Name: KEY

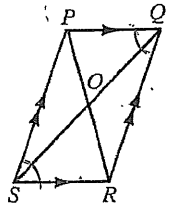
Sheet 1291: Cumulative Review Chapters 4–12

Chapters 4–7

Complete each statement with the word *always*, *sometimes*, or *never*.

1. A square is always a rhombus. (np→nq)
2. The inverse of a true statement is sometimes true.
3. Similar figures are sometimes congruent.
4. An acute triangle and an obtuse triangle are never similar.
5. The base angles of an isosceles trapezoid are always congruent.

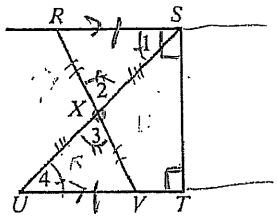
Refer to the diagram at the right. Tell whether each statement *must* be true.



MUST BE TRUE

6. $\overline{PQ} \cong \overline{SR}$ and $\overline{PS} \cong \overline{QR}$ — Yes
7. $\angle RSP \cong \angle PQR$ — Yes
8. $\angle SPQ \cong \angle PQR$ — No
9. $\angle SPQ$ and $\angle PQR$ are supp. $\measuredangle$. — Yes
10. $\overline{PR} \perp \overline{SQ}$ — No
11. $\overline{SO} \cong \overline{OQ}$ — Yes
12. $\overline{PS} \cong \overline{SR}$ — No

13. Refer to the diagram below.



Supply the reasons.

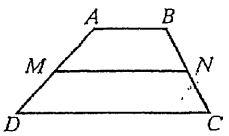
Given: $\overline{ST} \perp \overline{RS}$; $\overline{ST} \perp \overline{UT}$; $\overline{RS} \cong \overline{UV}$

Prove: $\overline{RV}$ and $\overline{US}$ bisect each other.

Proof: STATEMENTS	REASONS
1. $\overline{ST} \perp \overline{RS}$; $\overline{ST} \perp \overline{UT}$	Given
2. $\overline{RS} \parallel \overline{UT}$	Two lines ⊥ to a line are ∥
3. $\angle 1 \cong \angle 4$.	If two ∥ lines are cut by a transversal, then Alt. Int. ∠s are ≅
4. $\angle 2 \cong \angle 3$.	Vertical ∠s are ≅. (AIA Theorem)
5. $\overline{RS} \cong \overline{UV}$.	Given
6. $\triangle SXR \cong \triangle UXV$	AAS Theorem
7. $\overline{RX} \cong \overline{VX}$; $\overline{SX} \cong \overline{UX}$	CPCTC
8. $\overline{RV}$ and $\overline{US}$ bisect each other.	Definition of bisector

$\overline{MN}$ **is the median of trapezoid *ABCD*.**

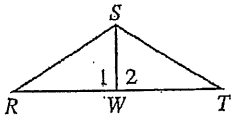
14. If $AM = 3$ and $BN = 2$, then $MD =$ 3 and $NC =$ 2.
15. If $AB = 10$ and $DC = 14$, then $MN =$? 12



14. 3, MEDIAN, 2, 3, 2

15. 10, 12, 14

16. If $\overline{SW}$ is an altitude of $\triangle RST$, then $m\angle 2 =$? 90°
17. If $\overline{SW}$ is a median of $\triangle RST$, then ? ≅ ? RW ≅ WT
18. If S is equidistant from R and T, then S lies on the ? of $\overline{RT}$. ⊥ BISECTOR



19. The lengths of two sides of a triangle are 10 and 13. The length of the third side must be greater than ? but less than ?

10 > 3 (13) — 3 — 23 — 10 13 < 23

20. If two angles of a triangle are congruent to two angles of another triangle, then the triangles are ? SIMILAR AA Postulate

21. The ratio of a side of a square to the perimeter of the square is ? 1:4

22. The measures of the angles of a quadrilateral are in the ratio 2:4:5:7. Find the measures of the angles.

$2x + 4x + 5x + 7x = 360°$

$18x = 360°$

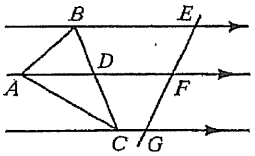
$x = 20°$

Answer: 40°, 80°, 100°, 140°

If you are interested, try this by using Theorems 7–3 and 7–4 on p. 269.

25. Given: $\overleftrightarrow{BE} \parallel \overleftrightarrow{AF} \parallel \overleftrightarrow{CG}$; $\overrightarrow{AF}$ bisects $\angle BAC$.

