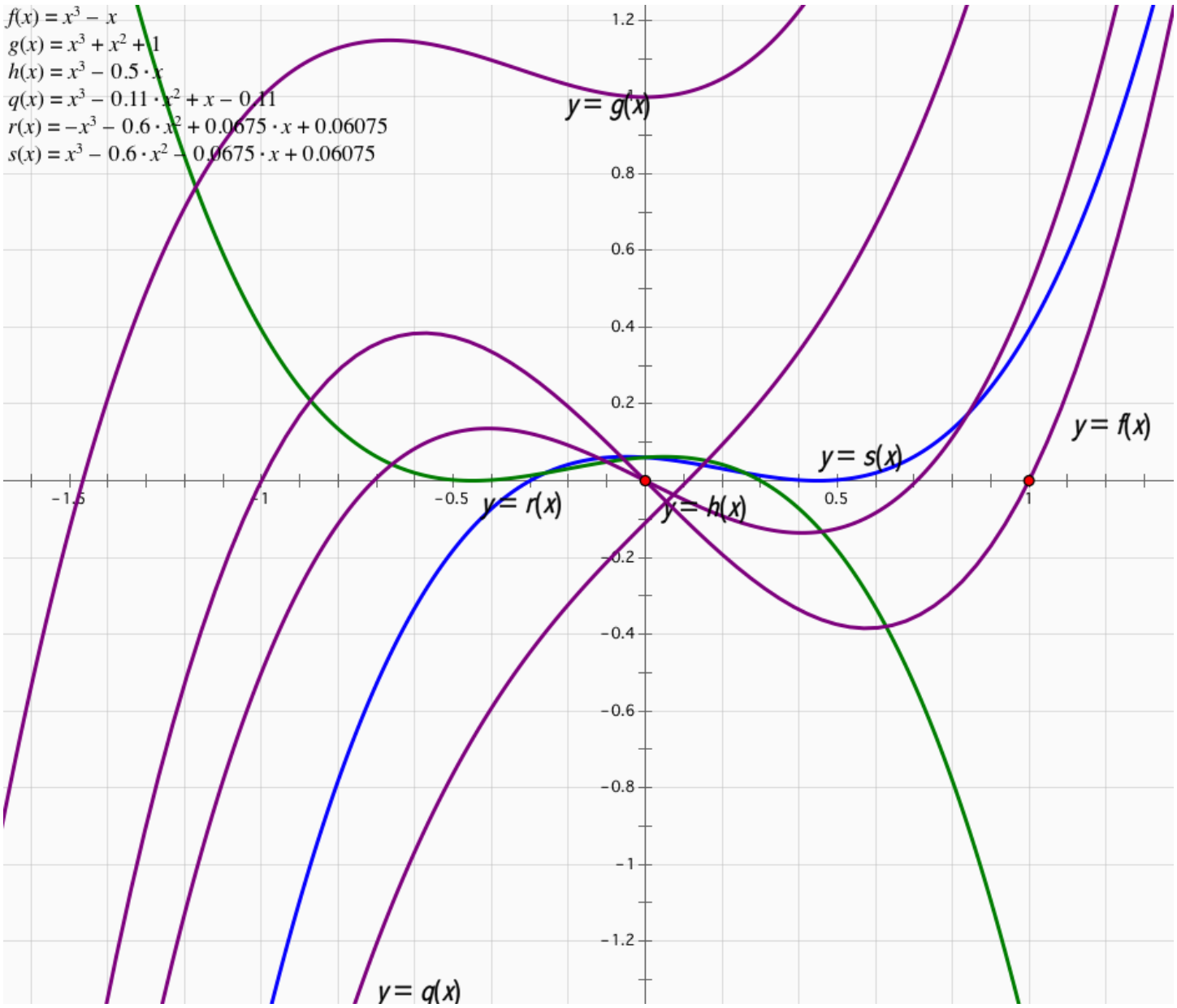
Sheet 452: Test Vectors for Finding Zeros

