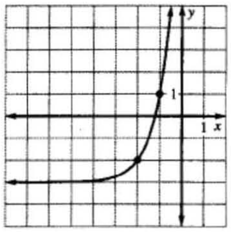
6.
domain: all real numbers
range: $y > 0$



7. domain: all real numbers; range: $y > 4$ $y > -3$



8. domain: all real numbers; range:

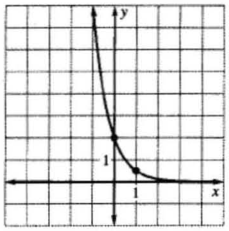


9. \$956.49 10. \$304,771.94

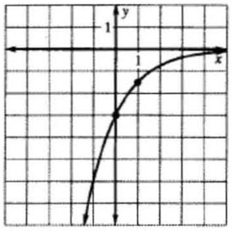
Lesson 8.2

1. exponential growth 2. exponential decay
3. exponential decay 4. exponential growth
5. exponential growth 6. exponential decay

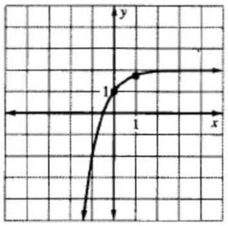
7.



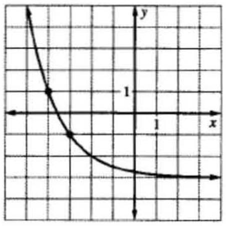
8.



13. domain: all real numbers; range: $y < 2$ $y > -3$



14. domain: all real numbers; range:

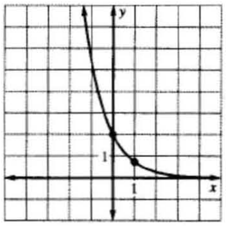


Lesson 8.3

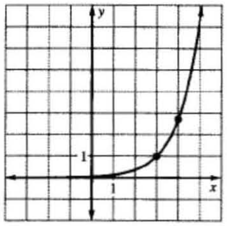
1. e^4 2. $20e^5$ 3. e^{6x} 4. $8e^9$ 5. e^3 6. $\frac{5}{e^2}$

7. 54.598 8. 1.396 9. 3.320 10. 1.637

11. domain: all real numbers; range: $y > 0$



12. domain: all real numbers; range: $y > 0$



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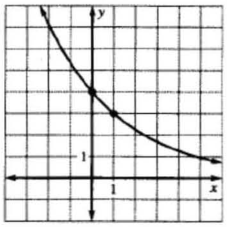
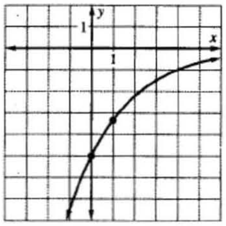
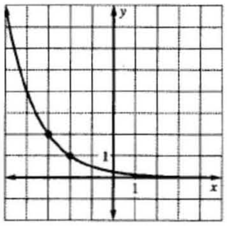


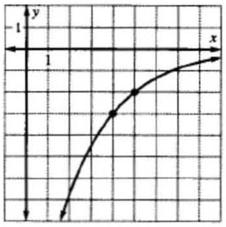
10.



11. domain: all real numbers; range: $y > 0$

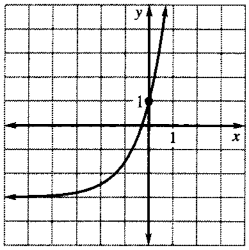


12. domain: all real numbers; range: $y < 0$

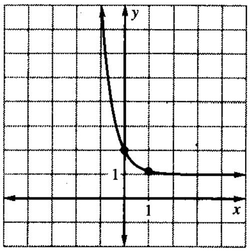


Chapter 8 *continued*

13. domain: all real numbers; range: $y > -3$



14. domain: all real numbers; range: $y > 1$

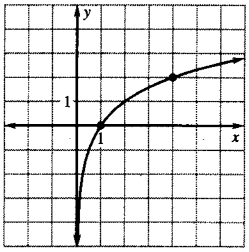


Lesson 8.4

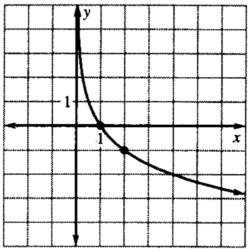
P. 160 $4^3=64$ $\log_4 64=3$

1. ~~$4^4 = 64$~~ **2.** $5^3 = 125$ **3.** $7^0 = 1$
4. $2^{-3} = \frac{1}{8}$ **5.** $8^1 = 8$ **6.** $\left(\frac{1}{3}\right)^{-1} = 3$ **7.** 5
8. 1 **9.** 0 **10.** $\frac{1}{2}$ **11.** -2 **12.** 5 **13.** x
14. 10 **15.** x **16.** $4x$

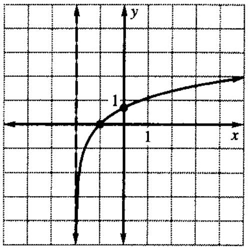
17. domain: $x > 0$
range: all real numbers



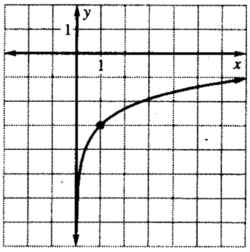
18. domain: $x > 0$
range: all real numbers



19. domain: $x > -2$
range: all real numbers



20. domain: $x > 0$
range: all real numbers



Lesson 8.5

1. 2.059 **2.** 2.57 **3.** 0.511 **4.** 2.322
5. $\log 9 + \log x$ **6.** $\log_2 6 + 3 \log_2 x$
7. $\log_6 2 - \log_6 3$ **8.** $\log_3 4 + \log_3 x - \log_3 5$
9. $\ln 2 + \ln x + \ln y$ **10.** $\ln 2 + 2 \ln x - \ln y$
11. $\log_4 60$ **12.** $\log \frac{x}{y}$ **13.** $\ln 2$
14. $\log_2 27$ **15.** $\log_2 x^9$ **16.** $\ln 3$ **17.** 3.096
18. 1.850 **19.** 4.087 **20.** 1.431

Lesson 8.6

1. 4 **2.** 2 **3.** -1 **4.** 1 **5.** 1.292
6. -1.609 **7.** 2 **8.** 1.091 **9.** 0
10. -1.242 **11.** 1 **12.** -2 **13.** 3 **14.** 9
15. 23 **16.** 1.745

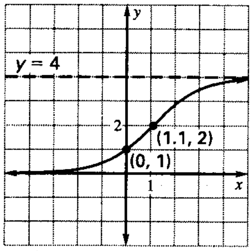
Lesson 8.7

1. $y = 7 \cdot 2^x$ **2.** $y = -6 \cdot 2^x$ **3.** $y = 3 \cdot 3^x$
4. $y = 0.25x^2$ **5.** $y = 0.75x^3$ **6.** $y = 2x^{0.5}$

Lesson 8.8

1. 2.5 **2.** 2.9 **3.** 2.1 **4.** 3.8 **5.** 1.4 **6.** 2.7

7.



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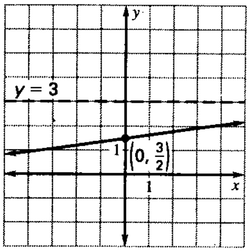

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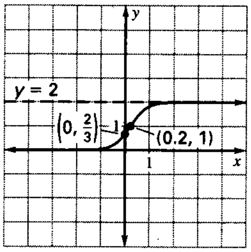

10. 2.0
11. -0.27
12. 0.54

Chapter 9

Lesson 9.1

1. inverse variation **2.** neither
3. direct variation **4.** direct variation
5. $y = \frac{20}{x}$; 5 **6.** $y = \frac{-9}{x}$; -2.25
7. $y = \frac{16}{x}$; 4 **8.** $z = 2xy$; 32
9. $z = -\frac{1}{3}xy$; $-\frac{16}{3}$ **10.** $z = 32xy$; 512

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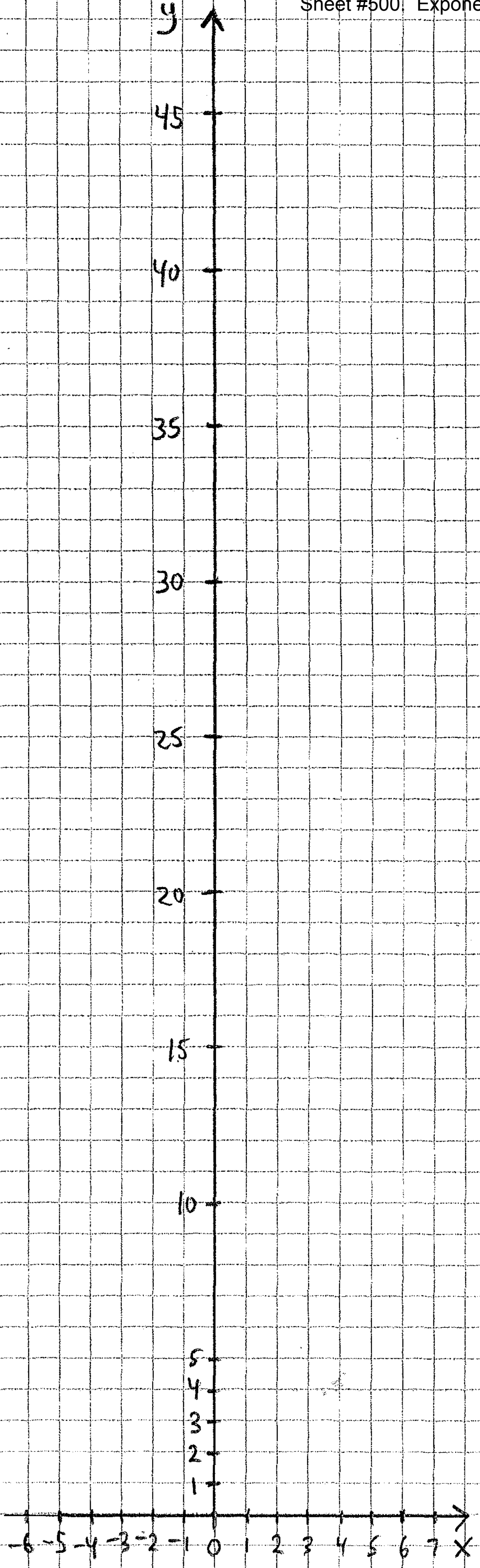
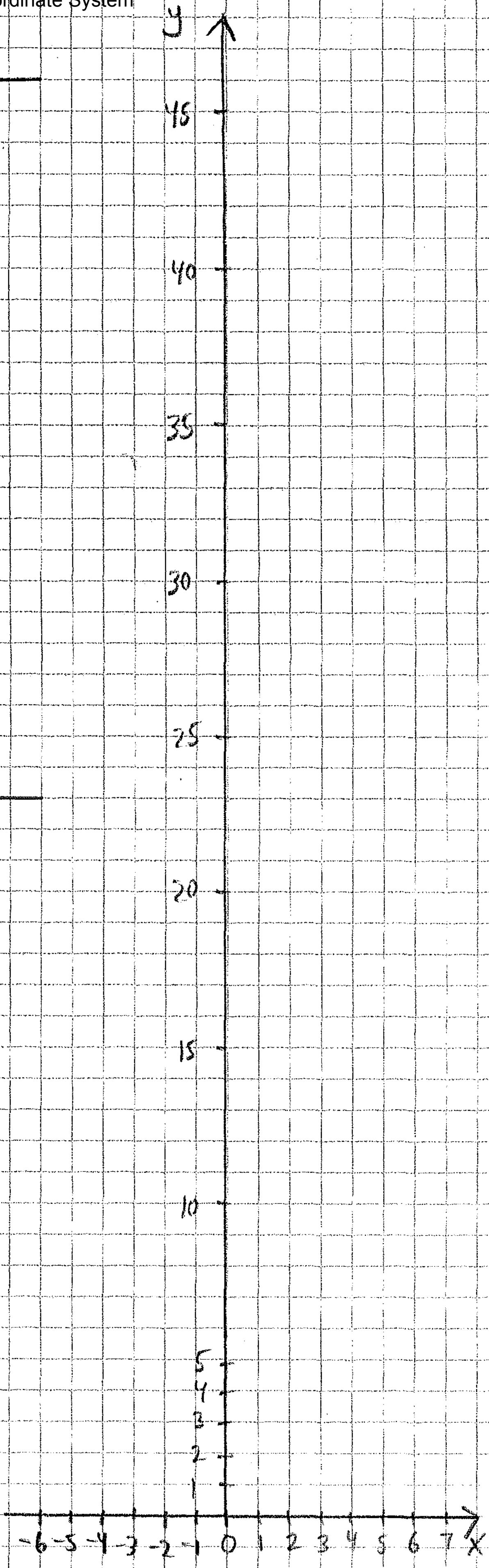
Sheet #500. Exponential Graphing Coordinate System

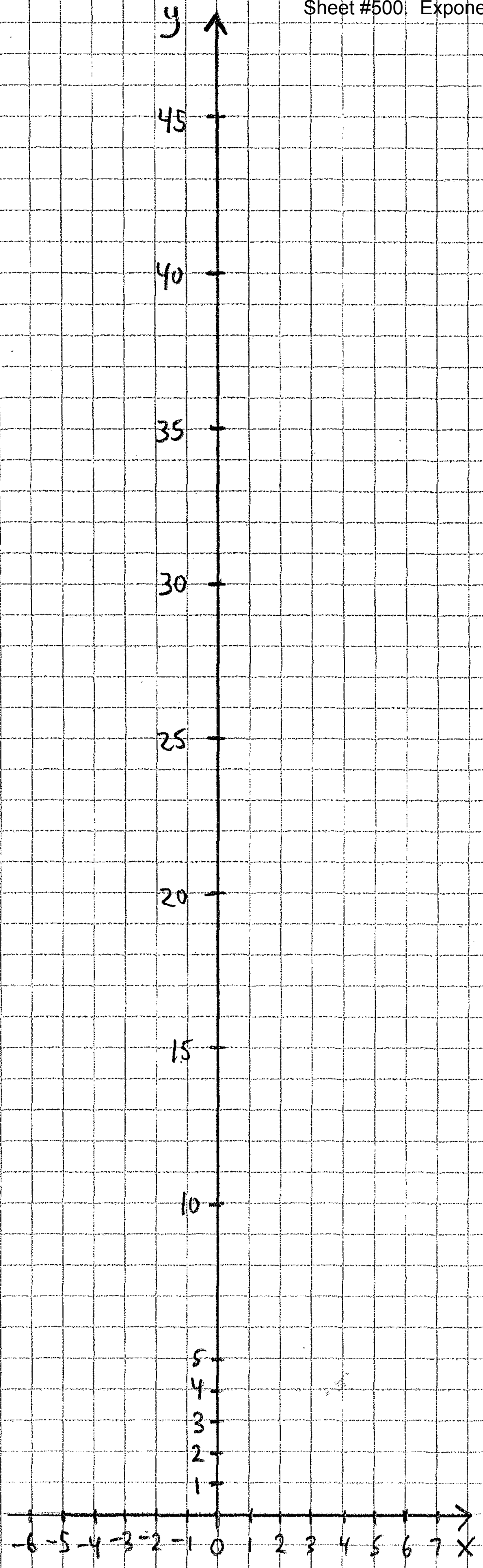


X	y

X	y

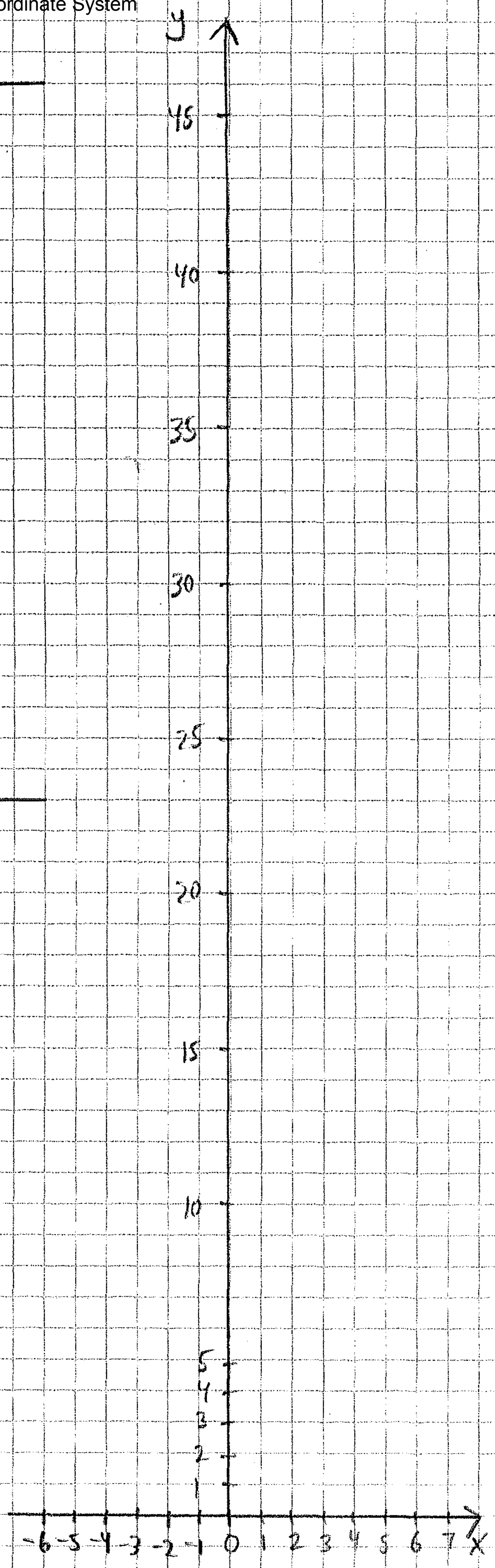


Sheet #500. Exponential Graphing Coordinate System



x	y

x	y



Sheet #501: Exponents Introduction
FMC 2nd Ch 3 Tools, p. 141

For Exercises 1–43, evaluate without a calculator.