Prove: $\frac{AB}{AC} = \frac{EF}{FG}$

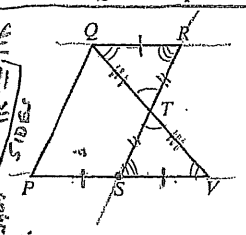


Name: KEY

26. Given: $\triangle QRT \cong \triangle VST$; S is the midpoint of $\overline{PV}$.
Prove: Quadrilateral $PQRS$ is a parallelogram.

WANT

WAYS TO PROVE
- both pairs are ||
- both pairs are ≅
- one pair both ≅ and ||
- both pairs of opp ∠s ≅.
- Diagonals Bisect

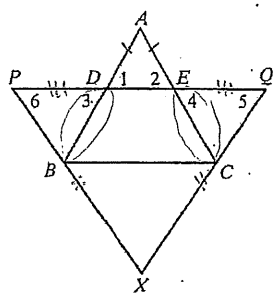


$\overline{PS} \parallel \overline{QR}$
$\overline{PS} \cong \overline{QR}$
Need AIA CPCTC
Start with $\overline{PS} \parallel \overline{QR}$

26.

STATEMENT	REASON
1. $\triangle QRT \cong \triangle VST$	1. Given
2. $\angle RQT \cong \angle SVT$	2. CPCTC "Corresp. parts of ≅ are ≅!" (S)
3. $\overline{PV} \parallel \overline{QR}$	3. If 2 lines are cut by a transversal and A.I.A are ≅, then lines are ||.
(4) $\overline{PS} \parallel \overline{QR}$	4. $\overline{PS}$ lies on $\overline{PV}$
5. S is mdpt of $\overline{PV}$	5. Given
6. $\overline{PS} \cong \overline{SV}$	6. Def. of midpt
7. $\overline{SV} \cong \overline{QR}$	7. CPCTC (of given △)
(8) $\overline{PS} \cong \overline{QR}$	8. Trans. Prop.
9. Quad PQRS is a ▱	9. If one pair of opp. sides are both ≅ and ||, then the quad is a ▱.

27. Given: $\overline{AD} \cong \overline{AE}$; $\overline{PX} \cong \overline{QX}$; $\overline{PD} \cong \overline{EQ}$
Prove: $\overline{BD} \cong \overline{CE}$

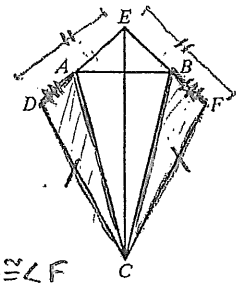


GET $\triangle PDB \cong \triangle QEC$
From ASA: $\angle 6 \cong \angle 5$ (ISOSC. △)
$\overline{PD} \cong \overline{EQ}$ (given)
$\angle 3 \cong \angle 4$ (Vert. Ang.)

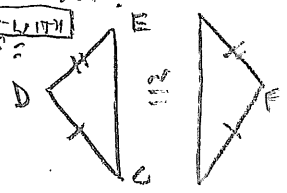
27.

	STATEMENT	REASON
	1. $\overline{AD} \cong \overline{AE}$	1. Given
	2. $\angle 1 \cong \angle 2$	2. Isosceles △ Thm (base angles ≅)
	3. $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$	3. Vertical angles are ≅
A	(4) $\angle 3 \cong \angle 4$	4. Trans. Prop. (twice)
S	(5) $\overline{PD} \cong \overline{EQ}$	5. Given
A	(6) $\angle 6 \cong \angle 5$	6. Isosc. △ Thm
	7. $\triangle PDB \cong \triangle QEC$	7. ASA Postulate
	8. $\overline{BD} \cong \overline{CE}$	8. CPCTC.

28. Given: $\overline{DC} \cong \overline{FC}$; $\overline{DE} \cong \overline{FE}$; $\overline{DA} \cong \overline{FB}$
Prove: $\triangle DAC \cong \triangle FBC$



Need $\angle D \cong \angle F$ and use SAS
Start with SSS:



28.

	STATEMENT	REASON
(S)	1. $\overline{DC} \cong \overline{FC}$	1. Given
S	2. $\overline{DE} \cong \overline{FE}$	2. Given
S	3. $\overline{EC} \cong \overline{EC}$	3. Reflexive Prop.
	4. $\triangle CDE \cong \triangle CFE$	4. SSS Postulate
(A)	5. $\angle D \cong \angle F$	5. CPCTC
(S)	6. $\overline{DA} \cong \overline{FB}$	6. Given
	7. $\triangle DAC \cong \triangle FBC$	7. SAS Postulate

Name: KEY

Chapters 8–10

Classify each statement as true or false.

