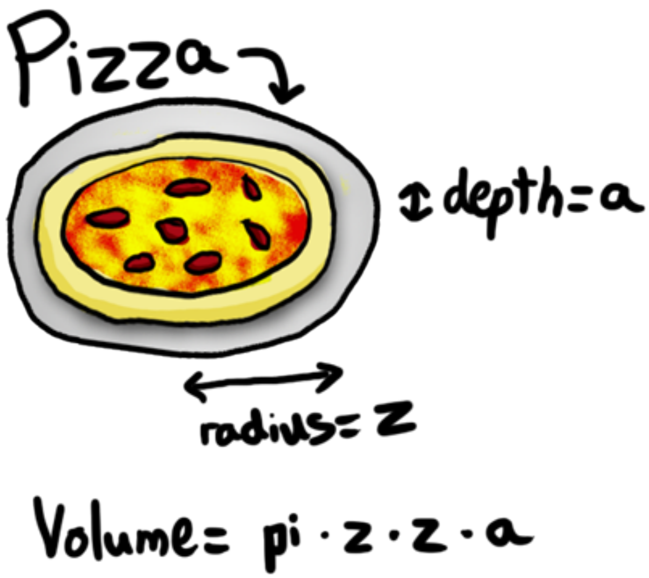
Full Name: ______________________________ Block/Period: ____
v. 1.0

AreaVolumePacket

Chapters 11 and 12

Area, Volume, and Solids



Spherical cow is a metaphor for highly simplified scientific models of complex real life phenomena.

When Radius of Circumscribed Sphere = 1.

Solid.	Side.	Radius inscribed.	Superficies.	Volume.
Tetrahedron .	1·6329932	0·3333333	4·6188023	0·5132002
Hexahedron .	1·1547005	0·5773503	8·0000000	1·5396006
Octohedron .	1·4142136	0·5773503	6·9282032	1·3333333
Dodecahedron	0·7136442	0·7946545	10·5146223	2·7851639
Icosahedron .	1·0514622	0·7946545	9·5745413	2·5361507

FORMULAS FROM GEOMETRY

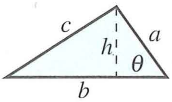
Triangle:

$h = a \sin \theta$

Area $= \frac{1}{2}bh$

(Laws of Cosines)

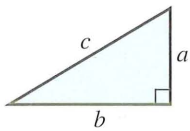
$c^2 = a^2 + b^2 - 2ab \cos \theta$



Right Triangle:

(Pythagorean Theorem)

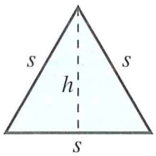
$c^2 = a^2 + b^2$



Equilateral Triangle:

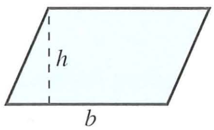
$h = \frac{\sqrt{3}s}{2}$

Area $= \frac{\sqrt{3}s^2}{4}$



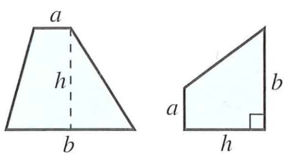
Parallelogram:

Area $= bh$



Trapezoid:

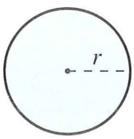
Area $= \frac{h}{2}(a + b)$



Circle:

Area $= \pi r^2$

Circumference $= 2\pi r$

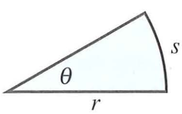


Sector of Circle:

(θ in radians)

Area $= \frac{\theta r^2}{2}$

$s = r\theta$



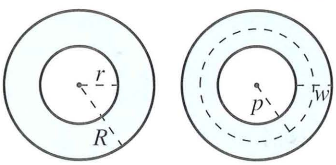
Circular Ring:

(p = average radius,

w = width of ring)

Area $= \pi(R^2 - r^2)$

$= 2\pi p w$



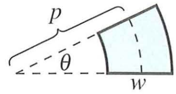
Sector of Circular Ring:

(p = average radius,

w = width of ring,

θ in radians)

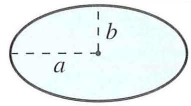
Area $= \theta p w$



Ellipse:

Area $= \pi ab$

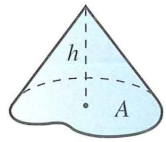
Circumference $\approx 2\pi \sqrt{\frac{a^2 + b^2}{2}}$



Cone:

(A = area of base)

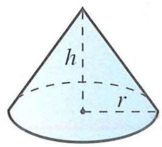
Volume $= \frac{Ah}{3}$



Right Circular Cone:

Volume $= \frac{\pi r^2 h}{3}$

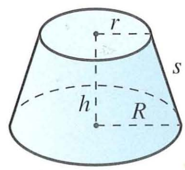
Lateral Surface Area $= \pi r \sqrt{r^2 + h^2}$



Frustum of Right Circular Cone:

Volume $= \frac{\pi(r^2 + rR + R^2)h}{3}$

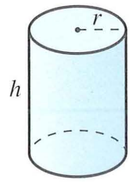
Lateral Surface Area $= \pi s(R + r)$



Right Circular Cylinder:

Volume $= \pi r^2 h$

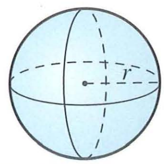
Lateral Surface Area $= 2\pi r h$



Sphere:

Volume $= \frac{4}{3}\pi r^3$

Surface Area $= 4\pi r^2$

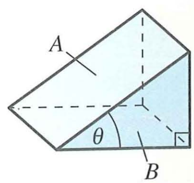


Wedge:

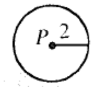
(A = area of upper face,

B = area of base)

$A = B \sec \theta$

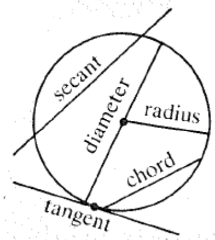


9–1 Basic Terms for Circles and Spheres



circle The set of all points in a plane at a given distance from a given point is a circle. ⊙P is the set of all points in a plane that are 2 units from P. The given point P is the center of the circle.

radius The given distance is the radius of the circle. A radius is also any segment joining the center of the circle to a point of the circle. (The plural of radius is radii.)



chord A segment whose endpoints lie on a circle is a chord.

diameter A chord that contains the center of a circle is a diameter. A diameter is also a length equal to twice a radius.

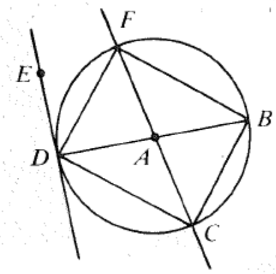
secant A line that contains a chord of a circle is a secant.

tangent A line in the plane of a circle that intersects the circle in exactly one point is a tangent. The point of tangency is the point of intersection.

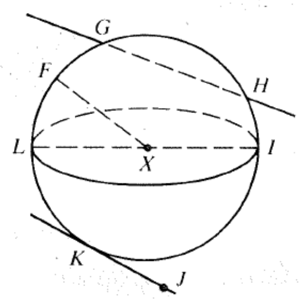
In ⊙A, name:

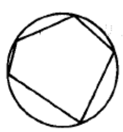
1. the center
2. two diameters
3. a point of tangency
4. four radii
5. a tangent
6. a secant
7. six chords

8. Why is $\overline{AC}$ not a chord of ⊙A?
9. Why is $\overleftrightarrow{BD}$ not a chord of ⊙A?



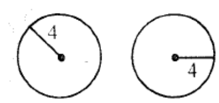
sphere The set of all points in space at a given distance from a given point is a sphere.
Many of the terms used with circles are also used with spheres.
For example, sphere X has
center: X
radii: $\overline{XL}, \overline{XF}, \overline{XI}$
chords: $\overline{GH}, \overline{LI}$
diameter: $\overline{LI}$
secants: $\overleftrightarrow{GH}, \overleftrightarrow{LI}$
tangent: $\overleftrightarrow{KJ}$
point of tangency: K

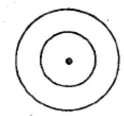




inscribed polygon A polygon is inscribed in a circle if each vertex of the polygon lies on the circle.

circumscribed circle A circle is circumscribed about a polygon if each vertex of the polygon lies on the circle.





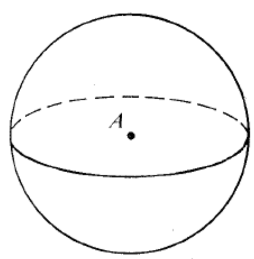
concentric circles Circles that lie in the same plane and have the same center are concentric.

concentric spheres Concentric spheres have the same center.

congruent circles / **congruent spheres** Circles (or spheres) are congruent if they have congruent radii.

In sphere A, draw:

10. a diameter, $\overline{BC}$
11. a chord, $\overline{DE}$
12. a tangent, $\overleftrightarrow{CF}$
13. a secant, $\overleftrightarrow{DG}$



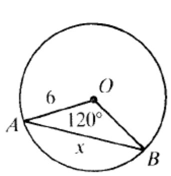
14. Draw two concentric circles. Draw a tangent to one of the circles. Is it tangent to the other circle?

15. Draw a large circle. Inscribe an isosceles triangle in the circle.

16. Draw a rectangle. Circumscribe a circle about the rectangle. (*Hint:* Draw the diagonals to find the center.)

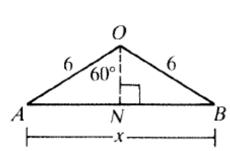
Example

Find the value of x.



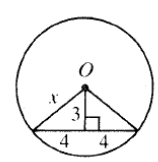
Solution

$OA = OB = 6$
Draw $\overline{ON} \perp \overline{AB}$. $\overline{ON}$ bisects $\overline{AB}$; $\overrightarrow{ON}$ bisects $\angle AOB$. Using the properties of 30°-60°-90° triangles, $ON = 3$ and $AN = 3\sqrt{3}$, so $x = 2 \cdot 3\sqrt{3} = 6\sqrt{3}$.