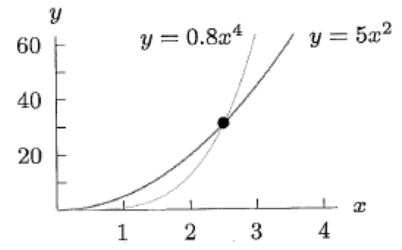
1. 4^3
2. $(-5)^2$
3. 11^2
4. 10^4
5. $(-1)^{12}$
6. $(-1)^{13}$
7. $\frac{5^3}{5^2}$
8. $\frac{5^3}{5}$
9. $\frac{10^8}{10^5}$
10. $\frac{6^4}{6^4}$
11. 8^0
12. $\sqrt{4}$
13. $\sqrt{4^2}$
14. $\sqrt{4^3}$
15. $\sqrt{4^4}$
16. $\sqrt{(-4)^2}$
17. $\frac{1}{7^{-2}}$
18. $\frac{2^7}{2^3}$
19. $(-1)^{445}$
20. -11^2
21. $(-2)3^2$
22. $\left(5^0\right)^3$
23. $2.1\left(10^3\right)$
24. $32^{1/5}$
25. $16^{1/2}$
26. $16^{1/4}$
27. $16^{3/4}$
28. $16^{5/4}$
29. $16^{5/2}$
30. $100^{5/2}$
31. $\sqrt[3]{-125}$
32. $\sqrt{(-4)^2}$
33. $(-1)^3\sqrt{36}$
34. $(0.04)^{1/2}$
35. $(-8)^{2/3}$
36. 3^{-1}
37. 3^{-2}
38. $3^{-3/2}$
39. 25^{-1}
40. 25^{-2}
41. $25^{-3/2}$
42. $(1/27)^{-1/3}$
43. $(0.125)^{1/3}$

If possible, evaluate the quantities in Exercises 78–86. Check your answers with a calculator.

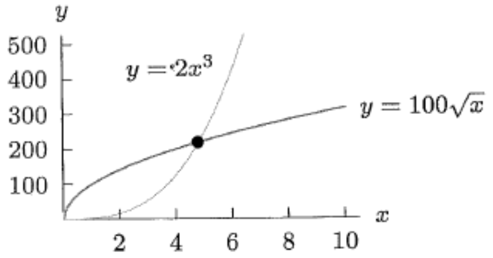
78. $(-32)^{3/5}$
79. $-32^{3/5}$
80. $-625^{3/4}$
81. $(-625)^{3/4}$
82. $(-1728)^{4/3}$
83. $64^{-3/2}$
84. $-64^{3/2}$
85. $(-64)^{3/2}$
86. $81^{5/4}$

In Exercises 93–94, use algebra to find the point of intersection.

93.



94.



Simplify the expressions in Exercises 44–77 and leave without radicals.

44. $\sqrt{x^4}$
45. $\sqrt{y^8}$
46. $\sqrt{w^8z^4}$
47. $\sqrt{x^5y^4}$
48. $\sqrt{16x^3}$
49. $\sqrt{49w^9}$
50. $\sqrt{25x^3z^4}$
51. $\sqrt{r^2}$
52. $\sqrt{r^3}$
53. $\sqrt{r^4}$
54. $\sqrt{36t^2}$
55. $\sqrt{64s^7}$
56. $\sqrt{50x^4y^6}$
57. $\sqrt{48u^{10}v^{12}y^5}$
58. $\sqrt{8m}\sqrt{2m^3}$
59. $\sqrt{6s^2t^3v^5}\sqrt{6st^5v^3}$

60. $(0.1)^2\left(4xy^2\right)^2$

61. $3\left(3^{x/2}\right)^2$

62. $\left(4L^{2/3}P\right)^{3/2}(P)^{-3/2}$

63. $7\left(5w^{1/2}\right)\left(2w^{1/3}\right)$

64. $\left(S\sqrt{16xt^2}\right)^2$

65. $\sqrt{e^{2x}}$

66. $(3AB)^{-1}\left(A^2B^{-1}\right)^2$

67. $e^{kt}\cdot e^3\cdot e$

68. $\sqrt{M+2}(2+M)^{3/2}$

69. $\left(3x\sqrt{x^3}\right)^2$

70. $x^e\left(x^e\right)^2$

71. $\left(y^{-2}e^y\right)^2$

72. $\dfrac{4x^{(3\pi+1)}}{x^2}$

73. $\dfrac{4A^{-3}}{(2A)^{-4}}$

74. $\dfrac{a^{n+1}3^{n+1}}{a^n3^n}$

75. $\dfrac{12u^3}{3\left(uv^2w^4\right)^{-1}}$

76. $\left(a^{-1}+b^{-1}\right)^{-1}$

77. $\left(\dfrac{35(2b+1)^9}{7(2b+1)^{-1}}\right)^2$ (Do not expand $(2b+1)^9$.)

In Exercises 87–92, solve for x.

87. $\dfrac{10x^5}{x^2}=2$

88. $\dfrac{5x^3}{x^5}=125$

89. $\sqrt{4x^3}=5$

90. $7x^4=20x^2$

91. $5x^{-2}=500$

92. $2(x+2)^3=100$

1	64
3	121
5	1
7	5
9	1,000
11	1
13	4
15	16
17	49
19	-1
21	-18
23	2100
25	4
27	8
29	1024
31	-5
33	-6
35	4
37	1/9
39	1/25
41	1/125
43	0.5
45	y^4
47	$x^{5/2}y^2$
49	$7w^{9/2}$
51	r
53	r^2
55	$8s^{7/2}$
57	$4\sqrt{3}u^5v^6y^{5/2}$
59	$6s^{3/2}t^4v^4$
61	3^{x+1}
63	$70w^{5/6}$
65	e^x
67	e^{kt+4}
69	$9x^5$
71	e^{2y}/y^4
73	$64A$
75	$4u^4v^2w^4$
77	$25(2b+1)^{20}$
79	-8
81	Not a real number
83	1/512
85	Not a real number
87	$x=0.585$
89	$x=1.842$
91	$x=\pm0.1$
93	$(2.5, 31.25)$

Sheet #511: Exponential Functions and Graphs
FMC 2nd Ch 3 Review, p.133

5. Without a calculator, match each of the formulas to one of the graphs in Figure 3.31.

(a) $y = 0.8^t$ **(b)** $y = 5(3)^t$
(c) $y = -6(1.03)^t$ **(d)** $y = 15(3)^{-t}$
(e) $y = -4(0.98)^t$ **(f)** $y = 82(0.8)^{-t}$

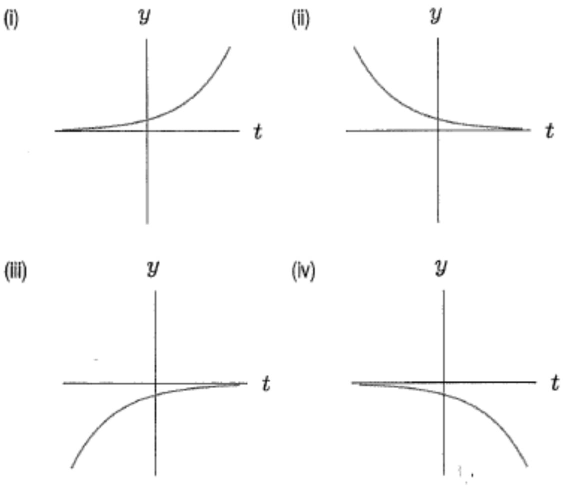


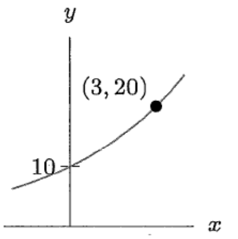
Figure 3.31

6. Without a calculator, match each of the following formulas to one of the graphs in Figure 3.31.

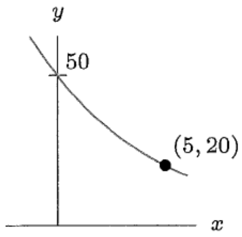
(a) $y = 8.3e^{-t}$ **(b)** $y = 2.5e^t$ **(c)** $y = -4e^{-t}$

For Exercises 7–11, find a formula for the exponential function.

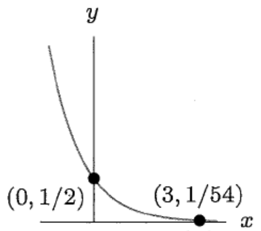
7.



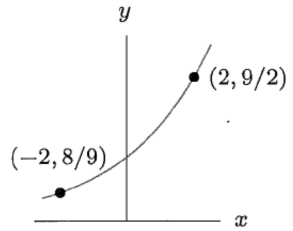
8.



9.



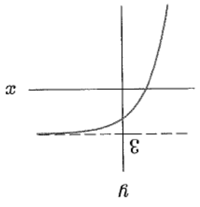
10.



17. Suppose that $f(x)$ is exponential and that $f(-3) = 54$ and $f(2) = \frac{2}{9}$. Find a formula for $f(x)$.

20. Graph a function with a horizontal asymptote of $y = 5$.

21. Use a graph to determine any horizontal asymptotes of $y = 3 - e^{-x}$.



5 (a) (ii)
(b) (i)
(c) (iv)
(d) (ii)
(e) (iii)
(f) (i)

7 $y = 10(1.260)^x$

9 $y = (1/2)(1/3)^x$

17 $f(x) = 2(1/3)^x$

21

Name: ______________________ Group Members: ____

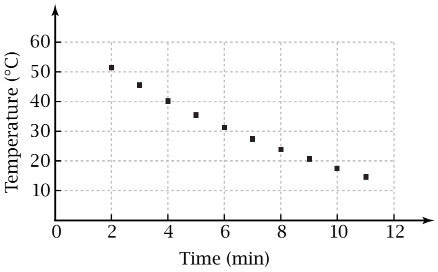
Exploration 8-4a: Coffee Data Residual Plot

Date: ____________

Objective: Use a residual plot to make conclusions about a cooling cup of coffee.

You pour a cup of coffee. Rather than drinking it right away, you let it cool, measuring its temperature at 1-minute intervals. This table and graph show y, the number of degrees above room temperature, as a function of x minutes since you poured the coffee.

x (min)	y (°C)
2	51.5
3	45.5
4	40.2
5	35.5
6	31.3
7	27.4
8	23.9
9	20.6
10	17.5
11	14.5



1. Based on the shape and concavity of the graph, explain why both an exponential function and a power function would be reasonable mathematical models for temperature as a function of time.

2. Explain why both exponential and power functions have reasonable endpoint behavior for large values of x but only the exponential function behaves correctly at the left end of the domain.

3. Confirm by exponential and power regression that an exponential function fits the data better than a power function. How do you decide?

4. Plot the exponential function of Problem 3 and the data on the same screen. Sketch the result on the given figure.

5. Calculate the residuals for each point using the exponential function from Problem 3. Make a residual plot using a window with an x-range of [0, 12] and a suitable range for the residuals. Show your graph to your instructor. ______________

6. After you have checked your residual plot with your instructor, sketch it here.

7. What does the fact that the residual plot shows a pattern indicate about how well the exponential function fits the data? What could you conclude if a residual plot showed no discernible pattern?

8. What did you learn as a result of doing this Exploration that you did not know before?

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Name: ______________________________

Sheet #513: Interest Choice Problem

v. 3

You have the opportunity to consider these choices for Certificates of Deposit (CDs):

Choice A: Deposit $1,500 at an annual rate of 5%.
Choice B: Deposit $1,000 at an annual rate of 10%.
Choice C: Deposit $2,000 with a zero coupon rate of 25%. Zero coupon means the interest is only paid at the end, i.e. once.

1. Consider 10-year CDs. Choices A and B are compounded yearly.
a) Guess the best, second best, third best (if best means more money).

b) Calculate the amount for each choice.

2. After how many years does the Choice B amount exceed the Choice A amount? (Compounded yearly.)

3. After how much time would Choice B have a larger amount than Choice A if the CDs were compounded *daily* instead? Answer in decimal to 3 decimals as well as in years and days.

1. $2443.30, $2593.70, $2500.00. 2. 9 yrs. 3. 8.111 yrs (8 yrs, 41 days).

Name: ________________________

Sheet 514: 100% Interest!

1 dollar, 100% annual interest, one year, variable n (# compound periods)

$y = 1 * (1 + 1/n)^\wedge(1*n)$

Make a table with increasing n. Calculator allowed.

n	$y = (1 + 1/n)^\wedge n$
1	(1+1)^1
2	
3	
10	
100	
1,000	
10,000	
1 * 10^10	

Sheet 515: History of the number *e*

Discovered by Jacob Bernoulli, lived 1654 - 1705,
when studying 100% interest

2.71828182845904523536028747135266249775724709369995...

