

Exploration

Euler's Formula for Polyhedrons

In this activity you will discover a relationship among the vertices, edges, and faces of a polyhedron. This relationship is called Euler's Formula for Polyhedrons, named after Leonhard Euler. Let's first build some of the polyhedrons you learned about in Lesson 10.1.



Activity

Toothpick Polyhedrons

You will need

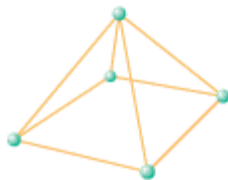
- toothpicks
- modeling clay, gumdrops, or dried peas

First, you'll model polyhedrons using toothpicks as edges and using small balls of clay, gumdrops, or dried peas as connectors.

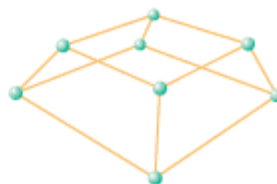
Step 1

Build and save the polyhedrons shown in parts a–d below and described in parts e–i on the top of page 531. You may have to cut or break some sticks. Share the tasks among the group.

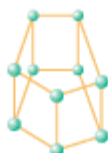
a.



b.



c.



d.



- e. Build a tetrahedron.
 f. Build an octahedron.
 g. Build a nonahedron.
 h. Build at least two different-shaped decahedrons.
 i. Build at least two different-shaped dodecahedrons.
- Step 2 Classify all the different polyhedrons your class built as prisms, pyramids, regular polyhedrons, or just polyhedrons.

Next, you'll look for a relationship among the vertices, faces, and edges of the polyhedrons.

- Step 3 Count the number of vertices (V), edges (E), and faces (F) of each polyhedron model. Copy and complete a chart like this one.

Polyhedron	Vertices (V)	Faces (F)	Edges (E)
⋮	⋮	⋮	⋮

- Step 4 Look for patterns in the table. By adding, subtracting, or multiplying V , F , and E (or a combination of two or three of these operations), you can discover a formula that is commonly known as Euler's Formula for Polyhedrons.

- Step 5 Now that you have discovered the formula relating the number of vertices, edges, and faces of a polyhedron, use it to answer each of these questions.
- Which polyhedron has 4 vertices and 6 edges? Can you build another polyhedron with a different number of faces that also has 4 vertices and 6 edges?
 - Which polyhedron has 6 vertices and 12 edges? Can you build another polyhedron with a different number of faces that also has 6 vertices and 12 edges?
 - If a solid has 8 faces and 12 vertices, how many edges will it have?
 - If a solid has 7 faces and 12 edges, how many vertices will it have?
 - If a solid has 6 faces, what are all the possible combinations of vertices and edges it can have?