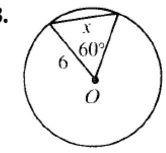


Find the value of x. O is the center of each circle.

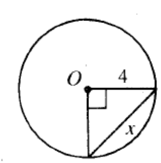
17.



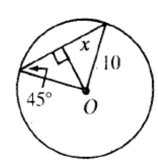
18.



19.

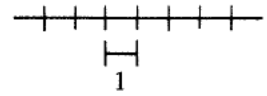


20.

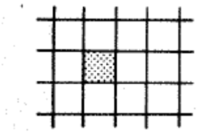


11–1 Areas of Rectangles

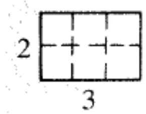
Postulate 17: **The area of a square** is the square of the length of a side. $A = s^2$.
Postulate 18: **Area Congruence.** If two figures are congruent, then they have the same area.
Postulate 19: **Area Addition.** The area of a region is the sum of its non-overlapping parts.
Theorem 1–1: **The area of a rectangle** = base x height. $A = bh$.



length: 1 unit



Area: 1 square unit



$A = 6$ square units

Example 1

a. Find the area of a rectangle with base 5 cm and height 4 cm.

b. The area of a rectangle is 36 cm^2. If its height is 9 cm, find the base.

Solution

a. $A = bh$
$A = 5 \cdot 4 = 20$ The area is 20 cm^2.

b. $A = bh$
$36 = 9 \cdot b$
$4 = b$ The base is 4 cm.

Exercises 1–8 refer to rectangles. Complete the table.

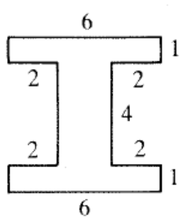
	1.	**2.**	**3.**	**4.**	**5.**
b	8 cm	15 cm	?	12 ft	$5\sqrt{3}$
h	7 cm	?	24 cm	?	$4\sqrt{3}$
A	?	60 cm^2	120 cm^2	144 ft^2	?

The perimeter of a polygon is the sum of the lengths of its sides.

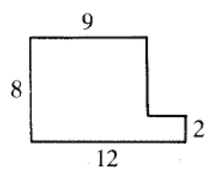
9. Find the side of a square if its area is 64 cm^2.
10. Find the perimeter of a square if its area is 144 cm^2.
11. Find the area of a square if its perimeter is 40 cm.
12. Find the area of a rectangle if its perimeter is 30 and its base is 8.

Consecutive sides of the figures are perpendicular. Find the area of each figure.

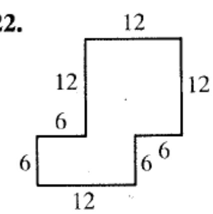
20.



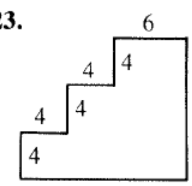
21.



22.



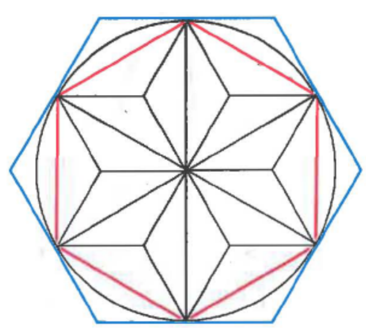
23.



Circle Area. All of the small triangles in the figure below are congruent.†

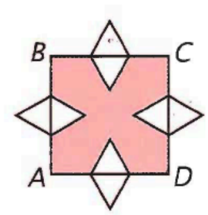
Given that the area of the red hexagon is 3 square units, find

35. the area of one of the small triangles.
36. the area of one of the equilateral triangles.



37. the area of the star.
38. the area of the blue hexagon.
39. Use your results to guess the approximate area of the circle.

SAT Problem. The figure below appeared in a problem on an SAT exam.

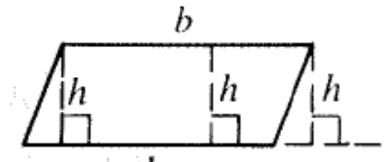


All of the triangles are congruent, the area of the shaded region is 84, and the area of square ABCD is 100.

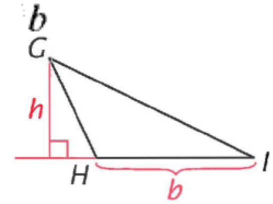
44. What is the area of one of the triangles?
45. What is the total area of the entire figure?

† *Wheels, Life, and Other Mathematical Amusements,* by Martin Gardner (W. H. Freeman and Company, 1983).

11–2 Areas of Parallelograms, Triangles, and Rhombuses

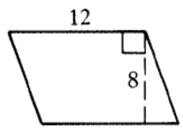


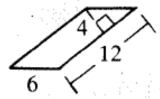
Any side of a parallelogram (or triangle) may be considered a base, *b*. An altitude is any segment perpendicular to the line containing a base from any point on the opposite side (or in a triangle, the opposite vertex). The length of an altitude is called the height, *h,* of the parallelogram (or triangle).