Given name *e* by Leonhard ______________ in 1737

He was a pioneering Swiss mathematician and physicist.
He made important discoveries in fields as diverse as infinitesimal calculus and graph theory

The number *e* is

not just irrational (as shown by L.E.)

it's not algebraic

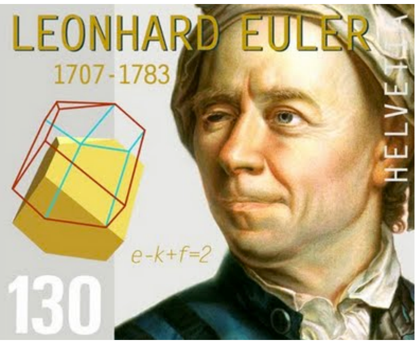
it is _____________________

proven

e Charles Hermite 1873

The number pi was proven

π Ferdinand von Lindemann 1882



Sheet #521: Table of Common Logarithms (base10)

1.000	0.00000000			2.00	0.3010300	3.00	0.4771213	4.00	0.6020600	5.00	0.6989700	6.00	0.7781513	7.00	0.8450980	8.00	0.9030900	9.00	0.9542425
1.001	0.00043408			2.01	0.3031961	3.01	0.4785665	4.01	0.6031444	5.01	0.6998377	6.01	0.7788745	7.01	0.8457180	8.01	0.9036325	9.01	0.9547248
1.002	0.00086772			2.02	0.3053514	3.02	0.4800069	4.02	0.6042261	5.02	0.7007037	6.02	0.7795965	7.02	0.8463371	8.02	0.9041744	9.02	0.9552065
1.003	0.00130093			2.03	0.3074960	3.03	0.4814426	4.03	0.6053050	5.03	0.7015680	6.03	0.7803173	7.03	0.8469553	8.03	0.9047155	9.03	0.9556878
1.004	0.00173371			2.04	0.3096302	3.04	0.4828736	4.04	0.6063814	5.04	0.7024305	6.04	0.7810369	7.04	0.8475727	8.04	0.9052560	9.04	0.9561684
1.005	0.00216606			2.05	0.3117539	3.05	0.4842998	4.05	0.6074550	5.05	0.7032914	6.05	0.7817554	7.05	0.8481891	8.05	0.9057959	9.05	0.9566486
1.006	0.00259798			2.06	0.3138672	3.06	0.4857214	4.06	0.6085260	5.06	0.7041505	6.06	0.7824726	7.06	0.8488047	8.06	0.9063350	9.06	0.9571282
1.007	0.00302947			2.07	0.3159703	3.07	0.4871384	4.07	0.6095944	5.07	0.7050080	6.07	0.7831887	7.07	0.8494194	8.07	0.9068735	9.07	0.9576073
1.008	0.00346053			2.08	0.3180633	3.08	0.4885507	4.08	0.6106602	5.08	0.7058637	6.08	0.7839036	7.08	0.8500333	8.08	0.9074114	9.08	0.9580858
1.009	0.00389117			2.09	0.3201463	3.09	0.4899585	4.09	0.6117233	5.09	0.7067178	6.09	0.7846173	7.09	0.8506462	8.09	0.9079485	9.09	0.9585639
1.010	0.00432137	1.10	0.0413927	2.10	0.3222193	3.10	0.4913617	4.10	0.6127839	5.10	0.7075702	6.10	0.7853298	7.10	0.8512583	8.10	0.9084850	9.10	0.9590414
1.011	0.00475116	1.11	0.0453230	2.11	0.3242825	3.11	0.4927604	4.11	0.6138418	5.11	0.7084209	6.11	0.7860412	7.11	0.8518696	8.11	0.9090209	9.11	0.9595184
1.012	0.00518051	1.12	0.0492180	2.12	0.3263359	3.12	0.4941546	4.12	0.6148972	5.12	0.7092700	6.12	0.7867514	7.12	0.8524800	8.12	0.9095560	9.12	0.9599948
1.013	0.00560945	1.13	0.0530784	2.13	0.3283796	3.13	0.4955443	4.13	0.6159501	5.13	0.7101174	6.13	0.7874605	7.13	0.8530895	8.13	0.9100905	9.13	0.9604708
1.014	0.00603795	1.14	0.0569049	2.14	0.3304138	3.14	0.4969296	4.14	0.6170003	5.14	0.7109631	6.14	0.7881684	7.14	0.8536982	8.14	0.9106244	9.14	0.9609462
1.015	0.00646604	1.15	0.0606978	2.15	0.3324385	3.15	0.4983106	4.15	0.6180481	5.15	0.7118072	6.15	0.7888751	7.15	0.8543060	8.15	0.9111576	9.15	0.9614211
1.016	0.00689371	1.16	0.0644580	2.16	0.3344538	3.16	0.4996871	4.16	0.6190933	5.16	0.7126497	6.16	0.7895807	7.16	0.8549130	8.16	0.9116902	9.16	0.9618955
1.017	0.00732095	1.17	0.0681859	2.17	0.3364597	3.17	0.5010593	4.17	0.6201361	5.17	0.7134905	6.17	0.7902852	7.17	0.8555192	8.17	0.9122221	9.17	0.9623693
1.018	0.00774778	1.18	0.0718820	2.18	0.3384565	3.18	0.5024271	4.18	0.6211763	5.18	0.7143298	6.18	0.7909885	7.18	0.8561244	8.18	0.9127533	9.18	0.9628427
1.019	0.00817418	1.19	0.0755470	2.19	0.3404441	3.19	0.5037907	4.19	0.6222140	5.19	0.7151674	6.19	0.7916906	7.19	0.8567289	8.19	0.9132839	9.19	0.9633155
1.020	0.00860017	1.20	0.0791812	2.20	0.3424227	3.20	0.5051500	4.20	0.6232493	5.20	0.7160033	6.20	0.7923917	7.20	0.8573325	8.20	0.9138139	9.20	0.9637878
1.021	0.00902574	1.21	0.0827854	2.21	0.3443923	3.21	0.5065050	4.21	0.6242821	5.21	0.7168377	6.21	0.7930916	7.21	0.8579353	8.21	0.9143432	9.21	0.9642596
1.022	0.00945090	1.22	0.0863598	2.22	0.3463530	3.22	0.5078559	4.22	0.6253125	5.22	0.7176705	6.22	0.7937904	7.22	0.8585372	8.22	0.9148718	9.22	0.9647309
1.023	0.00987563	1.23	0.0899051	2.23	0.3483049	3.23	0.5092025	4.23	0.6263404	5.23	0.7185017	6.23	0.7944880	7.23	0.8591383	8.23	0.9153998	9.23	0.9652017
1.024	0.01029996	1.24	0.0934217	2.24	0.3502480	3.24	0.5105450	4.24	0.6273659	5.24	0.7193313	6.24	0.7951846	7.24	0.8597386	8.24	0.9159272	9.24	0.9656720
1.025	0.01072387	1.25	0.0969100	2.25	0.3521825	3.25	0.5118834	4.25	0.6283889	5.25	0.7201593	6.25	0.7958800	7.25	0.8603380	8.25	0.9164539	9.25	0.9661417
1.026	0.01114736	1.26	0.1003705	2.26	0.3541084	3.26	0.5132176	4.26	0.6294096	5.26	0.7209857	6.26	0.7965743	7.26	0.8609366	8.26	0.9169800	9.26	0.9666110
1.027	0.01157044	1.27	0.1038037	2.27	0.3560259	3.27	0.5145478	4.27	0.6304279	5.27	0.7218106	6.27	0.7972675	7.27	0.8615344	8.27	0.9175055	9.27	0.9670797
1.028	0.01199311	1.28	0.1072100	2.28	0.3579348	3.28	0.5158738	4.28	0.6314438	5.28	0.7226339	6.28	0.7979596	7.28	0.8621314	8.28	0.9180303	9.28	0.9675480
1.029	0.01241537	1.29	0.1105897	2.29	0.3598355	3.29	0.5171959	4.29	0.6324573	5.29	0.7234557	6.29	0.7986506	7.29	0.8627275	8.29	0.9185545	9.29	0.9680157
1.030	0.01283722	1.30	0.1139434	2.30	0.3617278	3.30	0.5185139	4.30	0.6334685	5.30	0.7242759	6.30	0.7993405	7.30	0.8633229	8.30	0.9190781	9.30	0.9684829
1.031	0.01325867	1.31	0.1172713	2.31	0.3636120	3.31	0.5198280	4.31	0.6344773	5.31	0.7250945	6.31	0.8000294	7.31	0.8639174	8.31	0.9196010	9.31	0.9689497
1.032	0.01367970	1.32	0.1205739	2.32	0.3654880	3.32	0.5211381	4.32	0.6354837	5.32	0.7259116	6.32	0.8007171	7.32	0.8645111	8.32	0.9201233	9.32	0.9694159
1.033	0.01410032	1.33	0.1238516	2.33	0.3673559	3.33	0.5224442	4.33	0.6364879	5.33	0.7267272	6.33	0.8014037	7.33	0.8651040	8.33	0.9206450	9.33	0.9698816
1.034	0.01452054	1.34	0.1271048	2.34	0.3692159	3.34	0.5237465	4.34	0.6374897	5.34	0.7275413	6.34	0.8020893	7.34	0.8656961	8.34	0.9211661	9.34	0.9703469
1.035	0.01494035	1.35	0.1303338	2.35	0.3710679	3.35	0.5250448	4.35	0.6384893	5.35	0.7283538	6.35	0.8027737	7.35	0.8662873	8.35	0.9216865	9.35	0.9708116
1.036	0.01535976	1.36	0.1335389	2.36	0.3729120	3.36	0.5263393	4.36	0.6394865	5.36	0.7291648	6.36	0.8034571	7.36	0.8668778	8.36	0.9222063	9.36	0.9712758
1.037	0.01577876	1.37	0.1367206	2.37	0.3747483	3.37	0.5276299	4.37	0.6404814	5.37	0.7299743	6.37	0.8041394	7.37	0.8674675	8.37	0.9227255	9.37	0.9717396
1.038	0.01619735	1.38	0.1398791	2.38	0.3765770	3.38	0.5289167	4.38	0.6414741	5.38	0.7307823	6.38	0.8048207	7.38	0.8680564	8.38	0.9232440	9.38	0.9722028
1.039	0.01661555	1.39	0.1430148	2.39	0.3783979	3.39	0.5301997	4.39	0.6424645	5.39	0.7315888	6.39	0.8055009	7.39	0.8686444	8.39	0.9237620	9.39	0.9726656
1.040	0.01703334	1.40	0.1461280	2.40	0.3802112	3.40	0.5314789	4.40	0.6434527	5.40	0.7323938	6.40	0.8061800	7.40	0.8692317	8.40	0.9242793	9.40	0.9731279
1.041	0.01745073	1.41	0.1492191	2.41	0.3820170	3.41	0.5327544	4.41	0.6444386	5.41	0.7331973	6.41	0.8068580	7.41	0.8698182	8.41	0.9247960	9.41	0.9735896
1.042	0.01786772	1.42	0.1522883	2.42	0.3838154	3.42	0.5340261	4.42	0.6454223	5.42	0.7339993	6.42	0.8075350	7.42	0.8704039	8.42	0.9253121	9.42	0.9740509
1.043	0.01828431	1.43	0.1553360	2.43	0.3856063	3.43	0.5352941	4.43	0.6464037	5.43	0.7347998	6.43	0.8082110	7.43	0.8709888	8.43	0.9258276	9.43	0.9745117
1.044	0.01870050	1.44	0.1583625	2.44	0.3873898	3.44	0.5365584	4.44	0.6473830	5.44	0.7355989	6.44	0.8088859	7.44	0.8715729	8.44	0.9263424	9.44	0.9749720
1.045	0.01911629	1.45	0.1613680	2.45	0.3891661	3.45	0.5378191	4.45	0.6483600	5.45	0.7363965	6.45	0.8095597	7.45	0.8721563	8.45	0.9268567	9.45	0.9754318
1.046	0.01953168	1.46	0.1643529	2.46	0.3909351	3.46	0.5390761	4.46	0.6493349	5.46	0.7371926	6.46	0.8102325	7.46	0.8727388	8.46	0.9273704	9.46	0.9758911
1.047	0.01994668	1.47	0.1673173	2.47	0.3926970	3.47	0.5403295	4.47	0.6503075	5.47	0.7379873	6.47	0.8109043	7.47	0.8733206	8.47	0.9278834	9.47	0.9763500
1.048	0.02036128	1.48	0.1702617	2.48	0.3944517	3.48	0.5415792	4.48	0.6512780	5.48	0.7387806	6.48	0.8115750	7.48	0.8739016	8.48	0.9283959	9.48	0.9768083
1.049	0.02077549	1.49	0.1731863	2.49	0.3961993	3.49	0.5428254	4.49	0.6522463	5.49	0.7395723	6.49	0.8122447	7.49	0.8744818	8.49	0.9289077	9.49	0.9772662
1.050	0.02118930	1.50	0.1760913	2.50	0.3979400	3.50	0.5440680	4.50	0.6532125	5.50	0.7403627	6.50	0.8129134	7.50	0.8750613	8.50	0.9294189	9.50	0.9777236
1.051	0.02160272	1.51	0.1789769	2.51	0.3996737	3.51	0.5453071	4.51	0.6541765	5.51	0.7411516	6.51	0.8135810	7.51	0.8756399	8.51	0.9299296	9.51	0.9781805
1.052	0.02201574	1.52	0.1818436	2.52	0.4014005	3.52	0.5465427	4.52	0.6551384	5.52	0.7419391	6.52	0.8142476	7.52	0.8762178	8.52	0.9304396	9.52	0.9786369
1.053	0.02242837	1.53	0.1846914	2.53	0.4031205	3.53	0.5477747	4.53	0.6560982	5.53	0.7427251	6.53	0.8149132	7.53	0.8767950	8.53	0.9309490	9.53	0.9790929
1.054	0.02284061	1.54	0.1875207	2.54	0.4048337	3.54	0.5490033	4.54	0.6570559	5.54	0.7435098	6.54	0.8155777	7.54	0.8773713	8.54	0.9314579	9.54	0.9795484
1.055	0.02325246	1.55	0.1903317	2.55	0.4065402	3.55	0.5502284	4.55	0.6580114	5.55	0.7442930	6.55	0.8162413	7.55	0.8779470	8.55	0.9319661	9.55	0.9800034
1.056	0.02366392	1.56	0.1931246	2.56	0.4082400	3.56	0.5514500	4.56	0.6589648	5.56	0.7450748	6.56	0.8169038	7.56	0.8785218	8.56	0.9324738	9.56	0.9804579
1.057	0.02407499	1.57	0.1958997	2.57	0.4099331	3.57	0.5526682	4.57	0.6599162	5.57	0.7458552	6.57	0.8175654	7.57	0.8790959	8.57	0.9329808	9.57	0.9809119
1.058	0.02448567	1.58	0.1986571	2.58	0.4116197	3.58	0.5538830	4.58	0.6608655	5.58	0.7466342	6.58	0.8182259	7.58	0.8796692	8.58	0.9334873	9.58	0.9813655
1.059	0.02489596	1.59	0.2013971	2.59	0.4132998	3.59	0.5550944	4.59	0.6618127	5.59	0.7474118	6.59	0.8188854	7.59	0.8802418	8.59	0.9339932	9.59	0.9818186
1.060	0.02530587	1.60	0.2041200	2.60	0.4149733	3.60	0.5563025	4.60	0.6627578	5.60	0.7481880	6.60	0.8195439	7.60	0.8808136	8.60	0.9344985	9.60	0.9822712
1.061	0.02571538	1.61	0.2068259	2.61	0.4166405	3.61	0.5575072	4.61	0.6637009	5.61	0.7489629	6.61	0.8202015	7.61	0.8813847	8.61	0.9350032	9.61	0.9827234
1.062	0.02612452	1.62	0.2095150	2.62	0.4183013	3.62	0.5587086	4.62	0.6646420	5.62	0.7497363	6.62	0.8208580	7.62	0.8819550	8.62	0.9355073	9.62	0.9831751
1.063	0.02653326	1.63	0.2121876	2.63	0.4199557	3.63	0.5599066	4.63	0.6655810	5.63	0.7505084	6.63	0.8215135	7.63	0.8825245	8.63	0.9360108	9.63	0.9836263
1.064	0.02694163	1.64	0.2148438	2.64	0.4216039	3.64	0.5611014	4.64	0.6665180	5.64	0.7512791	6.64	0.8221681	7.64	0.8830934	8.64	0.9365137	9.64	0.9840770
1.065	0.02734961	1.65	0.2174839	2.65	0.4232459	3.65	0.5622929	4.65	0.6674530	5.65	0.7520484	6.65	0.8228216	7.65	0.8836614	8.65	0.9370161	9.65	0.9845273
1.066	0.02775720	1.66	0.2201081	2.66	0.4248816	3.66	0.5634811	4.66	0.6683859	5.66	0.7528164	6.66	0.8234742	7.66	0.8842288	8.66	0.9375179	9.66	0.9849771
1.067	0.02816442	1.67	0.2227165	2.67	0.4265113	3.67	0.5646661	4.67	0.6693169	5.67	0.7535831	6.67	0.8241258	7.67	0.8847954	8.67	0.9380191	9.67	0.9854265
1.068	0.02857125	1.68	0.2253093	2.68	0.4281348	3.68	0.5658478	4.68	0.6702459	5.68	0.7543483	6.68	0.8247765	7.68	0.8853612	8.68	0.9385197	9.68	0.9858754
1.069	0.02897771	1.69	0.2278867	2.69	0.4297523	3.69	0.5670264	4.69	0.6711728	5.69	0.7551123	6.69	0.8254261	7.69	0.8859263	8.69	0.9390198	9.69	0.9863238
1.070	0.02938378	1.70	0.2304489	2.70	0.4313638	3.70	0.5682017	4.70	0.6720979	5.70	0.7558749	6.70	0.8260748	7.70	0.8864907	8.70	0.9395193	9.70	0.9867717
1.071	0.02978947	1.71	0.2329961	2.71	0.4329693	3.71	0.5693739	4.71	0.6730209	5.71	0.7566361	6.71	0.8267225	7.71	0.8870544	8.71	0.9400182	9.71	0.9872192
1.072	0.03019479	1.72	0.2355284	2.72	0.4345689	3.72	0.5705429	4.72	0.6739420	5.72	0.7573960	6.72	0.8273693	7.72	0.8876173	8.72	0.9405165	9.72	0.9876663
1.073	0.03059972	1.73	0.2380461	2.73	0.4361626	3.73	0.5717088	4.73	0.6748611	5.73	0.7581546	6.73	0.8280151	7.73	0.8881795	8.73	0.9410142	9.73	0.9881128
1.074	0.03100428	1.74	0.2405492	2.74	0.4377506	3.74	0.5728716	4.74	0.6757783	5.74	0.7589119	6.74	0.8286599	7.74	0.8887410	8.74	0.9415114	9.74	0.9885590
1.075	0.03140846	1.75	0.2430380	2.75	0.4393327	3.75	0.5740313	4.75	0.6766936	5.75	0.7596678	6.75	0.8293038	7.75	0.8893017	8.75	0.9420081	9.75	0.9890046
1.076	0.03181227	1.76	0.2455127	2.76	0.4409091	3.76	0.5751878	4.76	0.6776070	5.76	0.7604225	6.76	0.8299467	7.76	0.8898617	8.76	0.9425041	9.76	0.9894498
1.077	0.03221570	1.77	0.2479733	2.77	0.4424798	3.77	0.5763414	4.77	0.6785184	5.77	0.7611758	6.77	0.8305887	7.77	0.8904210	8.77	0.9429996	9.77	0.9898946
1.078	0.03261876	1.78	0.2504200	2.78	0.4440448	3.78	0.5774918	4.78	0.6794279	5.78	0.7619278	6.78	0.8312297	7.78	0.8909796	8.78	0.9434945	9.78	0.9903389
1.079	0.03302144	1.79	0.2528530	2.79	0.4456042	3.79	0.5786392	4.79	0.6803355	5.79	0.7626786	6.79	0.8318698	7.79	0.8915375	8.79	0.9439889	9.79	0.9907827
1.080	0.03342376	1.80	0.2552725	2.80	0.4471580	3.80	0.5797836	4.80	0.6812412	5.80	0.7634280	6.80	0.8325089	7.80	0.8920946	8.80	0.9444827	9.80	0.9912261
1.081	0.03382569	1.81	0.2576786	2.81	0.4487063	3.81	0.5809250	4.81	0.6821451	5.81	0.7641761	6.81	0.8331471	7.81	0.8926510	8.81	0.9449759	9.81	0.9916690
1.082	0.03422726	1.82	0.2600714	2.82	0.4502491	3.82	0.5820634	4.82	0.6830470	5.82	0.7649230	6.82	0.8337844	7.82	0.8932068	8.82	0.9454686	9.82	0.9921115
1.083	0.03462846	1.83	0.2624511	2.83	0.4517864	3.83	0.5831988	4.83	0.6839471	5.83	0.7656686	6.83	0.8344207	7.83	0.8937618	8.83	0.9459607	9.83	0.9925535
1.084	0.03502928	1.84	0.2648178	2.84	0.4533183	3.84	0.5843312	4.84	0.6848454	5.84	0.7664128	6.84	0.8350561	7.84	0.8943161	8.84	0.9464523	9.84	0.9929951
1.085	0.03542974	1.85	0.2671717	2.85	0.4548449	3.85	0.5854607	4.85	0.6857417	5.85	0.7671559	6.85	0.8356906	7.85	0.8948697	8.85	0.9469433	9.85	0.9934362
1.086	0.03582983	1.86	0.2695129	2.86	0.4563660	3.86	0.5865873	4.86	0.6866363	5.86	0.7678976	6.86	0.8363241	7.86	0.8954225	8.86	0.9474337	9.86	0.9938769
1.087	0.03622954	1.87	0.2718416	2.87	0.4578819	3.87	0.5877110	4.87	0.6875290	5.87	0.7686381	6.87	0.8369567	7.87	0.8959747	8.87	0.9479236	9.87	0.9943172
1.088	0.03662890	1.88	0.2741578	2.88	0.4593925	3.88	0.5888317	4.88	0.6884198	5.88	0.7693773	6.88	0.8375884	7.88	0.8965262	8.88	0.9484130	9.88	0.9947569
1.089	0.03702788	1.89	0.2764618	2.89	0.4608978	3.89	0.5899496	4.89	0.6893089	5.89	0.7701153	6.89	0.8382192	7.89	0.8970770	8.89	0.9489018	9.89	0.9951963
1.090	0.03742650	1.90	0.2787536	2.90	0.4623980	3.90	0.5910646	4.90	0.6901961	5.90	0.7708520	6.90	0.8388491	7.90	0.8976271	8.90	0.9493900	9.90	0.9956352
1.091	0.03782475	1.91	0.2810334	2.91	0.4638930	3.91	0.5921768	4.91	0.6910815	5.91	0.7715875	6.91	0.8394780	7.91	0.8981765	8.91	0.9498777	9.91	0.9960737
1.092	0.03822264	1.92	0.2833012	2.92	0.4653829	3.92	0.5932861	4.92	0.6919651	5.92	0.7723217	6.92	0.8401061	7.92	0.8987252	8.92	0.9503649	9.92	0.9965117
1.093	0.03862016	1.93	0.2855573	2.93	0.4668676	3.93	0.5943926	4.93	0.6928469	5.93	0.7730547	6.93	0.8407332	7.93	0.8992732	8.93	0.9508515	9.93	0.9969492
1.094	0.03901732	1.94	0.2878017	2.94	0.4683473	3.94	0.5954962	4.94	0.6937269	5.94	0.7737864	6.94	0.8413595	7.94	0.8998205	8.94	0.9513375	9.94	0.9973864
1.095	0.03941412	1.95	0.2900346	2.95	0.4698220	3.95	0.5965971	4.95	0.6946052	5.95	0.7745170	6.95	0.8419848	7.95	0.9003671	8.95	0.9518230	9.95	0.9978231
1.096	0.03981055	1.96	0.2922561	2.96	0.4712917	3.96	0.5976952	4.96	0.6954817	5.96	0.7752463	6.96	0.8426092	7.96	0.9009131	8.96	0.9523080	9.96	0.9982593
1.097	0.04020663	1.97	0.2944662	2.97	0.4727564	3.97	0.5987905	4.97	0.6963564	5.97	0.7759743	6.97	0.8432328	7.97	0.9014583	8.97	0.9527924	9.97	0.9986952
1.098	0.04060234	1.98	0.2966652	2.98	0.4742163	3.98	0.5998831	4.98	0.6972293	5.98	0.7767012	6.98	0.8438554	7.98	0.9020029	8.98	0.9532763	9.98	0.9991305
1.099	0.04099769	1.99	0.2988531	2.99	0.4756712	3.99	0.6009729	4.99	0.6981005	5.99	0.7774268	6.99	0.8444772	7.99	0.9025468	8.99	0.9537597	9.99	0.9995655