Finding zeros of a cubic function on the interval [min, max] = [-1, 1]

```
/* Useful Test vectors for a, b, c, d for f(x) = ax^3 + bx^2 + cx + d.
        1 0 -1 0 ;                 -1 (min), 1 (max)
        1 1 0 1 ;                  -1.4656 (outside bounds)
        1 0 -0.5 0 ;               -0.707 (- to +), 0 (+ to -), 0.707
        1 -0.8 1 -0.8 ;            0.8 (- to +)
        1 -0.11 1 -0.11 ;          0.11 (- to +)
        1 -1 1 -1 ;                1 (max, - to +)
        1 -8 1 5 ;                 -0.702 (- to +), 0.913, 7.79 (outside)
        -1 2 0 -0.9 ;              -0.5895 (+ to -), 0.9077, 1.68 (outside)
        -1 4 0 -1 ;                -0.47283 (+ to -), 0.53740, 3.9354 (out)
        0 1 -0.9 0.2025 ;          0.45 (bounce, quadratic function)
        -1 -0.6 0.0675 0.06075 ;   -0.45 (bounce), 0.3 (+ to -)
        1 -0.6 -0.0675 0.06075 ;   -0.3 (- to +), 0.45 (bounce)
        4 5 -1 -2 ;                -1 (min, - to +), -0.84307, 0.59307
        1 1 9 1 ;                  -0.11236 (- to +)
        1 0.996 0 0 ;              -0.996, 0 (bounce)
      */
```

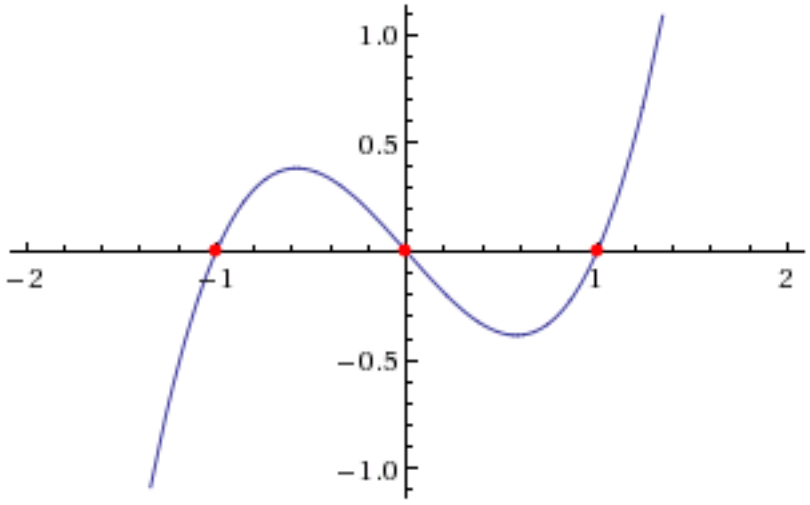

$f(x) = x^3 - x$
$g(x) = x^3 + x^2 + 1$
$h(x) = x^3 - 0.5 \cdot x$
$q(x) = x^3 - 0.11 \cdot x^2 + x - 0.11$
$r(x) = -x^3 - 0.6 \cdot x^2 + 0.0675 \cdot x + 0.06075$
$s(x) = x^3 - 0.6 \cdot x^2 - 0.0675 \cdot x + 0.06075$
y = g(x)
y = s(x)
y = f(x)
y = r(x)
y = h(x)
y = q(x)
1.2
0.8
0.6
0.4
0.2
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-0.5
0.5
1

1 0 -1 0

Input:

$$x^3 - x = 0$$

Root plot:



Alternate forms:

$$x^3 = x$$

$$(x-1)\,x\,(x+1) = 0$$

Solutions:

$$x = -1$$

$$x = 0$$

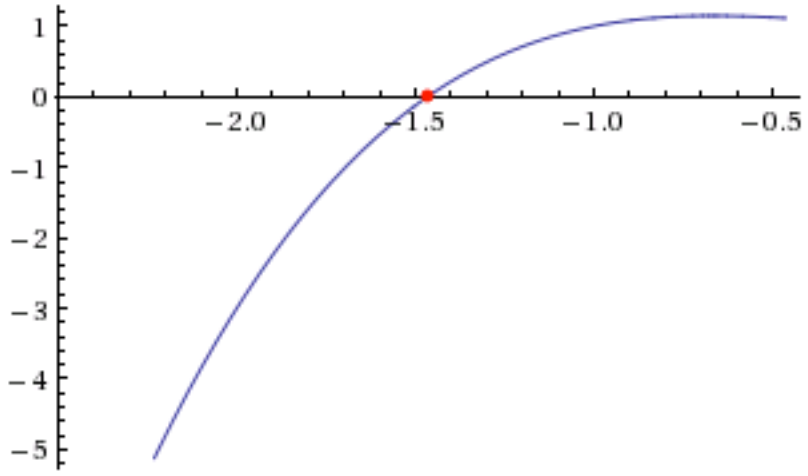
$$x = 1$$

1 1 0 1

Input:

$$x^3 + x^2 + 1 = 0$$

Root plot:



Alternate form:

$$x^3 + x^2 = -1$$

Real solution: Mor

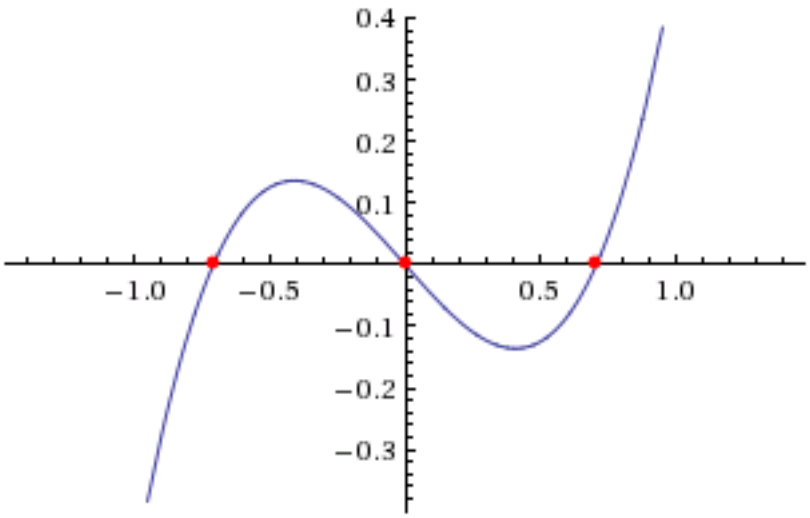
$$x = -1.4656$$

1 0 -0.5 0

Input:

$$x^3 - 0.5\,x = 0$$

Root plot:



Alternate form:

$$(x - 0.707107)\,x\,(x + 0.707107) = 0$$

Alternate form assuming x is real:

$$x^3 - 0.5\,x + 0.\,i = 0$$

Solutions:

$$x = -0.707107$$

$$x = 0$$

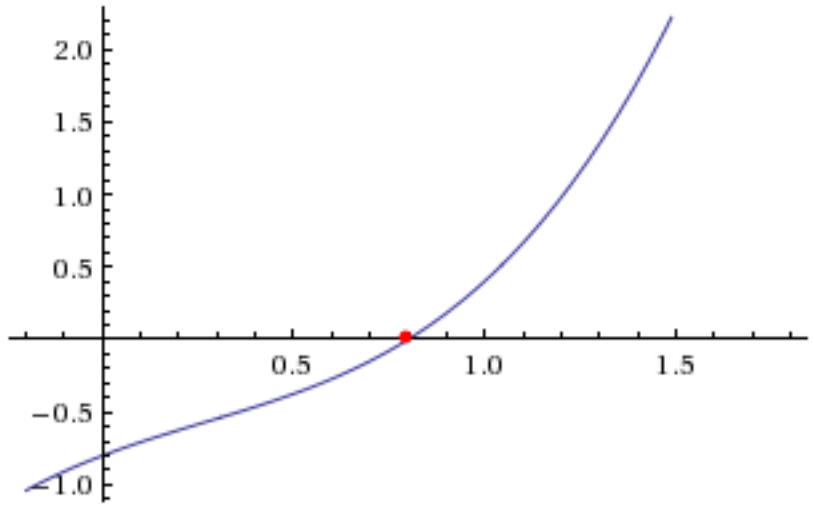
$$x = 0.707107$$

1 -0.8 1 -0.8

Input:

$$x^3 - 0.8\,x^2 + x - 0.8 = 0$$

Root plot:



Alternate forms:

$$(x - 0.266667)^3 + 0.786667\,(x - 0.266667) - 0.571259 = 0$$

$$(x - 0.8)\left(x^2 + 1.\right) = 0$$

Alternate form assuming x is real:

$$-(0.8 + 0.\,i) + x^3 - 0.8\,x^2 + x = 0$$

Real solution:

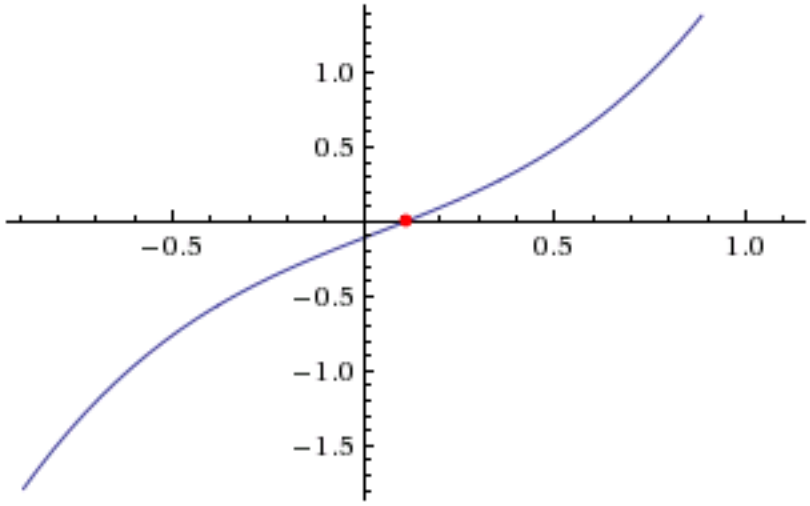
$$x = 0.8$$

1 -0.11 1 -0.11

Input:

$$x^3 - 0.11\,x^2 + x - 0.11 = 0$$

Root plot:



Alternate forms:

$$(x - 0.0366667)^3 + 0.995967\,(x - 0.0366667) - 0.0734319 = 0$$

$$(x - 0.11)\left(x^2 + 1.\right) = 0$$

Alternate form assuming x is real:

$$-(0.11 + 0.\,i) + x^3 - 0.11\,x^2 + x = 0$$

Real solution:

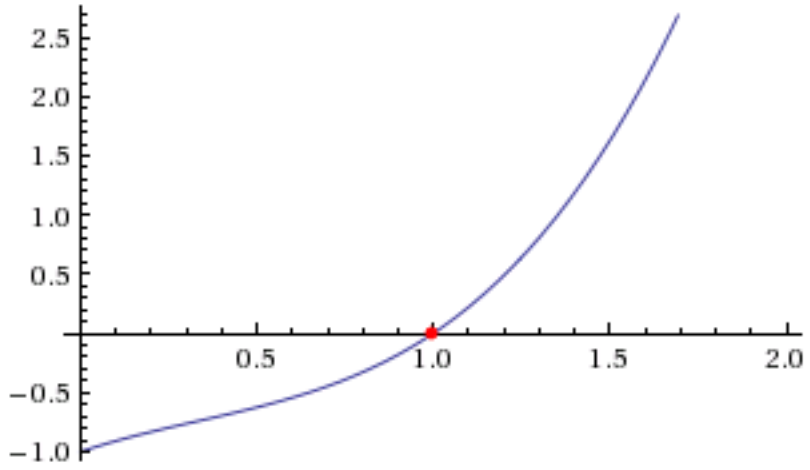
$$x = 0.11$$

1 -1 1 -1

Input:

$$x^3 - x^2 + x - 1 = 0$$

Root plot:



Alternate forms:

$$x^3 + x = x^2 + 1$$

$$(x - 1)\left(x^2 + 1\right) = 0$$

$$\left(x - \frac{1}{3}\right)^3 + \frac{2}{3}\left(x - \frac{1}{3}\right) - \frac{20}{27} = 0$$

Real solution:

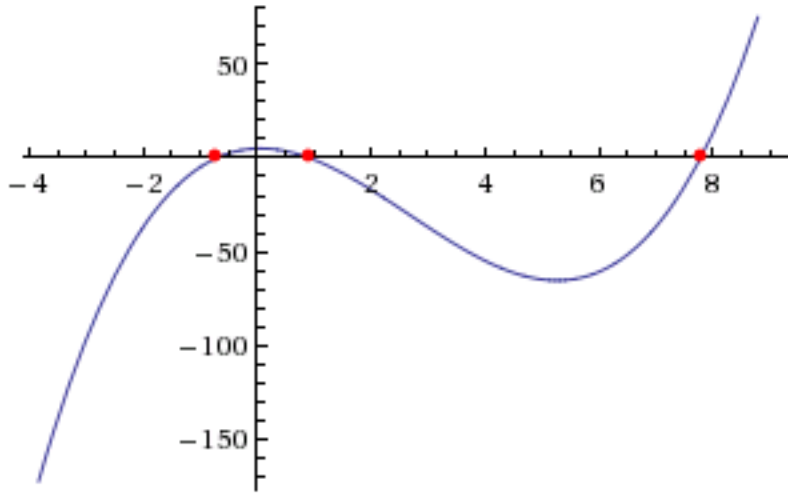
$$x = 1$$

1 -8 1 5

Input:

$$x^3 - 8x^2 + x + 5 = 0$$

Root plot:



Alternate forms:

$$x^3 + x + 5 = 8x^2$$

$$x^3 - 8x^2 + x = -5$$

$$\left(x - \frac{8}{3}\right)^3 - \frac{61}{3}\left(x - \frac{8}{3}\right) - \frac{817}{27} = 0$$

Solutions:

$$x = -0.70270$$

$$x = 0.91349$$

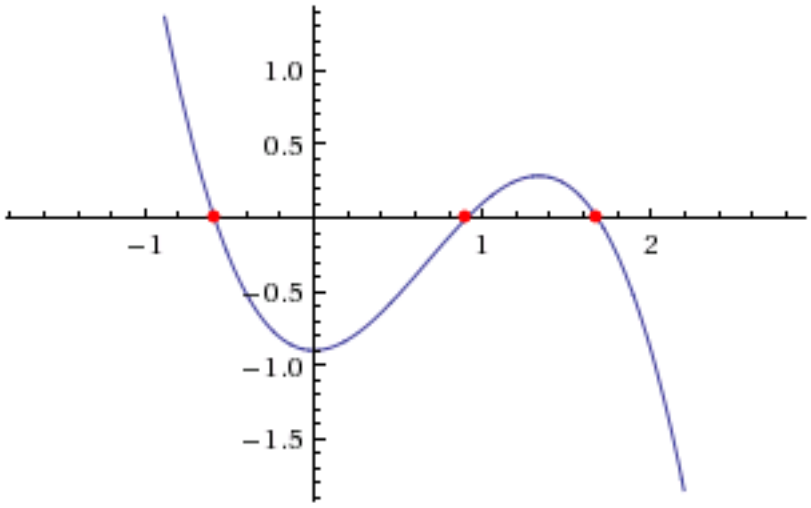
$$x = 7.7892$$

-1 2 0 -0.9

Input:

$$-x^3 + 2\,x^2 - 0.9 = 0$$

Root plot:



Alternate forms:

$$-(x-2)\,x^2 - 0.9 = 0$$

$$-(x - 1.68181)\,(x - 0.907728)\,(x + 0.589536) = 0$$

Solutions:

$$x = -0.589536$$

$$x = 0.907728$$

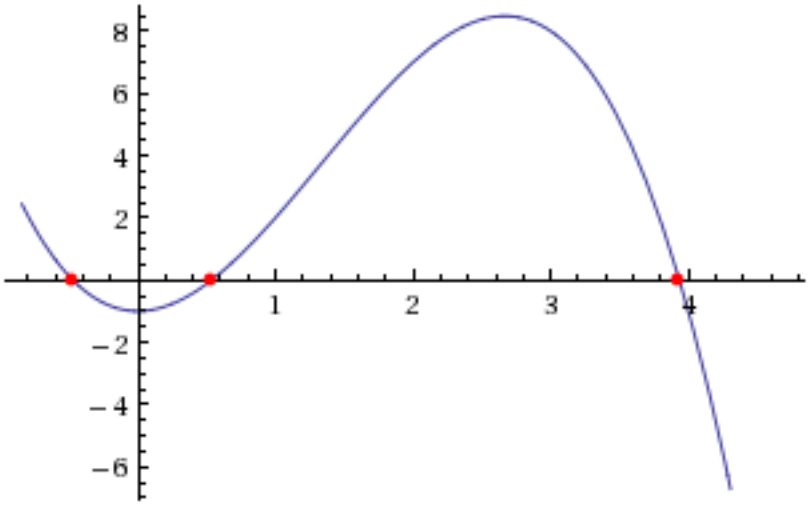
$$x = 1.68181$$

-1 4 0 -1

Input:

$$-x^3 + 4x^2 - 1 = 0$$

Root plot:



Alternate forms:

$$-(x-4)\,x^2 - 1 = 0$$

$$4x^2 - x^3 = 1$$

Solutions:

$$x = -0.47283$$

$$x = 0.53740$$

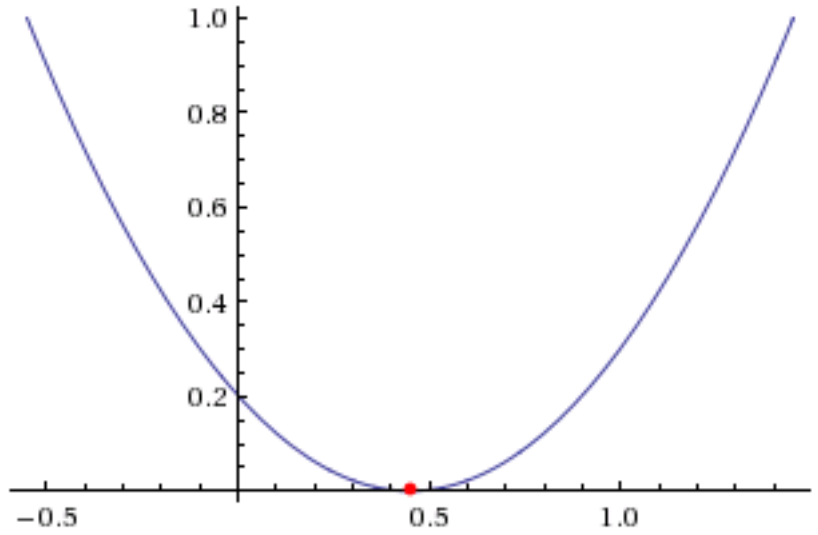
$$x = 3.9354$$

0 1 -0.9 0.2025 (quadratic)

Input:

$$x^2 - 0.9\,x + 0.2025 = 0$$

Root plot:



Alternate forms:

$$(x - 0.9)\,x + 0.2025 = 0$$

$$(x - 0.45)^2 + 0. = 0$$

$$x^2 - 0.9\,x + 0.2025 = 0$$

Alternate form assuming x is real:

$$(0.2025 + 0.\,i) + x^2 - 0.9\,x = 0$$

Solution:

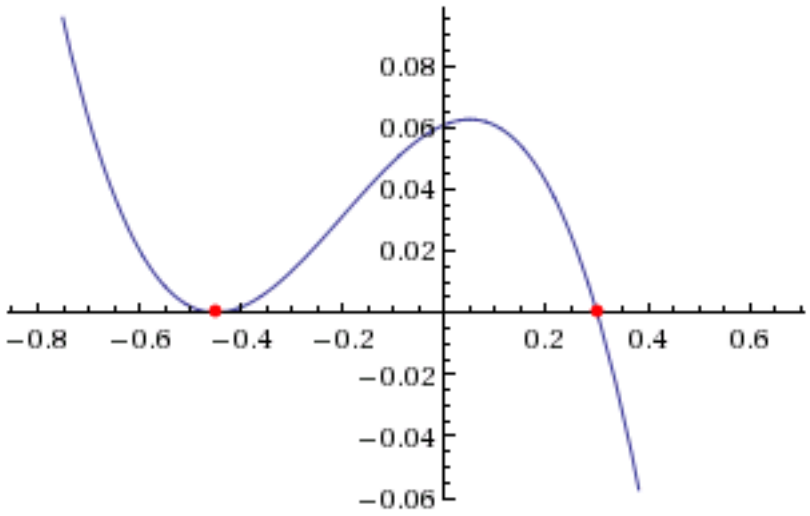
$$x = 0.45$$

-1 -0.6 0.0675 0.06075

Input:

$$-x^3 - 0.6\,x^2 + 0.0675\,x + 0.06075 = 0$$

Root plot:



Alternate forms:

$$0.06075 - x\,(x\,(x + 0.6) - 0.0675) = 0$$

$$-(x - 0.3)\left(x^2 + 0.9\,x + 0.2025\right) = 0$$

$$-(x + 0.2)^3 + 0.1875\,(x + 0.2) + 0.03125 = 0$$

Alternate form assuming x is real:

$$(0.06075 + 0.\,i) - x^3 - 0.6\,x^2 + 0.0675\,x = 0$$

Solutions:

$$x = -0.45$$

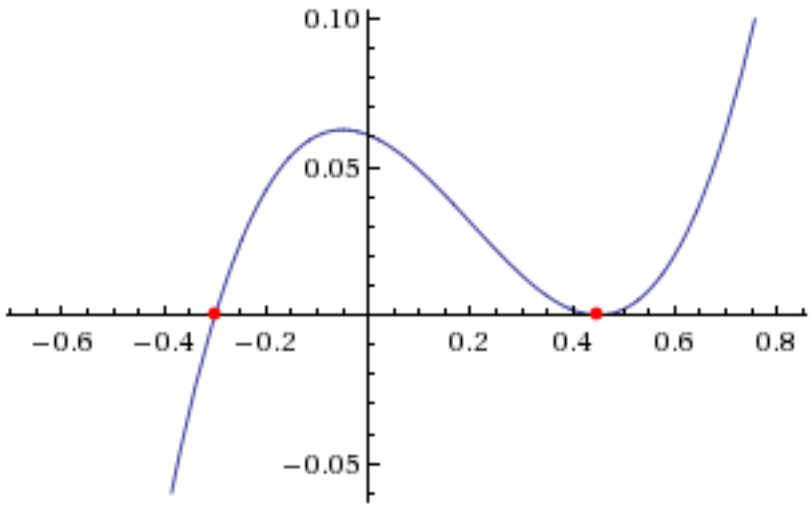
$$x = 0.3$$

1 -0.6 -0.0675 0.06075

Input:

$$x^3 - 0.6\,x^2 - 0.0675\,x + 0.06075 = 0$$

Root plot:



Alternate forms:

$$x\left((x - 0.6)\,x - 0.0675\right) + 0.06075 = 0$$

$$(x - 0.2)^3 - 0.1875\,(x - 0.2) + 0.03125 = 0$$

$$(x + 0.3)\left(x^2 - 0.9\,x + 0.2025\right) = 0$$

Alternate form assuming x is real:

$$(0.06075 + 0.\,i) + x^3 - 0.6\,x^2 - 0.0675\,x = 0$$

Solutions:

$$x = -0.3$$

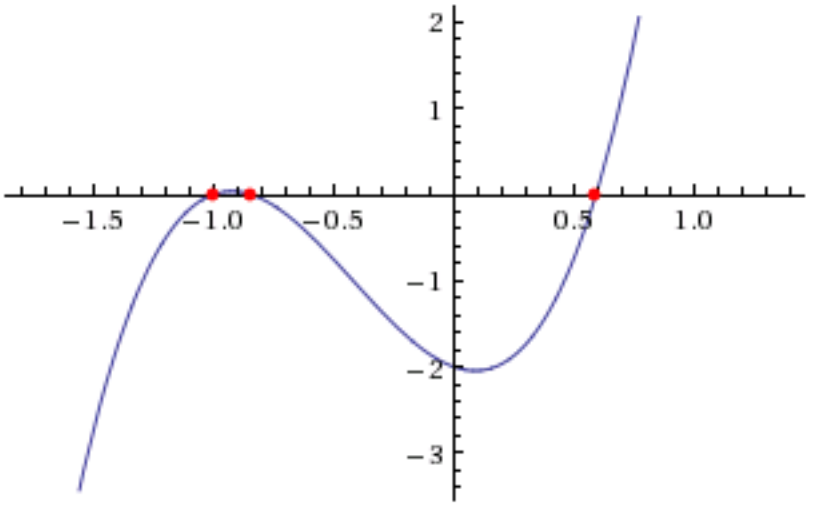
$$x = 0.45$$

4 5 -1 -2

Input:

$$4x^3 + 5x^2 - x - 2 = 0$$

Root plot:



Alternate forms:

$$x^2(4x + 5) = x + 2$$

$$(x + 1)(4x^2 + x - 2) = 0$$

$$4\left(x + \frac{5}{12}\right)^3 - \frac{37}{12}\left(x + \frac{5}{12}\right) - \frac{217}{216} = 0$$

Solutions:

$$x = -1$$

$$x = -0.84307$$

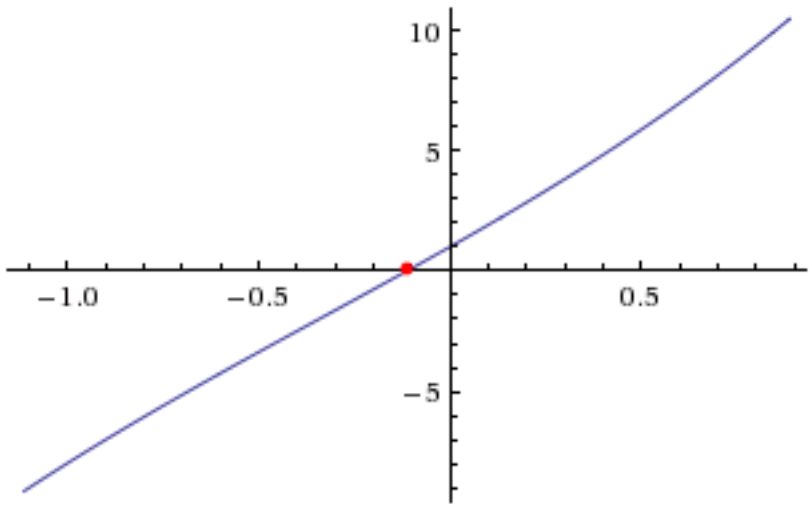
$$x = 0.59307$$

1191

Input:

$$x^3 + x^2 + 9x + 1 = 0$$

Root plot:



Alternate forms:

$$x^3 + x^2 + 9x = -1$$

$$x\left(x^2 + x + 9\right) + 1 = 0$$

$$\left(x + \frac{1}{3}\right)^3 + \frac{26}{3}\left(x + \frac{1}{3}\right) - \frac{52}{27} = 0$$

Real solution:

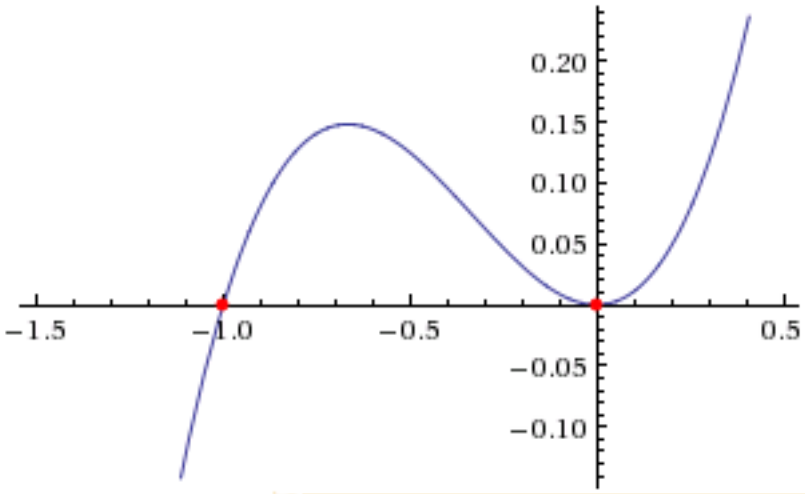
$$x = -0.11236$$

1 0.9996 0 0

Input:

$$x^3 + 0.9996\,x^2 = 0$$

Root plot:



Alternate forms:

$$x^2\,(x + 0.9996) = 0$$

$$x^2\,(x + 0.9996) = 0$$

Alternate form assuming x is real:

$$x^3 + 0.9996\,x^2 + 0.\,i = 0$$

Solutions:

$$x = -0.9996$$

$$x = 0$$