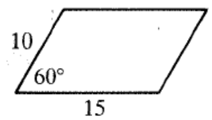


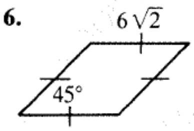
The area of a parallelogram equals the product of a base and the height to that base. $A = bh$.
The area of a triangle equals half the product of a base and the height to that base. $A = 1/2\ bh$.

Find the area of each parallelogram.

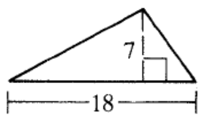
3.


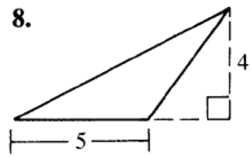
4.


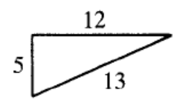
5.


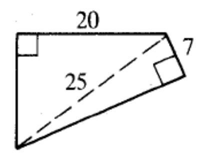
6.


Find the area of each figure.

7.


8.


9.


10.


11. Find the area of an isosceles triangle with sides 30, 30, and 24.

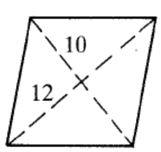
12. Find the area of an isosceles triangle with base 16 and perimeter 52.

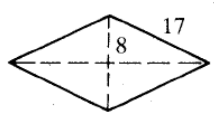
13. Find the area of an equilateral triangle with sides 12 cm.

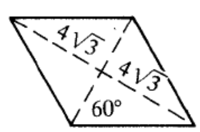
14. Find the area of an equilateral triangle with height $6\sqrt{3}$.

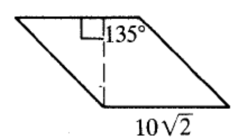
The area of a rhombus equals half the product of its diagonals. $A = \frac{1}{2}d_1d_2$. The parallelogram formula works, too.

Find the area of each rhombus.

15.


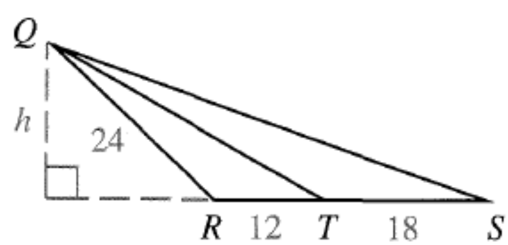
16.


17.


18.


21. Find the area of a rhombus with perimeter 100 and one diagonal 14.

29. a. Find the ratio of the areas of $\triangle QRT$ and $\triangle QTS$.
b. If the area of $\triangle QRS$ is 240, find the length of the altitude from S to $\overleftrightarrow{QR}$.



Heron's Theorem. Heron, a Greek mathematician who lived in Alexandria in the first century A.D., derived a formula for the area of a triangle in terms of the lengths of its sides.

His formula, sometimes called Heron's Theorem, says that the area of a triangle with sides a, b, and c is $\sqrt{s(s-a)(s-b)(s-c)}$, where s is half of the triangle's perimeter. Before you try the formula out, suppose that there are three triangles with the following sides.

Triangle 1: 5, 5, and 6.
Triangle 2: 5, 5, and 8.
Triangle 3: 5, 5, and 10.

The semiperimeter $s = 1/2\ (a + b + c)$.

1. Which triangle do you think has the greatest area?
2. Use Heron's Theorem to find the area of each triangle.
3. One of the "triangles" isn't really a triangle. Which one and why not?

Now try this. Suppose there are two triangles with the following sides.[†]

Triangle 4: 4, 6, and 8.
Triangle 5: 400, 600, and 1000.

4. Which do you think has the greater area?
5. Use Heron's Theorem to find it.

[†] *Riddles of the Sphinx*, by Martin Gardner (Mathematical Association of America, 1987).

Triangle Area from x-y Coordinates.

The area of a triangle with vertices at points (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

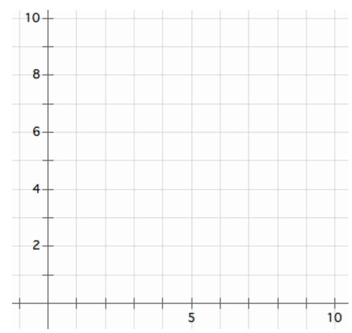
$$A = \pm\frac{1}{2}(x_2\,y_3 - x_3\,y_2 - x_1\,y_3 + x_3\,y_1 + x_1\,y_2 - x_2\,y_1).$$

$$\text{Area} = \pm 1/2 \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

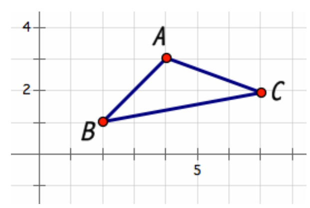
It must be positive.