Name:

Sheet 521X: USING A TABLE OF LOGARITHMS.

1. Add the entries for the two numbers. Find that entry in the table. Use the first 4 decimals only. No calculator.

a, 2.00 and 3.00.

b, 2.00 and 4.00

What is going on?

c, 1.34 and 2.38. Check with the calculator.

2. Multiply the entry and find the new entry in the table. Calculator OK.

a, 2.00 by 3.

What is going on?

b, 1.58 by 4.

3. a, What would be the entry for 10.0?

b, How do you construct an entry for 20.0? What is it?

c, What about 200?

4. Using the table, adding the powers of ten, find

a, $20 \cdot 30$

b, 15.8^4

5. Estimate, to 4 decimals, the entry for 5.384. to 6 decimals?

6. a, Adding logarithms can be used for ____________________.

b, Multiplying logarithms by a number can be used for ____________________.

Name: $\boxed{\text{KEY}}$

Sheet 521X: USING A TABLE OF LOGARITHMS

1. Add the entries for the two numbers. Find that entry in the table. Use the first 4 decimals only. No calculator.

a, 2.00 and 3.00.

$$\begin{array}{r} 0.3010 \\ +\,0.4771 \\ \hline 0.7781 \end{array}$$ is entry for $\boxed{6}$

b, 2.00 and 4.00

$$\begin{array}{r} 0.3010 \\ +\,0.6021 \\ \hline 0.9031 \end{array}$$ is entry for $\boxed{8}$

What is going on? If you <u>add</u> entries you <u>multiply</u> original numbers

c, 1.34 and 2.38. Check with the calculator.

$$\begin{array}{r} 0.1271 \\ 0.3766 \\ \hline 0.5037 \end{array}$$ 0.50379 is entry for $\boxed{3.19}$

$1.34 \cdot 2.38 = 3.1892$ ✓

2. Multiply the entry and find the new entry in the table. Calculator OK.

a, 2.00 by 3. $0.3010 \cdot 3 =$ ~~0.7781~~ 0.9030 is entry for $\boxed{8}$

What is going on? $2^3 = 8$. <u>MULTIPLYING</u> BY A NUMBER → RAISING TO A <u>POWER</u>

b, 1.58 by 4. $0.1986571 \cdot 4 = 0.7946284$

0.7944880 is entry for $\boxed{6.23}$

$1.58^4 \approx 6.2320$ ✓

3. a, What would be the entry for 10.0? $\boxed{1}$

b, How do you construct an entry for 20.0? What is it?

The entry for 10·2 would be $1 + 0.3010 = \boxed{1.3010}$

c, What about 200? $\boxed{2.3010}$

4. Using the table, adding the powers of ten, find

a, 20 · 30 $1.3010 + 1.4771 = 2.7781 \rightarrow 6 \cdot 10^2 = \boxed{600}$

b, 15.8^4 $1.198657 \cdot 4 = 4.7946$

$\rightarrow 10000 \cdot (6.23) \approx \boxed{62{,}300}$

QUESTIONS. 0.7315888 — 0.7307823

$0.7307823 + 0.4(0.0008065) = 0.7311049$

5. Estimate, to 4 decimals, the entry for 5.384. $\boxed{0.7311}$ to 6 decimals? $\boxed{0.731105}$

IT'S 0.4 OF DISTANCE FROM 5.38 to 5.39.

6. a, Adding logarithms can be used for $\boxed{\text{MULTIPLICATION}}$

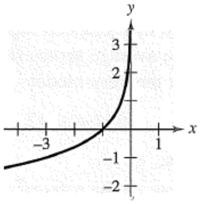
b, Multiplying logarithms by a number can be used for $\boxed{\text{TAKING POWERS}}$

Sheet 522: Logarithmic function applications

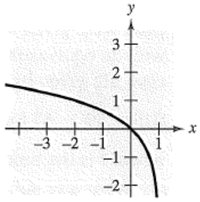
PC 5th Ch. 3-2, p. 307–308

In Exercises 45–50, use the graph of $y = \log_3 x$ to match the given function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

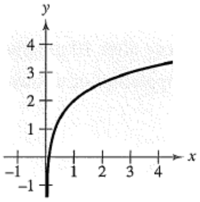
(a)



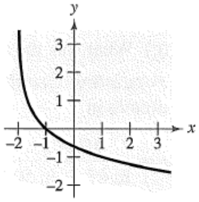
(b)



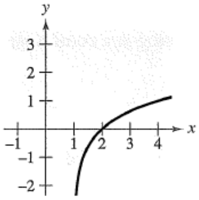
(c)



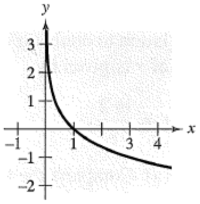
(d)



(e)



(f)



First try to find the formula without the choices below.
Write down asymptotes and points on the curve.

45. $f(x) = \log_3 x + 2$ **46.** $f(x) = -\log_3 x$

47. $f(x) = -\log_3(x + 2)$ **48.** $f(x) = \log_3(x - 1)$

49. $f(x) = \log_3(1 - x)$ **50.** $f(x) = -\log_3(-x)$

75. *Work* The work (in foot-pounds) done in compressing a volume of 9 cubic feet at a pressure of 15 pounds per square inch to a volume of 3 cubic feet is

$$W = 19{,}440(\ln 9 - \ln 3).$$

Find W.

76. *Sound Intensity* The relationship between the number of decibels β and the intensity of a sound I in watts per square meter is

$$\beta = 10 \log_{10}\left(\frac{I}{10^{-12}}\right).$$

(a) Determine the number of decibels of a sound with an intensity of 1 watt per square meter.

(b) Determine the number of decibels of a sound with an intensity of 10^{-2} watt per square meter.

(c) The intensity of the sound in part (a) is 100 times as great as that in part (b). Is the number of decibels 100 times as great? Explain.

3-2 answers
45c, 46f, 47d, 48e, 49b, 50a.
75. 21,357 foot-pounds
76. 120dB, 100dB, No.

Sheet #523: Logarithmic Functions

FMC 2nd Ch 4.3 p.165-166

1. Without a calculator, match the functions $y = 10^x$, $y = e^x$, $y = \log x$, $y = \ln x$ with the graphs in Figure 4.9.

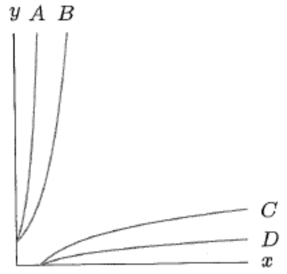


Figure 4.9

2. Without a calculator, match the functions $y = 2^x$, $y = e^{-x}$, $y = 3^x$, $y = \ln x$, $y = \log x$ with the graphs in Figure 4.10.

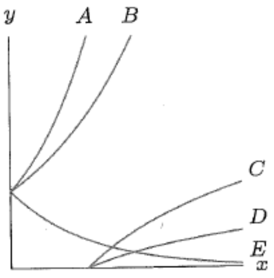


Figure 4.10

18. Match the graphs (a)–(c) to one of the functions $r(x)$, $s(x)$, $t(x)$ whose values are in the tables.
Find formulas for r(x), s(x) and t(x).

x	2	4	10
$r(x)$	1	1.301	1.699

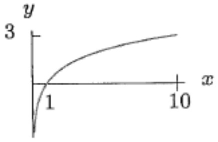
x	0.5	5	10
$s(x)$	~~-0.060~~	~~0.379~~	0.699

x	0.1	2	100
$t(x)$	-3	0.903	6

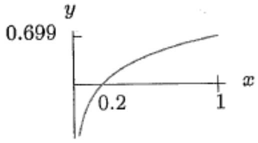
$\boxed{0.398}$

$\boxed{-0.602}$

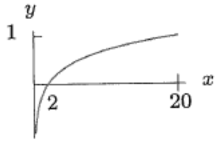
(a)



(b)

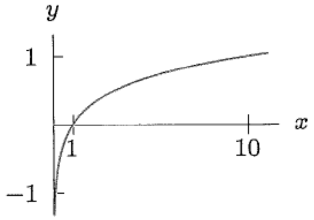


(c)



In Problems 20–25, find possible formulas for the functions using logs or exponentials.

20.



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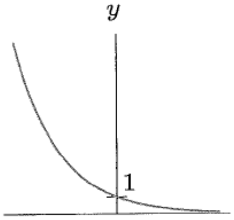

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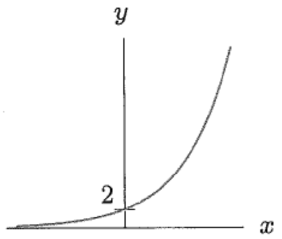

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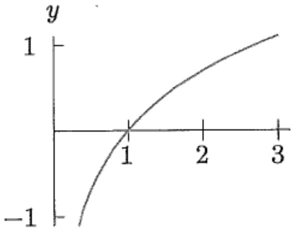

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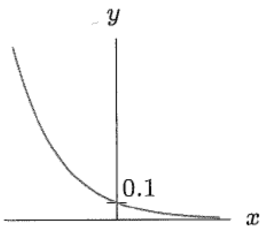

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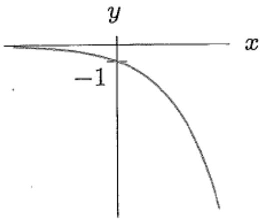


35. The magnitude of an earthquake is measured relative to the strength of a "standard" earthquake, whose seismic waves are of size W_0. The magnitude, M, of an earthquake with seismic waves of size W is defined to be

$$M = \log\left(\frac{W}{W_0}\right).$$

The value of M is called the *Richter scale* rating of the strength of an earthquake.