5. In right $\triangle ABC$, the tangent of $\angle B$ is $\frac{\text{leg opposite } \angle B}{\text{leg adjacent to } \angle B}$.

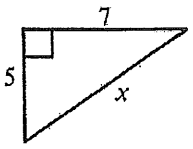
TRUE ✓

6. When two or more lines intersect in one point, the lines are concurrent.

TRUE ✓

Find the value of x.

8.

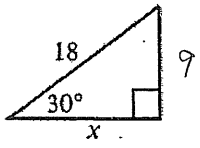


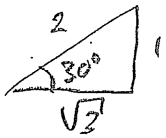
$x^2 = 5^2 + 7^2$

$x = \sqrt{25+49}$

$x = \sqrt{74}$ ✓

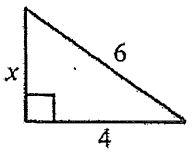
9.





$x = 9\sqrt{3}$ ✓ 15.5885

10.



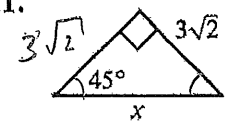
$x^2 + 4^2 = 6^2$

$x^2 = 36 - 16$

$x^2 = 20$

$x = 2\sqrt{5}$ ✓

11.

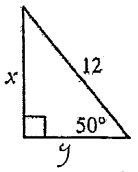


$x = (3\sqrt{2})\sqrt{2}$

$x = 6$

Find the value of x to the nearest tenth or to the nearest degree. Use a calculator or the table on page 311.

12.



$\frac{x}{12} = \sin 50°$

$x = 12 \sin 50°$

$x = 9.2$ ✓

$(\sin 50° = 0.7660)$

ALTERNATIVE

$\tan 50° = \frac{x}{y}$

$1.1918 = x/y$

$y = x/1.1918$

$y = 0.8391x$

$x^2 + y^2 = 12^2$

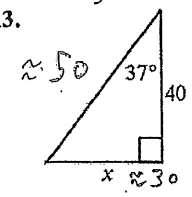
$x^2 + 0.7041x^2 = 12^2$

$1.7041x^2 = 144$

$x^2 = 84.5027$

$x = 9.2$

13.

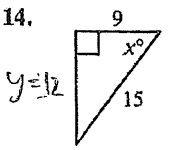


$\frac{x}{40} = \tan 37°$

$x = 40 \tan 37°$

$x = 30.1$ ✓ $(\tan 37° = 0.7536)$

14.



$9^2 + y^2 = 15^2$

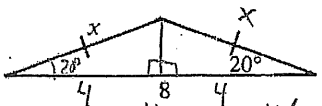
$y = \sqrt{144} = 12$

$\tan x = \frac{12}{9}$ $x = \tan^{-1}\left(\frac{4}{3}\right)$

FASTER: $x = \cos^{-1}\left(\frac{9}{15}\right) = 53°$

$x = 53°$ ✓ $(53.1°)$

15.



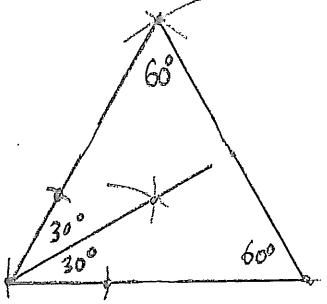
$\cos(20°) = \frac{4}{x}$ $x = 4/\cos(20°)$

$x = 4.3$ ✓

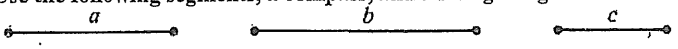
$(\cos 20° = 0.9397)$

For the following questions, use compass and straightedge, only.

20. Construct a 30° angle. EQUILATERAL Δ. BISECT ANGLE. (CONSTRUCTION 3)

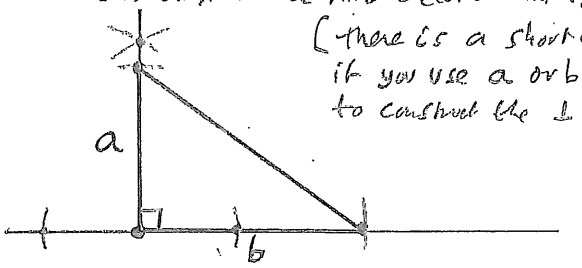


Use the following segments, a compass, and a straightedge.