Use this formula to find the area of the triangles with the given coordinates.

a. (0, 0), (10, 0), and (3, 10). Make a sketch on the left. How does the area make sense?



b. (2, 1), (4, 3), and (7, 2). Could you use $A = 1/2\ bh$? Why is this a challenge?



Wait, there is more!

The area Δ (sometimes also denoted σ) of a triangle ΔABC with side lengths a, b, c and corresponding angles A, B, and C is given by

$$\Delta = \frac{1}{2} b\,c \sin A \quad (1)$$
$$= \frac{1}{2} c\,a \sin B \quad (2)$$
$$= \frac{1}{2} a\,b \sin C \quad (3)$$
$$= \frac{1}{4}\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)} \quad (4)$$
$$= \frac{1}{4}\sqrt{2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4} \quad (5)$$
$$= \frac{a\,b\,c}{4R} \quad (6)$$
$$= r\,s, \quad (7)$$

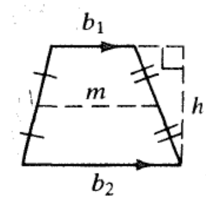
where R is the circumradius, r is the inradius, and $s = (a+b+c)/2$ is the semiperimeter (Kimberling 1998, p. 35; Trott 2004, p. 65).

and that's just the beginning of what you'll find at Wolfram MathWorld, http://mathworld.wolfram.com/TriangleArea.html.

11–3 Areas of Trapezoids

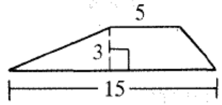
$A = 1/2\ h\ (b_1 + b_2)$.

$A = h\ m$



The area of a trapezoid equals half the product of the height and the sum of the bases.
Alternate forms are A = **height x average of the bases.** ***A* = height x median.**

Example 1. Find the area of this trapezoid.

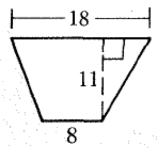


$$A = \tfrac{1}{2}h(b_1 + b_2) = \tfrac{1}{2} \cdot 3 \cdot (5 + 15) = 30$$

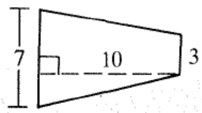
Set A.

Find the area of each trapezoid.

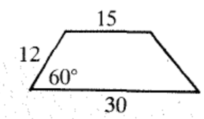
1.



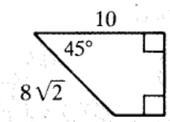
2.



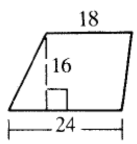
3.



4.



Example 2. Find the median and area.



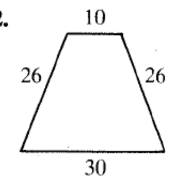
$$m = \tfrac{1}{2}(b_1 + b_2) = \tfrac{1}{2}(18 + 24) = 21$$
$$A = hm = (16)(21) = 336$$

Complete the table with the trapezoid measurements.

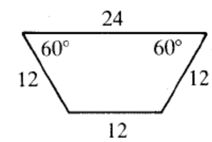
	5.	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**
b_1	15	25	8	42	?	?	$14x$
b_2	13	10	?	14	9	8	?
h	5	?	4	?	5	$6\sqrt{3}$	$7x$
A	?	140	46	336	?	$33\sqrt{3}$	$70x^2$
m	?	?	?	?	7	?	?

Find the area of each isosceles trapezoid.

12.

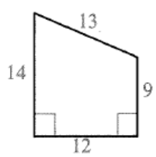


13.

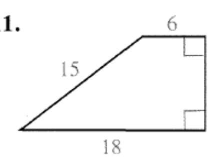


Set B. Find the areas.

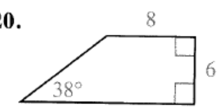
3.



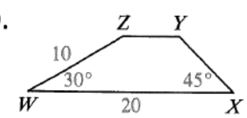
11.



20.



29.



Exact square roots, not decimal numbers

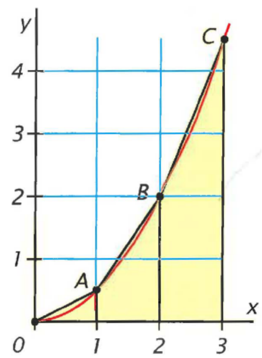
The curve below, called a parabola, has the equation $y = \frac{x^2}{2}$.

The x-coordinate of point A is 1; so its y-coordinate is $\frac{1^2}{2} = \frac{1}{2}$.

47. What are the coordinates of points B and C?

There are three trapezoids in the figure.

48. Add up their areas to find the area of the yellow region.

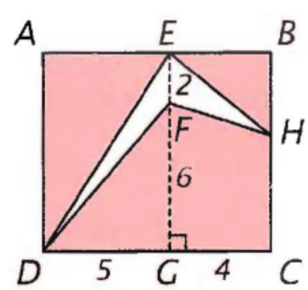


Your answer is the approximate area of the region between the parabola and the x-axis from 0 to 3.

49. Is your answer larger or smaller than the area under the parabola?

49 b. The answer from calculus is $A = x^3/6$ for any x. What is the answer for $x = 3$?