(a) Let M_1 and M_2 represent the magnitude of two earthquakes whose seismic waves are of sizes W_1 and W_2, respectively. Using log properties, find a simplified formula for the difference $M_2 - M_1$ in terms of W_1 and W_2.

(b) The 1989 earthquake in California had a rating of 7.1 on the Richter scale. How many times larger were the seismic waves in the 1906 earthquake in San Francisco which measured 8.4 on the Richter scale? Give your answer to the nearest integer.

Sheet 523 Answers.

1. A is $y = 10^x$, B is $y = e^x$, C is $y = \ln x$, D is $y = \log x$.

2. A is $y = 3^x$, B is $y = 2^x$, C is $y = \ln x$, D is $y = \log x$, E is $y = e^{-x}$.

20. A possible formula is $y = \log x$.

21. This graph could represent exponential decay, so a possible formula is $y = b^x$ with $0 < b < 1$.

22. This graph could represent exponential growth, with a y-intercept of 2. A possible formula is $y = 2b^x$ with $b > 1$.

23. A possible formula is $y = \ln x$.

24. This graph could represent exponential decay, with a y-intercept of 0.1. A possible formula is $y = 0.1b^x$ with $0 < b < 1$.

25. This graph could represent exponential "growth", with a y-intercept of -1. A possible formula is $y = (-1)b^x = -b^x$ for $b > 1$.

35. **(a)** We know $M_1 = \log\left(\frac{W_1}{W_0}\right)$ and $M_2 = \log\left(\frac{W_2}{W_0}\right)$. Thus,

$$\begin{aligned} M_2 - M_1 &= \log\left(\frac{W_2}{W_0}\right) - \log\left(\frac{W_1}{W_0}\right) \\ &= \log\left(\frac{W_2}{W_1}\right). \end{aligned}$$

(b) Let $M_2 = 8.4$ and $M_1 = 7.1$, so

$$M_2 - M_1 = \log\left(\frac{W_2}{W_1}\right)$$

becomes

$$\begin{aligned} 8.4 - 7.1 &= \log\left(\frac{W_2}{W_1}\right) \\ 1.3 &= \log\left(\frac{W_2}{W_1}\right) \end{aligned}$$

so

$$\frac{W_2}{W_1} = 10^{1.3} \approx 20.$$

Thus, the seismic waves of the 1906 earthquake were 20 times as large as those of the 1989 earthquake.

Sheet #524: Logarithmic Functions
FMC 2nd Ch 4.4 p.175

20. To study how recognition memory decreases with time, the following experiment was conducted. The subject read a list of 20 words slowly aloud, and later, at different time intervals, was shown a list of 40 words containing the 20 words that he or she had read. The percentage, P, of words recognized was recorded as a function of t, the time elapsed in minutes. Table 4.17 shows the averages for 5 different subjects.[10] This is modeled by $P = a \ln t + b$.

(a) Find $\ln t$ for each value of t, and then use regression on a calculator or computer to estimate a and b.

(b) Graph the data points and regression line on a coordinate system of P against $\ln t$.

(c) When does this model predict that the subjects will recognize no words? All words?

(d) Graph the data points and curve $P = a \ln t + b$ on a coordinate system with P against t, with $0 \leq t \leq 10{,}500$.

Table 4.17 *Percentage of words recognized*

t, min	5	15	30	60	120	240	480	720	1440	2880	5760	10,080
P%	73.0	61.7	58.3	55.7	50.3	46.7	40.3	38.3	29.0	24.0	18.7	10.3

$P = -7.787 \ln t + 86.283$

NAME: ____________________ Period: ______

Sheet #530. EVALUATING LOGARITHMIC EXPRESSIONS USING PROPERTIES

ANS. ↓

1. $\log_2(4 \cdot 16) =$
2. $\ln e^{-2} =$
3. $\log_2 4^3 =$
4. $\log_5 125 =$
5. $\log_3 9^4 =$
6. $\log \frac{1}{10} =$
7. $\ln \frac{1}{e^3} =$
8. $\log (0.01)^3 =$

ANSWERS.

1	2	3	4	5	6	7	8
6,	-2,	6,	3,	8,	-1,	-3,	-6

Sheet 531: Properties of Logarithms

PC 5th Ch. 3-3, p. 316

85. *Human Memory Model* Students participating in a psychological experiment attended several lectures and were given an exam. Every month for a year after the exam, the students were retested to see how much of the material they remembered. The average scores for the group can be modeled by the memory model

$$f(t) = 90 - 15\log_{10}(t+1), \qquad 0 \le t \le 12$$

where t is the time in months.

(a) What was the average score on the original exam ($t = 0$)?

(b) What was the average score after 6 months?

(c) What was the average score after 12 months?

(d) When will the average score decrease to 75?

(e) Use the properties of logarithms to write the function in another form.

(f) Sketch the graph of the function over the specified domain.

86. *Sound Intensity* The relationship between the number of decibels β and the intensity of a sound I in watts per square meter is

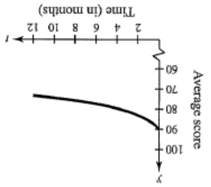
$$\beta = 10\log_{10}\left(\frac{I}{10^{-12}}\right).$$

Use the properties of logarithms to write the formula in simpler form, and determine the number of decibels of a sound with an intensity of 10^{-6} watt per square meter.

85. (a) 90 (b) 77 (c) 73 (d) 9 months

(e) $90 - \log_{10}(t+1)^{15}$

(f)



β=10[logI+12], 60dB

Name:____

Sheet # 532: Logarithms and Graphs

1. Rewrite the equation in exponential form:
$\log_2 64 = 6$.

2. Evaluate the expressions:
(a) $\log(10{,}000)$

(b) $\log_3(27)$

(c) $\ln e^5$

3. Simplify the expressions:
(a) $9^{\log_9 x}$

(b) $e^{\ln(\pi x)}$

4. Using calculator, evaluate to **three decimals,** rounding correctly:
(a) log(2)

(b) log(11)

(c) log(2) + log(11)

(d) log(22)

What is going on here?

5. Expand the expressions *completely*:
(a) $\log_4\left(\frac{2x}{z}\right)$

(b) $\ln(17x^5y^2)$

(c) $\ln\left(\sqrt{e^4}\,x\right)$

6. Condense the expressions into *one* logarithm:

(a) $\log(x^5) + \log(y)$

(b) $\ln(5) - 7\ln(x) + 2\ln(y)$

(c) $\log(x^3) + \log(x^2) + \log(x)$

7. Consider $f(x) = \log_8(x+3)$.
(a) Find $f(61)$. That is, find y when $x = 61$.

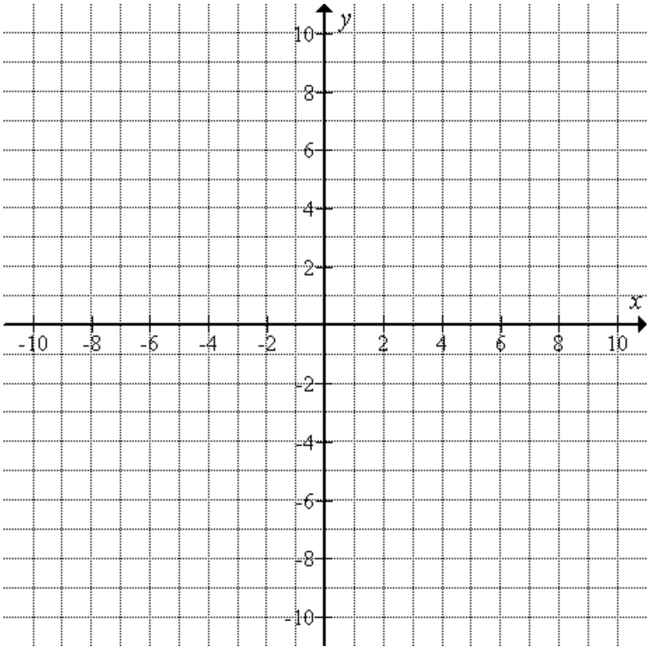
(b) Write the equation of the asymptote.

8. (a) Complete the table.

x	$y = \log_4 x$
1/16	
1/ 4	
1	
4	
16	

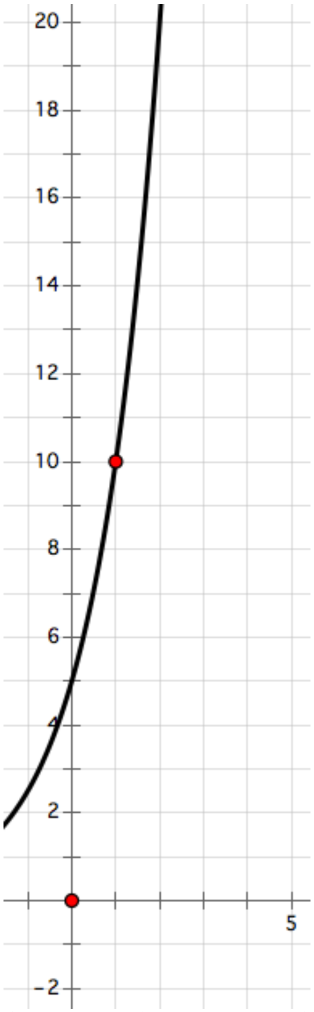
(b) Graph the function y in part (a) using accurate points to show the full curve.

(c) Graph the asymptote and label it "asymptote."

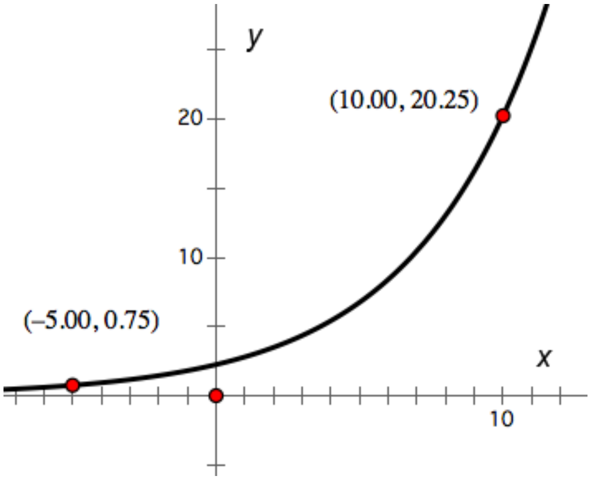


9. Write possible formulas for the graphs. Use exponential functions in the form $y = A \cdot b^x$.

(a)



(b) Answer in any way. Then answer without decimals and without radicals (nth roots), if you have not already done so.



Name: KEY

Sheet # 532: Logarithms and Graphs

1. Rewrite the equation in exponential form: $\log_2 64 = 6$.

$$\boxed{2^6 = 64}$$

2. Evaluate the expressions:

(a) $\log(10{,}000) = \boxed{4}$ [because $10^4 = 10{,}000$]

(b) $\log_3(27) = \boxed{3}$ [because $3^3 = 27$]

(c) $\ln e^5 = \boxed{5}$

3. Simplify the expressions:

(a) $9^{\log_9 x} = \boxed{x}$

(b) $e^{\ln(\pi x)} = \boxed{\pi x}$

4. Using calculator, evaluate to **three decimals,** rounding correctly:

(a) $\log(2) = \boxed{0.301}$

(b) $\log(11) = \boxed{1.04}$

(c) $\log(2) + \log(11) = \boxed{1.342}$

(d) $\log(22) = \boxed{1.342}$

What is going on here?

PROPERTY OF LOGARITHM:

$$\boxed{\log(uv) = \log(u) + \log(v)}$$

5. Expand the expressions *completely*:

(a) $\log_4\left(\dfrac{2x}{z}\right)$

$$\boxed{\log_4(2) + \log_4(x) - \log_4(z)}$$

[where $\log_4(2) = 1/2$ because $4^{1/2} = 2$]

(b) $\ln(17x^5y^2)$

$$\boxed{\ln(17) + 5\ln x + 2\ln y}$$

(c) $\ln\left(\sqrt{e^4}x\right) =$

$\ln\left(\sqrt{e^4}\right) + \ln(x)$

$= \ln(e^2) + \ln(x) = \boxed{2 + \ln x}$

6. Condense the expressions into *one* logarithm:

(a) $\log(x^5) + \log(y)$

$$\boxed{\log(x^5 \cdot y)}$$

(b) $\ln(5) - 7\ln(x) + 2\ln(y)$ $\to \ln(x^7)$ $\to \ln(y^2)$

$$\boxed{\ln\left(\frac{5y^2}{x^7}\right)}$$

(c) $\log(x^3) + \log(x^2) + \log(x)$

$\log(x^3 \cdot x^2 \cdot x) = \boxed{\log(x^6)}$

7. Consider $f(x) = \log_8(x+3)$.

(a) Find $f(61)$. That is, find y when $x = 61$.

$y = f(61) = \log_8(61+3) =$

$= \log_8(64) = \boxed{2}$ because $8^2 = 64$.

(b) Write the equation of the asymptote.

$\boxed{x = -3}$ because the asy. of $y = \log_b(x - H)$ is $x = H$. In this case $H = -3$.

[Note: ↙ -3 is not a correct answer]

8. (a) Complete the table.

x	$y = \log_4 x$
1/16	−2
1/4	−1
1	0
4	1
16	2

because...

$4^{-2} = 1/4^2 = 1/16$

$4^{-1} = 1/4$

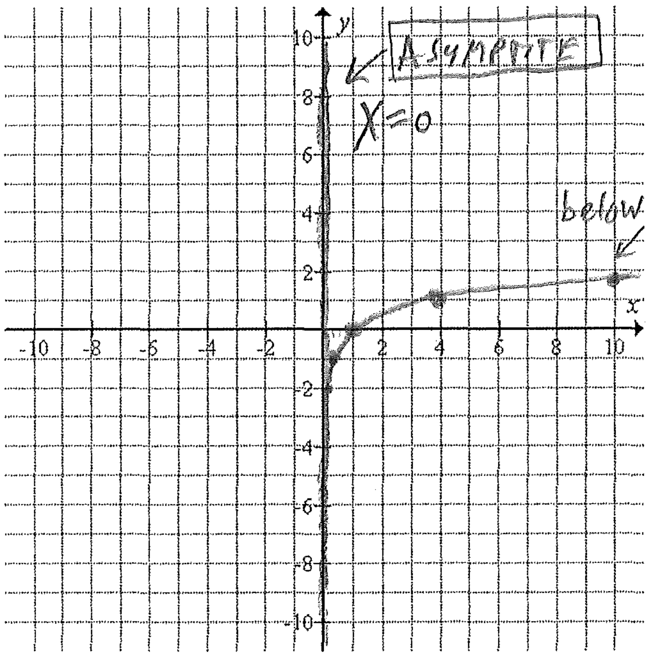
$4^0 = 1$

$4^1 = 4$

$4^2 = 16$

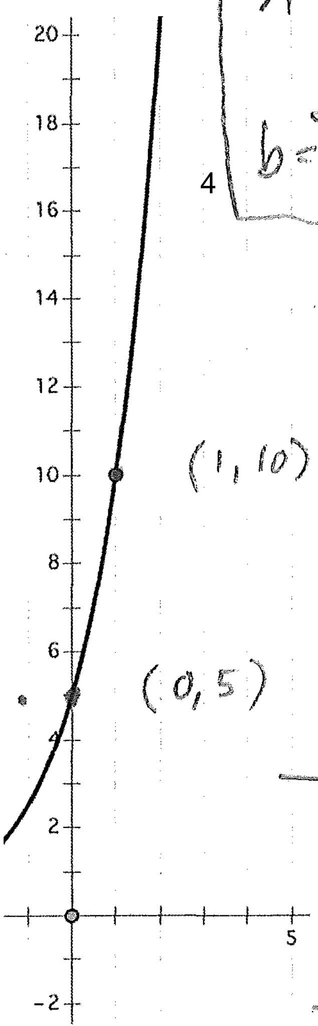
√ (b) Graph the function y in part (a) using accurate points to show the full curve.