23. Construct a right triangle with legs of lengths a and b.

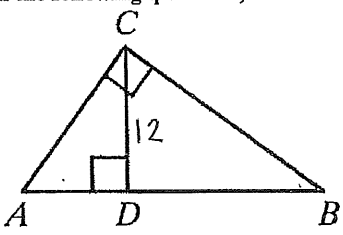
CONSTRUCTION 5: given a point on a line, construct the perpendicular to the line at the given point. THEN copy SEGMENT a AND b (CONSTRUCTION 1). [there is a shortcut if you use a orb to construct the ⊥ line]



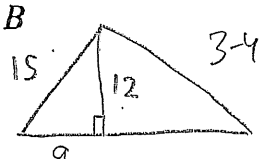
If you are interested, try this by using Construction 14 on page 397.

24. Construct a segment of length d such that $ad = be$.

In the following questions, CD = 12. (Write so in the diagram.)



31. If $AC = 15$, find AD.



3-4-5 Δ

AD = 9 ✓

32. If $AD = 4$, find DB.

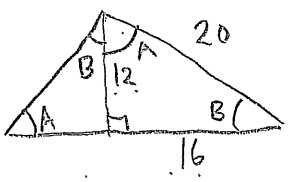
$\tan B = \frac{4}{12}$ $\frac{4}{12} = \frac{12}{DB}$

$\tan B = \frac{12}{DB}$ $DB = \frac{12 \cdot 12}{4}$

DB = 36 ✓ 4 DB

33. If $DB = 16$, find BC.

3-4-5 Δ



BC = 20 ✓

34. If $AB = 26$ and $AD < BD$, find AD.

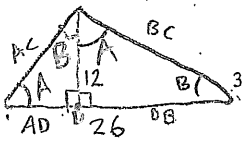
$\tan A = \frac{12}{AD}$ $\frac{12}{AD} = \frac{DB}{12}$

$\tan A = \frac{DB}{12}$ $DB \cdot AD = 144$.

FACTORS OF 144 THAT ADD TO 26: 8 AND 18.

$AD + DB = 26$

AD = 8 ✓



KEY

Name: _____

Chapters 11–12

Find each of the following.

1. The volume of a sphere with radius 10 cm.

$V = \frac{4}{3}\pi (10\text{ cm})^3 = \boxed{\frac{4000}{3}\pi\text{ cm}^3}$ ✓

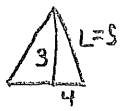
3. The volume of a cylinder with height 10 and ***diameter*** 10.

$r = 5$ $V = \pi r^2 h = \pi (5)^2 \cdot 10 = \boxed{250\pi}$ ✓

4. The volume of a regular square pyramid with base edges 8 and height 3.

$V = \frac{1}{3}(8)^2 \cdot 3 = \boxed{64}$ ✓ $B = 8^2 = 64$

5. The total area of the pyramid in Exercise 4.



$L.A. = \frac{1}{2}P \cdot L = \frac{1}{2}(4 \cdot 8) \cdot 5 = \frac{1}{2} \cdot 32 \cdot 5 = 80$

$T.A. = 80 + 64 = \boxed{144}$ ✓

6. The lateral area of a cone with slant height 14 and radius 9.

$L.A. = \frac{1}{2}PL = \pi r \cdot L = \pi \cdot 9 \cdot 14 = \boxed{126\pi}$ ✓

9. The area of a rhombus with diagonals 8 and 18.

$A = \frac{1}{2}d_1 d_2 = \frac{1}{2} 8 \cdot 18 = 144/2 = \boxed{72}$ ✓

11. The area of a sphere with radius 5.

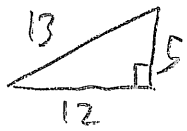
$A = 4\pi R^2 = 4 \cdot \pi \cdot 25 = \boxed{100\pi}$

20. The ratio of the volumes of two similar prisms is 64/27. Find the ratio of their lateral areas.

$\frac{V_1}{V_2} = \frac{64}{27}$ $\frac{R_1}{R_2} = \frac{\sqrt[3]{64}}{\sqrt[3]{27}} = \frac{4}{3}$ $\frac{A_1}{A_2} = \boxed{\frac{16}{9}}$ ✓

[RATIO: 4/3. AREA $4^2/3^2 = 16/9$. VOLUME $4^3/3^3 = 64/27$.]

21. Find the area of a right triangle with hypotenuse 13 and leg 5.



5-12-13 Δ

$A = \frac{1}{2} 5 \cdot 12 = \boxed{30}$ ✓

25. Find the height of a trapezoid with bases 9 cm and 15 cm long and area 60 cm².

$A = \frac{1}{2}(b_1 + b_2) \cdot h$

$60 = \frac{1}{2}(9 + 15) \cdot h$

$60 = 12h$

$\boxed{h = 5\text{ cm}}$ ✓

26. A right triangular prism has height 6 and base edges 5, 5, and 8. Find the total area.

$H = 6$

TWO 3-4-5 Δ's

$B = 2 \cdot \frac{1}{2}(3 \cdot 4) = 12$

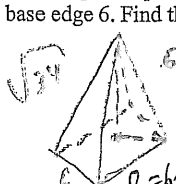
$2B = 24$

$L.A. = P \cdot L = (5 + 5 + 8) \cdot 6 = 18 \cdot 6 = 108$

$T.A. = \boxed{132}$ ✓

$= 12 + 12 + 108$.