52. Find the area of the shaded region.



11–4 & 11–5 Areas of Regular Polygons and Circles

$A = 1/2\ ap$
$p = ns$

The area of a regular polygon equals half the product of the apothem and the perimeter. The **apothem**, a, is the distance from the center to a side.

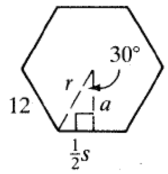
The **perimeter** $p = ns$, where n is the number of sides, s.
The **radius**, r, is the distance from the center to a vertex.

The right triangle above has hypotenuse r, legs $s/2$ and a, and the angle adjacent to a is $180°/n$. Why?

Find the perimeter and the area of each regular polygon described.

1. A regular polygon with apothem 98 and side 39. Challenge: what is n?

2. Find the exact area of the regular hexagon.

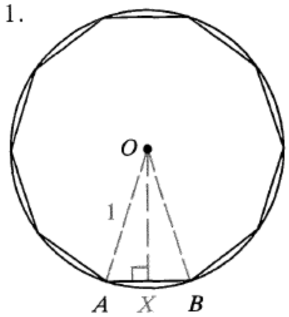


23. What is the perimeter and area of a $r = 1$
a) 180-gon?
$s = 2\sin(180°/n) = ?$
$p =$
$a = ?$
$A =$

b) 18,000-gon?

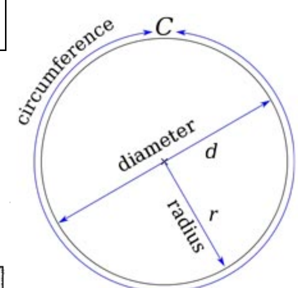
c) What do you see?

17. A regular decagon is shown inscribed in a circle with radius 1.
a. Explain why $m\angle AOX = 18$.
b. Use a calculator or the table on page 311 to evaluate OX and AX below.
$\sin 18° = \frac{AX}{1}$, so $AX \approx$ __?__.
$\cos 18° = \frac{?}{?}$, so $OX \approx$ __?__.
c. Perimeter of decagon $\approx$ __?__
d. Area of $\triangle AOB \approx$ __?__
e. Area of decagon $\approx$ __?__



Circles
A circle can be thought of as a regular polygon with many many sides.
As n approaches infinity, p approaches the **circumference**, C, and a approaches r.
The circumference is proportional to the radius and the proportionality constant is defined to be 2π.
The area $A = 1/2\ ap$ approaches $1/2\ rc = 1/2\ r\ (2\pi r)$. **The area of a circle** is π times radius-squared.

$C = 2\pi r$
$A = \pi r^2$



Leave answers in terms of π, unless asked for a decimal. Don't use 3.14 anymore.

Complete the table.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**
Radius	5	8	$3\sqrt{2}$	?	?	?	?	?
Circumference	?	?	?	12π	100π	?	?	?
Area	?	?	?	?	?	16π	121π	64π

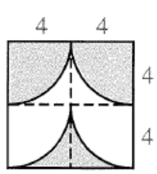
3.14159265358979323846264338
327950288419716939937510582
097494459230781640628620899
862803482534211706798214808
651328230664709384460955058
223172535940812848111745028
410270193852110555964462294
895493038196442881097566593

15. The diameter ($d = 2r$) of a bicycle wheel is 28 inches.
a. How far does the bicycle travel in inches with each revolution of the wheel?

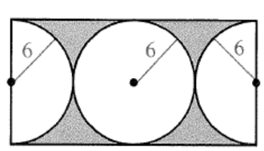
b. How many revolutions will it take to go 1 mile (63,360 in.)?

Find the shaded areas in terms π.

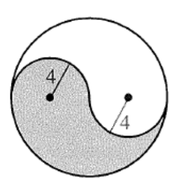
24.



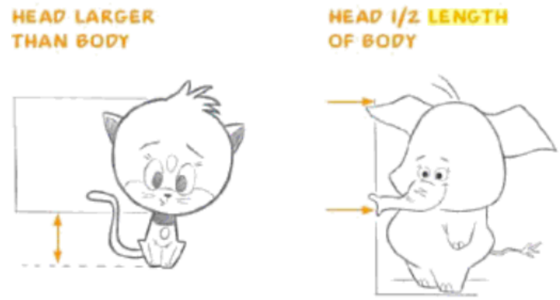
26.



27.



29. Draw a square and its inscribed and circumscribed circles. Find the ratio of the areas of these two circles.



11–7 Ratios of Areas

If the scale factor of two similar figures is $a:b$, then
(1) the ratio of the perimeters is $a:b$.
(2) the ratio of the areas is $a^2:b^2$.

Example 2

a. The scale factor of two similar figures is 3:5. Find the ratio of the perimeters and the ratio of the areas.

b. The ratio of the areas of two similar figures is 1:4. Find the ratio of their perimeters.