√ (c) Graph the asymptote and label it "asymptote."



9. Write possible formulas for the graphs. Use exponential functions in the form $y = A \cdot b^x$.

(a)



A = vertical ✓
Stetch =
y intercept.
b = base.

y INTERCEPT
= A = 5

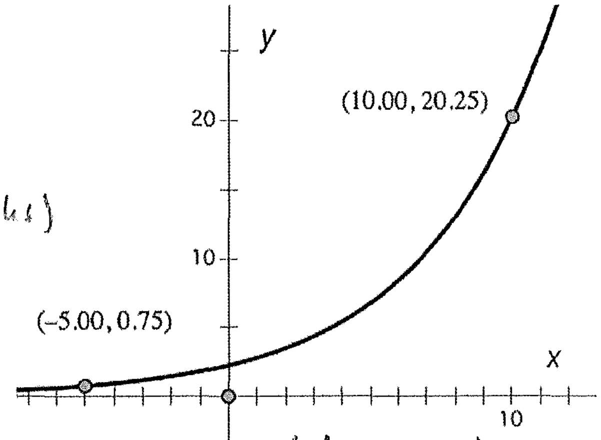
(1, 10) $10 = A b^1$

(0, 5) $5 = A b^0$

$2 = b$

$\boxed{y = 5 \cdot 2^x}$

(b) Answer in any way. Then answer without decimals and without radicals (nth roots), if you have not already done so.



9b, TWO EQUATIONS
TWO UNKNOWNS. HINTS:

① START WITH LARGEST X VALUE.

② DIVIDE EQUATIONS, RATHER THAN USE SUBSTITUTION (Which also Works)

$y = Ab^x$

$20.25 = A b^{10}$

$0.75 = A b^{-5}$

DIVIDE

$27 = 1 \cdot b^{10-(-5)}$

$27 = b^{15}$

$\sqrt[15]{27} = b$

$b = (27)^{1/15} = (3^3)^{1/15} = 3^{(3/15)} = 3^{1/5}$

Use (10, 20.25) TO FIND A.

$20.25 = A(3^{1/5})^{10}$ $20.25 = A\,3^2$

$A = 20.25/9 = 2.25$

$\boxed{y = 2.25(3^{1/5})^x}$

$\boxed{y = 2.25\,3^{x/5}}$

Sheet# 540

NAME: ______________

EXP-LOG QUESTIONS

Period: ______

NO CALCULATOR ALLOWED. ALL ANSWERS IN TERMS OF LOG(X), LN(X) OR, preferred: JUST DECIMALS; INTEGERS.

1. FIND X

$$5e^{2x+1} = 3$$

a, EXACTLY (NO TI-83)

b, if know $\ln(3) = 1.099$. $\ln(5) = 1.609$ (NO TI-83, 3 DECIMALS)

2. $\ln(1/2) = c \ln(2)$. FIND c.

3. $\ln(\sqrt{x^7}) = a \ln(x)$. FIND. a

4. $\log_3(7) = g \cdot \ln(7)$ FIND g.

$\boxed{\text{KEY}}$ Sheet #540

EXP-LOG QUESTIONS

NAME: $\boxed{\text{KEY}}$

1. FIND X

$$5e^{2x+1} = 3$$

a, EXACTLY (NO TI-83)

$$\ln(e^{2x+1}) = \ln(3/5)$$

$$2x+1 = \ln(3/5)$$

$$\frac{2x}{2} = \frac{\ln(3/5)-1}{2}$$

$$\boxed{x = \frac{\ln(3/5)-1}{2}}$$

b, if know $\ln(3) = 1.099$. $\ln(5) = 1.609$ (NO TI-83, 3 DECIMALS)

$$x = \frac{\ln(3) - \ln(5) - 1}{2} = \frac{0.099 - 1.609 - 1}{2}$$

$$= \frac{-1.51}{2} = \boxed{-0.755}$$

2. $\ln(1/2) = c \ln(2)$. FIND c.

$$\ln(1/2) = -\ln(2) \qquad \boxed{c = -1}$$

2. $\frac{1}{2} = 2^c \qquad c = -1$

3. $\ln(\sqrt{x}) = a \ln(x)$. FIND. a

$$\ln(x^{1/2}) = \frac{1}{2}\ln(x) \qquad \boxed{a = \frac{1}{2}}$$

3. ALTERNATIVE 1:

$$e^{\ln(\sqrt{x})} = e^{a\ln(x)}$$

$$\sqrt{x} = e^{\ln(x^a)}$$

$$\sqrt{x} = x^a$$

$$a = 1/2.$$

ALTERNATIVE 2.

$$u = \sqrt{x} \quad u^2 = x$$

$$\ln(u) = a \ln(u^2)$$

$$\ln(u) = a \cdot 2 \cdot \ln(u)$$

$$1 = 2a$$

$$a = 1/2.$$

4. $\log_3(7) = g \cdot \ln(7)$ FIND g.

$$\ln(7)/\ln(3) = g\ln(7)$$

$$\boxed{g = 1/\ln(3)}$$

NAME: ______________________

Sheet #S41:

Period: ____

SOLVING EXPONENTIAL & LOGARITHMIC EQUATIONS

1. $e^{2x} = e^{x+3}$

2. $\log(2x+1) = \log(15)$

3A) $10^{x} = 14$

3B) $2e^{3x} + 1 = 5$

4. $\log(2x) = 3$

5. $4\log(x+2) = 8$

6. $e^{2x} + 3 = 10$

7A) $3\log(x+1) = 9$

7B) $\log_3(2x) = 4$

8. $\frac{1}{2} = e^{0.01t}$

9. CONSIDER $y = 3e^{kt}$

a) FIND K if $(10, 2)$ is on the graph.

b) Find y if $t = 15$.

c) FOR WHAT VALUE OF t IS $y = \frac{3}{2}$?
(half of the y value WHEN $t=0$)

NAME: SOLUTIONS **KEY**

Period: ____

Sheet #541:

SOLVING EXPONENTIAL & LOGARITHMIC EQUATIONS

1. $e^{2X} = e^{X+3}$

$2X = X + 3$

$\boxed{X = 3}$

2. $\log(2X+1) = \log(15)$

$2X + 1 = 15$

$2X = 14$

$\boxed{X = 7}$

3A) $10^{X} = 14$

$\boxed{X = \log(14)}$

$X \approx 1.146$

3B) $2e^{3X} + 1 = 5$

$2 \cdot e^{3X} = 4$

$e^{3X} = 2$

$3X = \ln(2)$

$\boxed{X = \frac{1}{3}\ln(2)}$ $X \approx 0.231$

4. $\log(2X) = 3$

$10^{\log(2X)} = 10^{3}$

$2X = 1000$

$\boxed{X = 500}$

5. $4\log(X+2) = 8$

$\log(X+2) = 2$

$10^{\log(X+2)} = 10^{2}$

$X + 2 = 100$

$\boxed{X = 98}$

6. $e^{2X} + 3 = 10$

$e^{2X} = 7$

$2X = \ln(7)$

$\boxed{X = \frac{1}{2}\ln(7)}$ $X \approx 0.973$

7A) $3\log(X+1) = 9$

$\log(X+1) = 3$

$10^{\log(X+1)} = 10^{3}$

$X + 1 = 1000$

$\boxed{X = 99}$ x = 999

7B) $\log_3(2X) = 4$

$3^{\log_3(2X)} = 3^{4}$

$2X = 81$

$\boxed{X = 81/2}$ x = 40.5

8. $\frac{1}{2} = e^{0.01t}$

$\ln(\frac{1}{2}) = 0.01t$

$\boxed{t = \ln(1/2)/0.01}$ $t = -100\ln(2)$

$t \approx 69.315$

9. CONSIDER $y = 3e^{kt}$

a, FIND K if (10, 2) is on the graph.

b, Find y if $t = 15$.

c, FOR WHAT VALUE OF t IS $y = \frac{3}{2}$? (half of the y value when $t=0$)

a) $2 = 3e^{k \cdot 10}$

$\frac{2}{3} = e^{10k}$

$\ln(\frac{2}{3}) = 10k$

$k = \frac{1}{10}\ln(2/3)$

$\boxed{k \approx -0.040547}$

b, $y(15) = 3e^{k \cdot 15}$

$\boxed{y \approx 1.633}$

c, $\frac{3}{2} = 3e^{kt}$

solve for t = half life

$\frac{1}{2} = e^{kt}$

$\ln(1/2) = kt$

$t = \ln(1/2)/k$

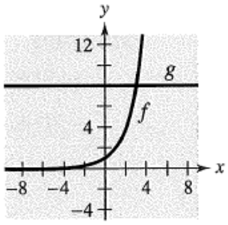
$\boxed{t \approx 17.095}$

Sheet 542: Exponential and Logarithmic Equations 2

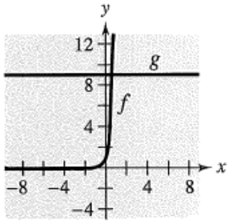
PC 5th Ch. 3-4, p. 324-325

In Exercises 31–34, approximate the point of intersection of the graphs of f and g. Then solve the equation $f(x) = g(x)$ algebraically.

31. $f(x) = 2^x$
$g(x) = 8$

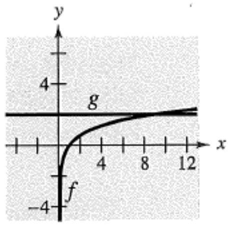


32. $f(x) = 27^x$
$g(x) = 9$

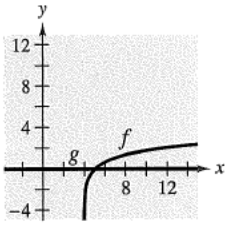


Write down your visual approximation before solving.

33. $f(x) = \log_3 x$
$g(x) = 2$



34. $f(x) = \ln(x - 4)$
$g(x) = 0$



Finance **In Exercises 111 and 112, find the time required for a \$1000 investment to double at interest rate r, compounded continuously.**

111. $r = 0.085$ **112.** $r = 0.12$

Finance **In Exercises 113 and 114, find the time required for a \$1000 investment to triple at interest rate r, compounded continuously.**

113. $r = 0.085$ **114.** $r = 0.12$

31.(3, 8) 32.($\frac{2}{3}$, 9) 33.(9, 2) 34.(5, 0) **111.** 8.2 years 112. 5.8 years **113.** 12.9 years 114. 9.2 years.

Algebra II-10

Sheet #551 - Ch5 Review

Name ______________________________ Period ____________

Note: Make sure you can do each question with or without multiple choices given. Be prepared to do problems without the calculator (recognizing graphs and answering in terms of logs instead of decimals).

Write as the sum or difference of logarithms with no exponents.

1. $\log 57x$

2. $\log n^5$

3. $\log n^3 m^8$

Write as a single logarithm.

4. $\log n - \log 90$

5. $2\log m + 7\log n$

6. Use the formula $\log_b M = \dfrac{\log_a M}{\log_a b}$ to find $\log_5 137$ to the nearest thousandth.

Solve.

7. $2^{3x} = 64$

8. $5 = 2e^{1+x}$

9. $\log_2(4 - 5x) = 2$

10. The future value of an investment of P dollars earning an annual interest of r can be calculated with the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where t is the number of years of the investment and n is the number of compounding periods per year. Find the future value of \$2000 if it is invested for 4 years at an annual interest rate of 10% compounded every 3 months.

11. Which of the following functions could have the graph sketched here?

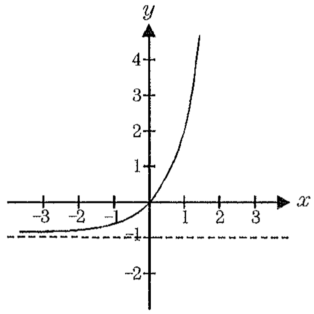
A) $y = 3^{x-1}$

B) $y = 3^x - 1$

C) $y = 3^{1-x}$

D) $y = 3^{-x} - 1$

E) $y = e^{3x} - 1$



12. Which of the following functions could have the graph sketched here?

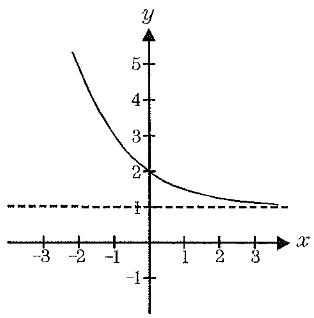
A) $f(x) = (\frac{1}{2})^x - 1$

B) $f(x) = 2^x + 1$

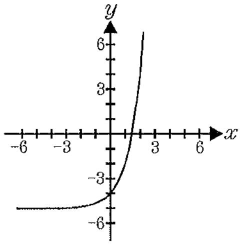
C) $f(x) = 2^{-x} + 1$

D) $f(x) = 3^{-x}$

E) $f(x) = 3^x - 1$



13. Which of the following could be the graph of $f(x) = 3^{-x} + 5$?

A)


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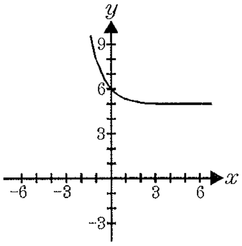

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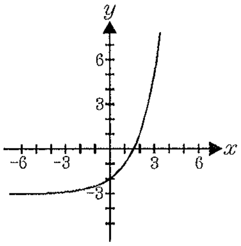
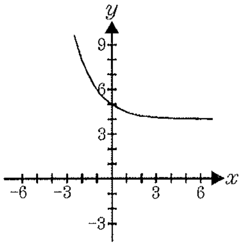
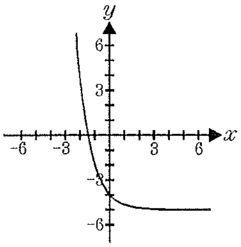
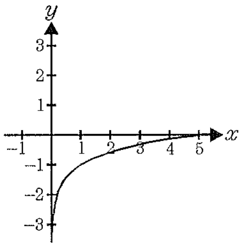
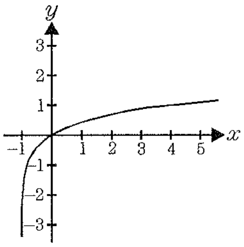


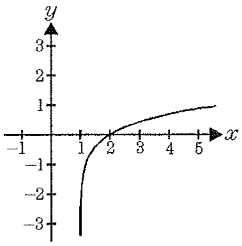
D)


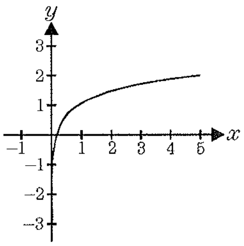
E)


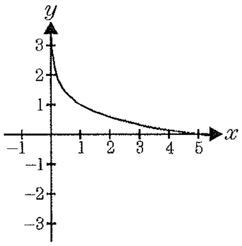
14. Which of the following is the graph of $f(x) = \log_5(x+1)$?

A)


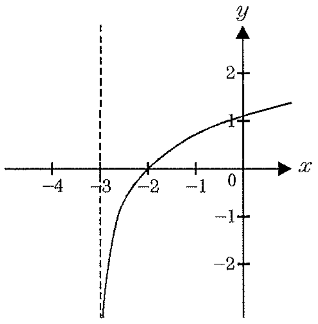
B)


C)


D)


E)


15. Which of the following functions could have the graph sketched here?



A) $f(x) = -3 + \ln x$ B) $f(x) = 3 + \ln x$
C) $f(x) = \ln(x-3)$ D) $f(x) = \ln(x+3)$
E) $f(x) = 2 + \ln(x-3)$

16. Which of the following is equal to $e^{\ln 5 + \ln x}$?

A) $\log \frac{5}{x}$ B) x^{50} C) 5^{10x}
D) e^{5x} E) $5x$

17. Which of the following is equal to $e^{2\ln(x-2)+3\ln y}$?

A) $y^3(x-2)^2$ B) $2^{3y(x-2)}$
C) $e^{6+\ln(x+y-2)}$ D) $\ln 3y^2(x-2)$
E) $\dfrac{\ln_3 y}{\ln_2(x-2)^4}$

18. Choose the expression equivalent to $\ln\left(\frac{8x^2}{3y}\right)$.

A) $\ln 8 - \ln 3 + 2\ln x - \ln y$
B) $\dfrac{\ln 8 + \ln x^2}{\ln 3 + \ln y}$
C) $\ln(8x^2) + \ln(3y)$
D) $2\ln(8x) - \ln(3y)$
E) $\ln\left(\frac{8}{3}\right) + \ln\left(\frac{x}{y}\right)^2$

19. Solve for x: $2^{5x-1} = 3$

A) $\frac{8}{3}$

B) $\frac{1}{5}\ln 6$

C) $\dfrac{1}{5}\left[\dfrac{\ln 3}{\ln 2} + 1\right]$

D) $\frac{1}{5}\left[1 - \ln(\frac{2}{3})\right]$

E) $\frac{1}{5}\left[\ln(\frac{2}{3} + 1)\right]$

20. A fish population is increasing at a rate of 5% a year. How many fish will there be in 9 years, to the nearest thousand fish, if there are 17 thousand now?