27. A regular square pyramid has lateral edge $\sqrt{34}$ *(square root of 34)* and base edge 6. Find the volume. NOT SLANT HEIGHT.



$B = 6 \cdot 6 = 36$.

$(\sqrt{34})^2 = (3\sqrt{2})^2 + H^2$

$34 = 18 + H^2$

$16 = H^2$ $H = 4$

$V = \frac{1}{3}Bh = \frac{1}{3}(36)(4)$

$\boxed{V = 48}$ ✓

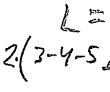
28. A cone with radius 6 has lateral area 60π. Find the volume.

$A = \frac{1}{2}PL = \pi r L$

$60\pi = \pi \cdot 6 \cdot L$

$L = 10$

2(3-4-5 Δ)



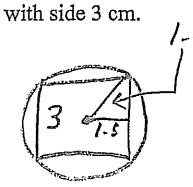
$H = 8$

$V = \frac{1}{3}B \cdot H = \frac{1}{3}\pi 6^2 \cdot 8$

$V = 12 \cdot 8\pi$

$\boxed{V = 96\pi}$ ✓

29. Find the circumference of the circle that can be circumscribed about a square with side 3 cm.



$C = 2\pi r$

$C = 2\pi(1.5\sqrt{2})$

$\boxed{C = 3\sqrt{2}\pi\text{ cm}}$ ✓

30. The lateral areas of two similar square pyramids are 20 m² and 125 m², respectively. The volume of the smaller pyramid is 8 m³. Find the volume of the larger pyramid.

RATIO OF LENGTHS = $2.5\ (\sqrt{6.25})$

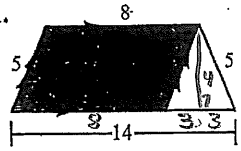
RATIO OF AREAS: $125/20 = 6.25\ (2.5^2)$

RATIO OF VOLUMES: $(2.5)^3 = 15.625$.

$V_{LARGE} = V_{SMALL}(15.625) = 8(15.625) = \boxed{125\text{ m}^3}$ ✓

Find the area of each shaded region.

31.

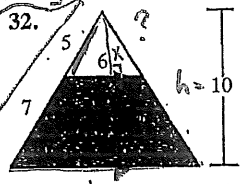


USE SMALL Δ TO FIND HEIGHT.

H = 4 (3-4-5 Δ)

$A = 8 \cdot 4 = \boxed{32}$ ✓

32.



32. TRIANGLES PROPORTIONAL

LARGE Δ BASE = b.

$\frac{5}{12} = \frac{6}{b}$

$5b = 12 \cdot 6$

$b = 12 \cdot 6/5$

$b = 72/5 = 14.4$

LARGE TRIANGLE: $A = \frac{1}{2}(b)(h)$

$A = \frac{1}{2}(\frac{72}{5})(10) = 72$.

SMALL Δ HEIGHT = x (NOT 4 SINCE NOT A 3-4-5 Δ)

$\frac{x}{6} = \frac{10}{b}$

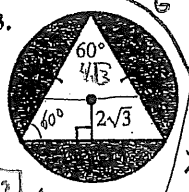
$\frac{x}{6} = \frac{10}{(72/5)}$

$x = \frac{10 \cdot 6}{(72/5)} = \left(\frac{60}{1}\right)\left(\frac{5}{72}\right) = \frac{25}{6}$

SMALL Δ: $A = \frac{1}{2}(6)(\frac{25}{6}) = 12.5$

SHADED Δ: $A = 72 - 12.5 = \boxed{59.5}$ ✓

33.



38. Find the edge of a cube with total area 150 cm².

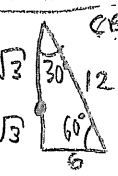
38. $A = 6s^2$

$150 = 6s^2$

$s^2 = 25$

$\boxed{s = 5\text{ cm}}$ ✓

CENTROID IS 1/3 HEIGHT FROM BASE.



$A_\Delta = \frac{1}{2}(12)(6\sqrt{3}) = 36\sqrt{3}$

$A_\odot = \pi(4\sqrt{3})^2 = 48\pi$

$\boxed{A_{SHADED} = 48\pi - 36\sqrt{3}}$ ✓