Solution

a. $a:b = 3:5$
ratio of perimeters $= a:b = 3:5$
ratio of areas $= a^2:b^2 = 9:25$

b. ratio of areas $= a^2:b^2 = 1:4$,
so $a:b = 1:2$.
ratio of perimeters $= a:b = 1:2$

Set A.

The table refers to similar figures. Complete the table.

	6.	7.	8.	9.	10.	11.	12.
Scale factor	3:4	5:2	$x:7y$	?	?	?	?
Ratio of perimeters	?	?	?	8:7	2:13	?	?
Ratio of areas	?	?	?	?	?	36:1	4:9

17. A hexagon with sides 5 cm, 6 cm, 7 cm, 8 cm, 9 cm, and 10 cm has area 123 cm^2. If a similar hexagon has longest side of 40 cm, what is its area?

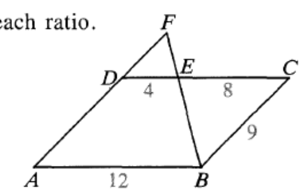
☺

23. *ABCD* is a parallelogram. Find each ratio.

a. $\frac{\text{Area of } \triangle DEF}{\text{Area of } \triangle ABF}$

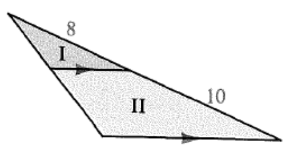
b. $\frac{\text{Area of } \triangle DEF}{\text{Area of } \triangle CEB}$

c. $\frac{\text{Area of } \triangle DEF}{\text{Area of trap. } DEBA}$

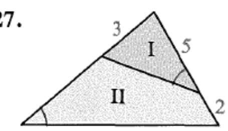


Find the ratio of the areas of regions I and II.

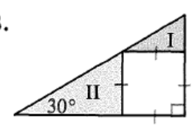
26.



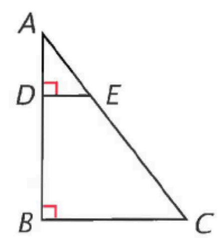
27.



28.



SAT Problem. The figure at the right appeared in a problem on an SAT exam. It was given that the area of $\triangle ABC = 54$ and that $AD = \frac{1}{3}AB$.



The answers to some SAT problems cannot be determined from the information given.

32. Can any conclusion be drawn about $\triangle ADE$ and $\triangle ABC$? Explain.

33. Is it correct to assume that $DE = \frac{1}{3}BC$? Why or why not?

34. Is it possible to figure out the lengths of AB and BC?

35. Is it possible to figure out the lengths of AD and DE?

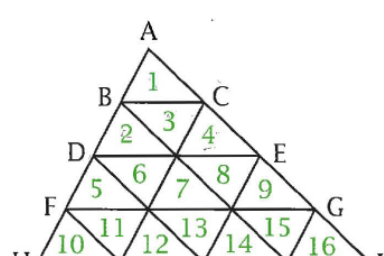
36. Is it possible to figure out the area of $\triangle ADE$? Explain.

Set B.

Triangle Ratios. The figure at the right from an old geometry book shows a triangle divided into 16 congruent triangles.*

Find each of the following ratios.

1. $\frac{BC}{DE}$.
2. $\frac{\alpha\triangle ABC}{\alpha\triangle ADE}$.
3. $\frac{BC}{FG}$.
4. $\frac{\alpha\triangle ABC}{\alpha\triangle AFG}$.
5. $\frac{DE}{FG}$.
6. $\frac{\alpha\triangle ADE}{\alpha\triangle AFG}$.
7. $\frac{DE}{HI}$.
8. $\frac{\alpha\triangle ADE}{\alpha\triangle AHI}$.
9. State the theorem illustrated by exercises 1 through 8.



***First Steps in Geometry*, by G. A. Wentworth and G. A. Hill (Ginn, 1901).

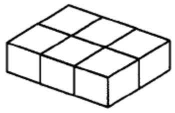
Sheet 1201: Nothing But Net Name: ______________________________

Questions 1–8 by T.S. Denison

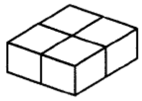
A ***net*** is a pattern that can be folded to cover a solid figure. The area of the unfolded net equals the surface area of the solid figure.

Match the net with its solid.

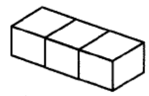
1. _____



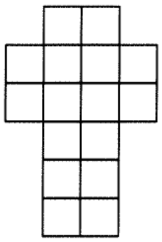
2. _____



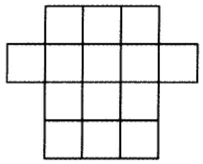
3. _____



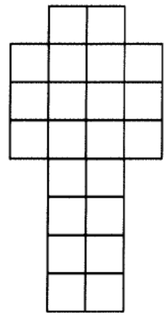
A.



B.



C.