A) 41 B) 45 C) 26 D) 85 E) 23

21. A radioactive element has a half-life of 60 days. What percentage of the original sample is left after 45 days?

A) 25.00% B) 35.91% C) 59.46%
D) 66.23% E) 75.00%

22. A deposit of $3,000 is made into a fund with an annual interest rate of 12 percent. Find the time (in years) necessary for the investment to triple if the interest is compounded continuously. Round your answer to 2 decimal places.

A) 8.58 B) 9.16 C) 7.23 D) 12 E) 6

23. A mold culture doubles its mass every seven days. Find the growth model for a plate seeded with 0.9 grams of mold.

A) $y = 0.9e^{0.09902t}$

B) $y = 0.9e^{0.12183t}$

C) $y = 0.9e^{0.38541t}$

D) $y = 0.9e^{0.45128t}$

E) $y = 0.9e^{0.81818t}$

Algebra II-10 Sheet #551 - Ch5 Review

Answer List

1. $\log 57 + \log x$
2. $5 \log n$
3. $3 \log n + 8 \log m$
4. $\log \frac{n}{90}$
5. $\log m^2 n^7$
6. $\frac{\log 137}{\log 5} = 3.057$
7. $x = 2$
8. $x = -1 + \ln 2.5 \approx -0.08$
9. $x = 0$
10. \$2969.01
11. B
12. C
13. B
14. B
15. D
16. E
17. A
18. A
19. C
20. C
21. C
22. B
23. A

SOLUTIONS

10. $A = \$2000\left(1 + \frac{0.10}{4}\right)^{16}$ (4 TIMES/YEAR)(4 YEARS)

19. $2^{5x-1} = 3$

$\boxed{\text{METHOD 1:}}$ $\log_2(2^{5x-1}) = \log_2(3)$

$5x - 1 = \log_2(3)$

$x = \frac{1}{5}(\log_2(3) + 1)$

$x = \frac{1}{5}(LN(3)/LN(2) + 1) \approx 0.517$

CHANGE OF BASE

$\boxed{\text{METHOD 2:}}$

PROPERTY OF LOG(POWER).

$LN(2^{5x-1}) = LN(3)$

$[5x-1] \cdot LN(2) = LN(3) \longrightarrow$

20. $y = 17e^{0.05t}$

$y(9) = 17e^{0.05(9)} = 26.7 > 26$ thousand

21. $\boxed{\text{METHOD 1}}$:

$y = 100\% \cdot \left(\frac{1}{2}\right)^{t/60} = 100\% \cdot \left(\frac{1}{2}\right)^{45/60} = 59.46\%$

$t/60 = \#$ of half lives

$\boxed{\text{METHOD 2:}}$

LONG METHOD.

$y = 100\% \, e^{-kt}$

$50\% = 100\% \, e^{-k \cdot 60}$. K UNKNOWN.

$LN(50/100) = -k \cdot 60$ $K = -LN(1/2)/60 = 0.011552453$

$y(45) = 100\% \cdot e^{-0.011552453 \cdot 45} = 59.46\%$

22. $9000 = 3000e^{0.12t} \rightarrow LN(3) = 0.12t \rightarrow t = LN(3)/0.12 = 9.16$ years.

23. INITIAL VALUE = 0.9 g.

$y = 0.9e^{kt}$. Solve for k: $1.8 = 0.9e^{k \cdot 7}$. $2 = e^{k \cdot 7}$ $k = LN(2)/7 =$ 0.09902/day

Algebra II

Sheet #551

Page 5

SOLUTION $\boxed{\text{KEY}}$: See Also Answer Key

Name $\boxed{\text{KEY}}$ Period ____________

Note: Make sure you can do each question with or without multiple choices given. Be prepared to do problems without the calculator (recognizing graphs and answering in terms of logs instead of decimals).

Write as the sum or difference of logarithms with no exponents.

1. $\log 57x$ $= \boxed{\log(57) + \log(x)}$

2. $\log n^5$ $= \boxed{5\log(n)}$

3. $\log n^3 m^8$ $= \boxed{3\log(n) + 8\log(m)}$

LOG OF POWERS: MULTIPLY BY EXPONENT.

Write as a single logarithm.

4. $\log n - \log 90$ $\boxed{\log\left(\frac{n}{90}\right)}$

5. $2\log m + 7\log n$ $\boxed{\log\left(\frac{m^2}{n^7}\right)}$

DON'T FORGET $2\log m = \log m^2$.

6. Use the formula $\log_b M = \dfrac{\log_a M}{\log_a b}$ to find $\log_5 137$ to the nearest thousandth.

$\log(137)/\log(5) \cong \boxed{3.057}$

or use $LN(137)/L(5) \cong 3.057$.

Solve.

7. $2^{3x} = 64$

METHOD 2

$LN(2^{3X}) = LN(64)$

$3X LN(2) = LN(64)$

$X = \dfrac{LN(64)/LN(2)}{3}$

METHOD 1

$\log_2(2^{3X}) = \log_2(64)$

$3X = \log_2(2^6)$

$3X = 6$

$\hookrightarrow \boxed{X = 2}$

8. $5 = 2e^{1+x}$

LN OF BOTH SIDES.

$LN\left(\frac{5}{2}\right) = LN(e^{1+x})$

$LN(5/2) = 1 + X$

$X = \boxed{LN(5/2) - 1} \cong -0.084$

9. $\log_2(4 - 5x) = 2$ RAISE BOTH SIDES BY BASE 2.

$2^{\log_2(4-5x)} = 2^2$

$4 - 5x = 4$

$\boxed{X = 0}$

10. The future value of an investment of P dollars earning an annual interest of r can be calculated with the formula $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$, where t is the number of years of the investment and n is the number of compounding periods per year. Find the future value of \$2000 if it is invested for 4 years at an annual interest rate of 10% compounded every 3 months.

$A = \$2000\left(1 + \frac{0.10}{4}\right)^{4 \cdot 4}$

n: # TIMES/yr = 12/3 = 4/yr.

t = 4yrs.

A = \$2000

r = 10/100 = 0.1/yr.

$A = 2000(1.025)^{16} = \boxed{\$2969.01}$

$\rightarrow y = e^x - K$ HAS HORIZ. ASYMPTOTE $y = K$

11. Which of the following functions could have the graph sketched here?

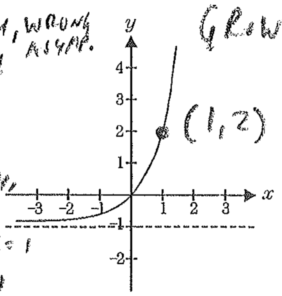
A) $y = 3^{x-1}$ ✗ GROWTH, WRONG ASYMP.

[✓ B) $y = 3^x - 1$] GROWTH

C) $y = 3^{1-x}$ ✗ DECAY

D) $y = 3^{-x} - 1$ ✗ DECAY

E) $y = e^{3x} - 1$ ✗ GROWTH, WRONG POINT X = 1



GROWTH

CLUES:

- HORIZONTAL ASYMPTOTE: $y = -1$
- IF $X = 1$, $y = 3^1 - 1 = 2$.

12. Which of the following functions could have the graph sketched here?

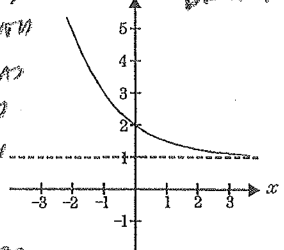
A) $f(x) = (\frac{1}{2})^x - 1$ ✗ DECAY

B) $f(x) = 2^x + 1$ ✗ GROWTH

[✓ C) $f(x) = 2^{-x} + 1$] DECAY

D) $f(x) = 3^{-x}$ ✗ DECAY

E) $f(x) = 3^x - 1$ ✗ GROWTH



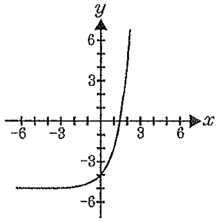
DECAY

CLUE:

- HORIZONTAL ASYMPTOTE $y = 1$

13. Which of the following could be the graph of $f(x) = 3^{-x} + 5$?

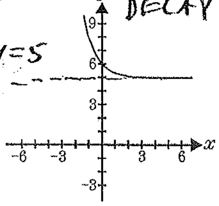
A)



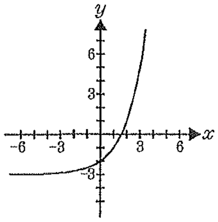
[✓ B)]

DECAY

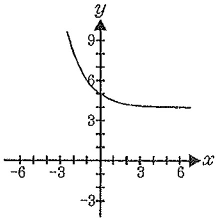
$y=5$



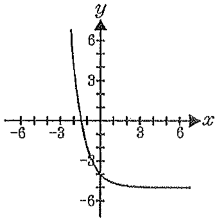
C)



D)



E)

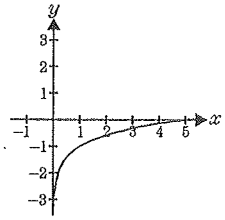


DOMAIN : all x
RANGE: $y > 5$

→ $y = \log(x - H)$ HAS VERTICAL ASYMPTOTE $X = H$.

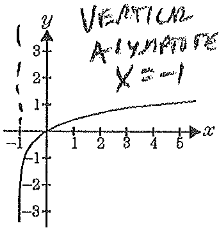
14. Which of the following is the graph of $f(x) = \log_5(x+1)$?

A)

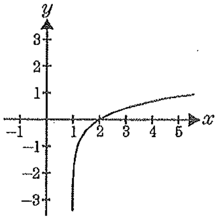


[✓ B)]

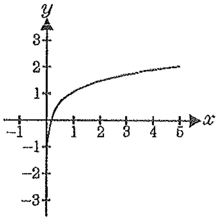
VERTICAL ASYMPTOTE $X = -1$



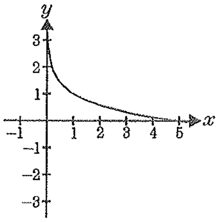
C)



D)



E)

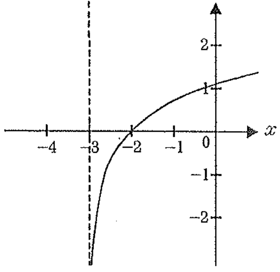


DOMAIN : $X > -1$

RANGE : all y.

15. Which of the following functions could have the graph sketched here?

V.Asy $X=-3$



A) $f(x) = -3 + \ln x$
B) $f(x) = 3 + \ln x$
C) $f(x) = \ln(x-3)$
[✓ D) $f(x) = \ln(x+3)$]
E) $f(x) = 2 + \ln(x-3)$

→ $e^{\ln(x)} = x$ and $\ln(e^x) = x$.

16. Which of the following is equal to $e^{\ln 5 + \ln x}$?

A) $\log \frac{5}{x}$
B) x^{50}
C) 5^{10x}
D) e^{5x}
[✓ E) $5x$]

$e^{\ln(5 \cdot x)} = 5x$

17. Which of the following is equal to $e^{2\ln(x-2) + 3\ln y}$?

[✓ A) $y^3(x-2)^2$]
B) $2^{3y(x-2)}$
C) $e^{6 + \ln(x+y-2)}$
D) $\ln 3y^2(x-2)$
E) $\dfrac{\ln_3 y}{\ln_2 (x-2)^4}$

$e^{\ln[(x-2)^2 \cdot y^3]} = (x-2)^2 y^3$

18. Choose the expression equivalent to $\ln\left(\dfrac{8x^2}{3y}\right)$.

[✓ A) $\ln 8 - \ln 3 + 2\ln x - \ln y$]
B) $\dfrac{\ln 8 + \ln x^2}{\ln 3 + \ln y}$
C) $\ln(8x^2) + \ln(3y)$
D) $2\ln(8x) - \ln(3y)$
E) $\ln\left(\dfrac{8}{3}\right) + \ln\left(\dfrac{x}{y}\right)^2$

19 – 23 See Answer Key.

Sheet # 552: NOTES ON

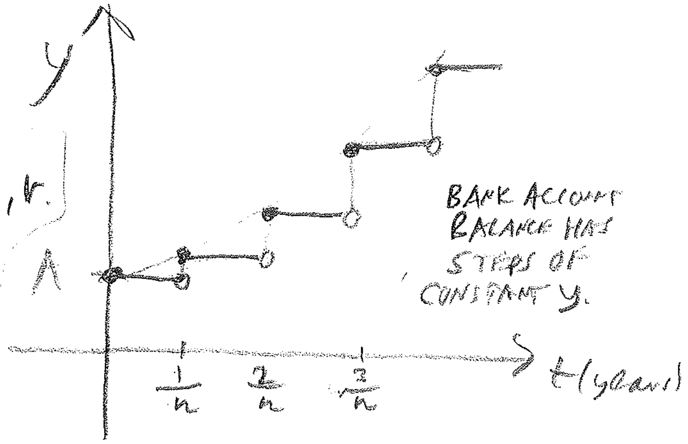
EXPONENTIAL MODELS

t = TIME IN YEARS.
A = INITIAL (PRINCIPAL) AMOUNT
r = ANNUAL (INTEREST RATE/100%.

1\. a. COMPOUNDED.

$$y = A\left(1 + \frac{r}{n}\right)^{nt}$$

COMPOUNDED n TIMES PER YEAR.
TOTAL PAYMENTS = nt.



BANK ACCOUNT BALANCE HAS STEPS OF CONSTANT y.

b, CONTINUOUS, GIVEN ANNUAL RATE, r.

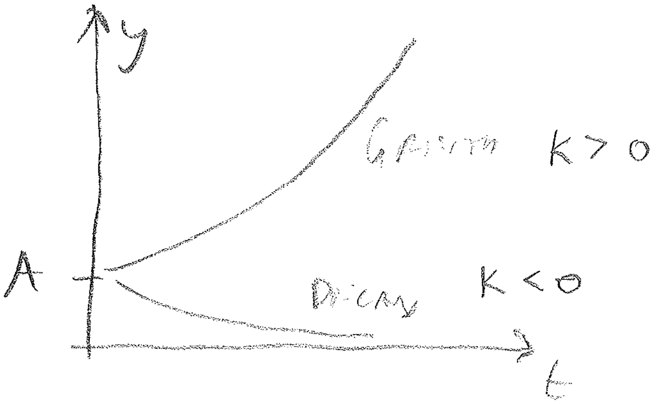
$$y = A(1+r)^t$$

2\. EXP. GROWTH / DECAY

$$y = Ae^{kt}$$

K = CONTINUOUS EXPONENTIAL $\frac{\text{DECAY}}{\text{GROWTH}}$ RATE CONSTANT

THIS MODEL CAN ALWAYS BE USED FOR CONTINUOUS GROWTH & DECAY.



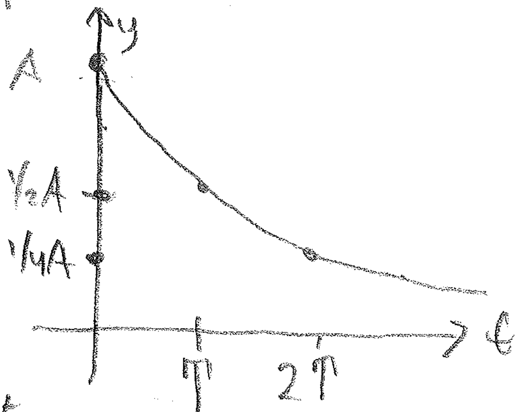
3\. DECAY.

$$y = A\left(\frac{1}{2}\right)^{t/T}$$

T = half life
t/T = # of half lives.

THIS MODEL IS ALSO CONTINUOS,

USEFUL FOR HALF LIFE.



4\. GROWTH $y = A(2)^{t/T}$

T = doubling time.

Name: ________________________ Period: ____

Sheet #556: Exponential Equations and Exponential Models

Possibly helpful formulas: $y = A\left(1+\frac{r}{n}\right)^{nt}$ $y = Ae^{kt}$ $y = A\left(\frac{1}{2}\right)^{t/T}$

1. Solve for *x*. Answer to three decimals.
   $2^{x+1} = 5$

2. Solve for *t*. Answer to three decimals.
   $e^{0.1t} = 4$

3. If the continuous exponential growth rate is 8%/hour and you start with an amount of 300 units. How many units do you have after 5 hours? (Answer to the nearest completed unit.)

4. A student began taking the AP History test at 8am ($t = 0$ hours). The student had 100% brain energy (*y* in percent) at that time. By 12noon, the student had 40% brain energy left. Use an exponential model to find the following.
   a. Find the half life (*T* in hours) of brain energy. Answer to the nearest 10th of an hour.

   b. Predict the brain energy left at 3pm. Answer to the nearest percent.

Name: $\boxed{\text{KEY}}$ Period: ____

Sheet #556: Exponential Equations and Exponential Models

Possibly helpful formulas: $y = A\left(1+\frac{r}{n}\right)^{nt}$ $y = Ae^{kt}$ $y = A\left(\frac{1}{2}\right)^{t/T}$

1. Solve for x.
   $2^{x+1} = 5$

$LN(2^{X+1}) = LN(5)$

$(X+1)LN(2) = LN(5)$

$X+1 = LN(5)/LN(2)$

$$\boxed{X = \frac{LN(5)}{LN(2)} - 1 \approx 1.322}$$

2. Solve for t.
   $e^{0.1t} = 4$

$LN(e^{0.1t}) = LN(4)$

$0.1t = LN(4)$

$$\boxed{t = LN(4)/0.1 \approx 13.863}$$

3. If the continuous exponential growth rate is 8%/hour and you start with an amount of 300 units. How many units do you have after 5 hours?

$y = Ae^{kt}$ $K = 8\%$, $A = 300$ UNITS.

$y = 300e^{0.08t}$

$t = 5$

$y = 300e^{0.08\cdot 5} = \boxed{300e^{0.4}} \approx 447.547$ $\boxed{447 \text{ UNITS}}$

(OR 448 UNITS)

4. A student began taking the AP History test at 8am ($t = 0$ hours). The student had 100% brain energy (y in percent) at that time. By 12noon, the student had 40% brain energy left. Use an exponential model to find the following.
   a. Find the half life (T in hours) of brain energy.

METHOD 2

$y = Ae^{kt}$

FIND K

$40 = 100e^{K\cdot 4}$

$K = LN(0.4)/4 \approx -0.22907$ per hour.

THEN FIND T

$50 = 100e^{-0.22907\cdot T}$

$LN(1/2) = -0.22907\cdot T$

$T = \frac{LN(1/2)}{-0.22907} \approx 3.026$ hours

$\boxed{\approx 3.0 \text{ hours}}$

   b. Predict the brain energy left at 3pm.

$t = 7$ hours

$y = 100e^{-0.22907\cdot 7} \approx 20.119\%$

$\left\{\text{OR } y \approx 100\left(\frac{1}{2}\right)^{(7/3.0)} \approx 20\%\right\}$

$\boxed{\approx 20\%}$

Sheet 557: Exponential and Logarithmic Models 2

PC 5th Ch. 3-5, p. 335-338

Finance **In Exercises 7–14, complete the table for a savings account in which interest is compounded continuously.**

	Initial Investment	*Annual % Rate*	*Time to Double*	*Amount After 10 Years*
7.	\$1000	12%		
8.	\$20,000	$10\frac{1}{2}$%		
9.	\$750		$7\frac{3}{4}$ yr	
10.	\$10,000		12 yr	
11.	\$500			\$1505.00
12.	\$600			\$19,205.00
13.		4.5%		\$10,000.00
14.		8%		\$20,000.00

In Exercises 25–30, complete the table for the radioactive isotope.

	Isotope	*Half-life (years)*	*Initial Quantity*	*Amount After 1000 Years*
25.	^{226}Ra	1620	10 g	
26.	^{226}Ra	1620		1.5 g
27.	^{14}C	5730		2 g
28.	^{14}C	5730	3 g	
29.	^{239}Pu	24,360		2.1 g
30.	^{239}Pu	24,360		0.4 g

41. *Bacteria Growth* The number of bacteria N in a culture is modeled by

$N = 250e^{kt}$

where t is the time in hours. If $N = 280$ when $t = 10$, estimate the time required for the population to double in size.

Chemistry **In Exercises 58–63, use the acidity model given by pH $= -\log_{10}[H^+]$, where acidity (pH) is a measure of the hydrogen ion concentration $[H^+]$ (measured in moles of hydrogen per liter) of a solution.**

58. Find the pH if $[H^+] = 2.3 \times 10^{-5}$.

59. Find the pH if $[H^+] = 11.3 \times 10^{-6}$.

60. Compute $[H^+]$ for a solution in which pH $= 5.8$.

61. Compute $[H^+]$ for a solution in which pH $= 3.2$.

62. A certain fruit has a pH of 2.5 and an antacid tablet has a pH of 9.5. The hydrogen ion concentration of the fruit is how many times the concentration of the tablet?

63. If the pH of a solution is decreased by 1 unit, the hydrogen ion concentration is increased by what factor?

	Initial Investment	*Annual % Rate*	*Time to Double*	*Amount after 10 Years*
7.	\$1000	12%	5.78 yr	\$3,320.12
9.	\$750	8.9438%	7.75 yr	\$1,834.37
11.	\$500	11.0%	6.3 yr	\$1,505.00
13.	\$6376.28	4.5%	15.4 yr	\$10,000.00
15.	\$112,087.09			

	Isotope	*Half-life (Years)*	*Initial Quantity*	*Amount After 1000 Years*
25.	^{226}Ra	1620	10 g	6.52 g
27.	^{14}C	5730	2.26 g	2 g
29.	^{239}Pu	24,360	2.16 g	2.1 g

41. 61.16 hours

58. 4.64

59. 4.95

60. 1.58×10^{-6} moles/liter

61. $10^{-3.2} \approx 6.3 \times 10^{-4}$ moles per liter

62. 10^7

63. 10

Name:____

Sheet #558: Exponential Models 3

Possibly helpful formulas: $y = A(1 + r/n)^{nt}$ $y = Ae^{kt}$ $y = A(2)^{t/T}$ $y = A(0.5)^{t/T}$

1. Solve for the unknowns to 3 significant figures. Use algebra. You may check with the calculator.

   (a) $30 = 60e^{-0.05t}$

   (b) $120 = Ae^{0.7}$

2. The number of bacteria in a culture, y, is modeled by $y = 400e^{kt}$, where t is the time in hours.

   (a) Find k if $(t, y) = (3, 600)$ is on the graph of y. Write k to six decimals.

   (b) Using part (a), find the number of bacteria y after $t = 4$ hours. Round down.

   (c) Find the doubling time, T, to 4 decimals.

   (d) Find the value of t for which $y = 1000$ bacteria.

3. The radioactive isotope Rubikskubium-299, ^{299}Rk, could have a half life of 3 attoseconds. If there were 40 femtograms of the isotope at zero attoseconds, how much of the isotope is there after the following time intervals?

   (a) 12 attoseconds (answer exactly).

   (b) 14 attoseconds (answer to two decimals).

4. A beetle population is growing exponentially. If the continuous exponential growth ***rate constant*** is 20% per day and there are 1,000 beetles now, how many beetles will there be after 14 days? Round down to the nearest beetle.

5. A student began taking the ACT at 8am ($t = 0$ hours). The student had 100% brain energy (y in percent) at that time. By 12:30pm ($t = 4.5$ hours), the student had 30% brain energy left. Use a continuous exponential decay model to find the following.

   (a) Find the half life (T in hours) of brain energy. Answer to the nearest 10th of an hour.

   (b) Predict the brain energy that was left at 10am. Answer to the nearest percent.

Answers. 1a: 13.9. 1b: 59.6; 2a: 0.135155/h. 2b: 686. 2c: 5.1285h. 2d: 6.7796h; 3a: 2.5fg 3b: 1.57fg; 4: 16,444; 5a: 2.6h. 5b: 59%.

Name: $\boxed{\text{KEY}}$

Sheet #558: Exponential Models 3

Score		

Possibly helpful formulas: $y = A(1+r/n)^{nt}$ $y = Ae^{kt}$ $y = A(2)^{t/T}$ $y = A(0.5)^{t/T}$

1. Solve for the unknowns to 3 significant figures. Use algebra. You may check with the calculator.

(a) $30 = 60e^{-0.05t}$

$\ln\left(\frac{30}{60}\right) / -0.05 = -0.693 / -0.05 = \boxed{13.9}$

[3 figures ⟷ 1 decimal here]

(b) $120 = Ae^{0.7}$

$\frac{120}{e^{0.7}} = \boxed{59.6}$

2. The number of bacteria in a culture, y, is modeled by $y = 400e^{kt}$, where t is the time in hours.

(a) Find k if $(t, y) = (3, 600)$ is on the graph of y. Write k to ~~five~~ 6 decimals.

$600 = 400e^{k \cdot 3}$

$\ln(1.5)/3 = \boxed{0.135155}/h$

(b) Using part (a), find the number of bacteria y after $t = 4$ hours.

$686.8 = \boxed{686}$ bacteria

(c) Find the doubling time, T.

$2 = e^{kT}$

5.12853h

$T = \ln(2)/k = \boxed{5.1285\ h}$

(d) Find the value of t for which $y = 1000$ bacteria.

$1000 = 400e^{kt}$

$t = \ln(2.5)/k =$

$= \boxed{6.7796\ h}$

6.77955h

3. The radioactive isotope Rubikskubium-299, ^{299}Rk, could have a half life of 3 attoseconds. If there were 40 femtograms of the isotope at zero attoseconds, how much of the isotope is there after the following time intervals?

(a) ~~18~~ 12 attoseconds (exactly).

$\frac{t}{T} = \frac{12}{3} = 4$ HALF LIVES. $\frac{40}{2^4} = \frac{40}{16} = \boxed{2.5\ fg}$

(b) ~~25~~ 14 attoseconds (to two decimals).

1.5749

$y = 40(0.5)^{14/3} = \boxed{1.57\ fg}$

4. A beetle population is growing exponentially. If the continuous exponential growth ***rate constant*** is 20% per day and there are 1,000 beetles now, how many beetles will there be after 14 days? Round to the nearest beetle.

$k = 0.20$

$y = 1000\, e^{0.2 \cdot 14}$

16,444.6

$\boxed{16{,}444}$

5. A student began taking the ACT at 8am ($t = 0$ hours). The student had 100% brain energy (y in percent) at that time. By 12:30pm ($t = 4.5$ hours), the student had 30% brain energy left. Use a continuous exponential decay model to find the following.

(a) Find the half life (T in hours) of brain energy. Answer to the nearest 10th of an hour.

$y = 100(0.5)^{t/T}$

$30 = 100\,(0.5)^{4.5/T}$

$\ln(0.3) = \frac{4.5}{T}\ln(0.5)$

$T = 4.5\ln(0.5)/\ln(0.3) = 2.59$

$\boxed{2.6\ h}$

ALT: $y = Ae^{kt}$

$k = \ln(0.3)/4.5$

-0.267550

$T = \ln(1.5)/k$

$= 2.5907$

(b) Predict the brain energy that was left at 10am. Answer to the nearest percent

$y = 100(0.5)^{2/2.590725}$

$= 58.56$

$= \boxed{59\%}$

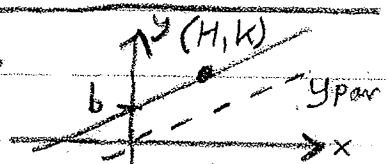
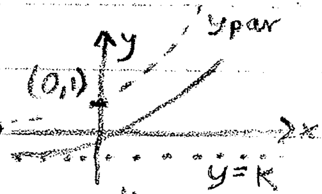
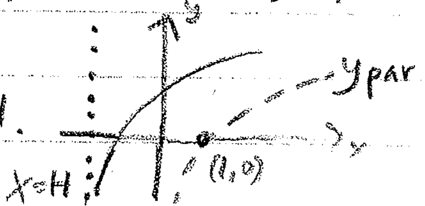
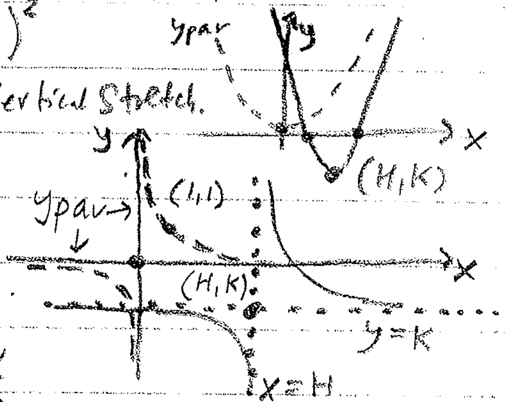
L65

[Note $100(0.5)^{2/2.6} = 58.7 \approx 59\%$]

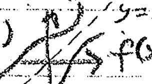
Functions: One-page Summary

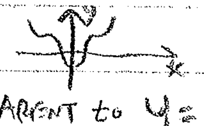
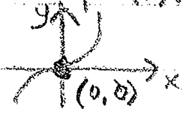
SHEET N150 8/29/07
9/28/07 V.2.
8/22/08 V.3

H, K = HORIZ + VERT. SHIFTS. A = VERT. STRETCH. B = HORIZ. SHRINK

	PARENT (y_{par})	CHILD FUNCTIONS
LINE	$y=mx$	$y-K=m(x-H)$  point: (H,K). slope: m. y-intercept: b = K-mh
EXP-ONENTIAL	$y=e^x$	$y-K=Ae^{Bx}$ Horizontal asymptote y=K. Continuous rate constant = B. 
LOG-ARITHMIC	$y=\ln(x)$	$y-K=\ln(x-H)$ Vertical asymptote x=H. 
QUAD-RATIC	$y=x^2$	$y-K=A(x-H)^2$ Vertex: (H,K). A = Vertical Stretch. 
RECI-PROCAL	$y=\frac{1}{x}$	$y-K=\frac{A}{x-H}$ Vert. asy: x=H Horiz asy: y=K
SINE	$y=\sin(x)$	$y-K=A\sin[B(x-H)]$ $\lvert A\rvert$ = Amplitude. $B=\frac{2\pi}{T}$. T=period=Horiz Stretch.

FUNCTION. x (DOMAIN) → $y=f(x)$ (RANGE), y UNIQUE. ZEROS: x when $f(x)=0$.

INVERTIBLE (IF 1-TO-1). $f(g(x))=g(f(x))=x$  $y=f(x) \leftrightarrow x=f^{-1}(y)$.

SYMMETRY. EVEN: $f(-x)=f(x)$.  ODD: $f(-x)=-f(x)$ 

REFLECT ABOUT X-AXIS: $y=f(x)$ PARENT to $y=-f(x)$ CHILD 

EXP-LOG. $e^u\cdot e^v=e^{u+v}$. $(e^u)^v=e^{u\cdot v}$. $\ln(u\cdot v)=\ln(u)+\ln(v)$. $\ln(u^v)=v\cdot\ln(u)$.

CONTINUOUS GROWTH $y=Ae^{|B|t}$ $y=A\cdot(S)^t$. $y=A\left(\frac{1}{2}\right)^{t/T}$ $y=A(2)^{t/T}$

DECAY $y=Ae^{-|B|t}$ B = Contin. rate constant S = base T = half life or doubling time

COMPOUNDED (SEQUENCE / PIECEWISE FUNCTION WITH JUMPS): $y=A\left(1+\frac{r}{100}\right)^t$. r = annual interest rate (%)

TRIG. CARTESIAN: $x=r\cdot\cos\theta$, $y=r\cdot\sin\theta$. POLAR: $r=\sqrt{x^2+y^2}$, $\theta=\tan^{-1}\left(\frac{y}{x}\right)$.