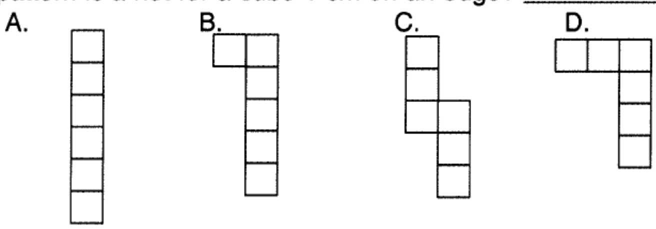
Volume = side^3 for a cube.

11 – 18. Given each cube represents 1 cm^3, find the volumes for solids 1–8. Write the answers next to each solid, *V* = ___ cm^3.

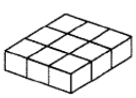
If each square represents 1 cm^2, find the surface area of each solid.

4. Area #1 = _____ cm^2 5. Area #2 = _____ cm^2 6. Area #3 = _____ cm^2

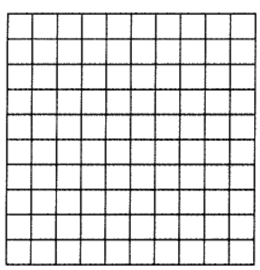
7. Which pattern is a net for a cube 1 cm on an edge? _____________



8. On the grid draw a net for the rectangular prism shown and calculate its surface area.



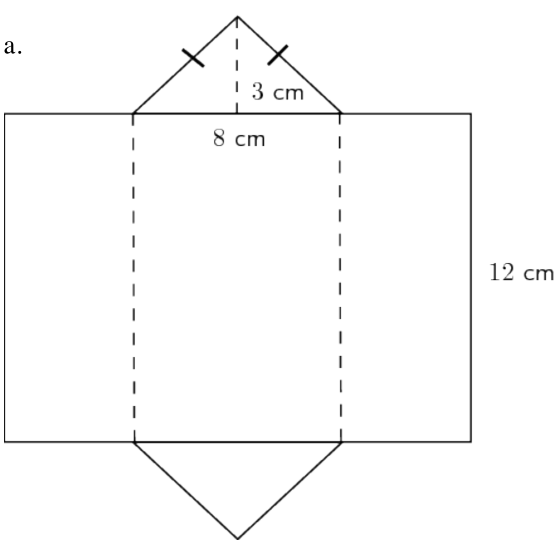
Net:



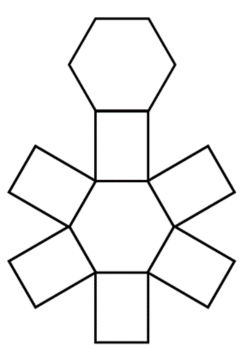
Area:

9. Find the surface area. Name the shape.

a.



b.

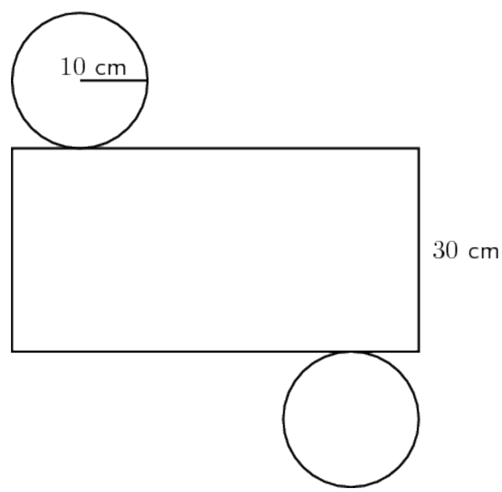


Rectangle Sides = 2 cm and 3cm

Volume = base area x height for a prism

19 a, b, c. Find the volumes for solids 9 a, b, c. Write the answer next to each solid, *V* = ___ cm^3.

c.



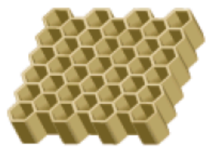
12–1 & 12–2 Prisms and Pyramids

A solid formed by polygons that enclose a single region of space is called a **polyhedron**. A **prism** is a polyhedron with two parallel congruent polygon **bases**.

Set A. Match the names to the shapes.

10. Tomb of Egyptian rulers

11. Honeycomb



12. Die

13. Stop sign

14. Holder for a scoop of ice cream

16. Moon

18. Box of breakfast cereal

20. Plastic bowl with lid

15. Wedge or doorstop

17. Can of tuna fish

19. Book

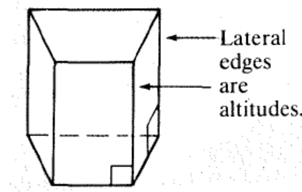
21. Pup tent

A. Cylinder
B. Cone
C. Square prism
D. Square pyramid
E. Sphere
F. Triangular pyramid
G. Octagonal prism
H. Triangular prism
I. Trapezoidal prism
J. Rectangular prism
K. Heptagonal pyramid
L. Hexagonal prism
M. Hemisphere

22. Ingot of silver

UNITS. Length (m), Area is squared (m^2), Volume is cubed (m^3). Know this!

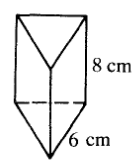
The faces that are not bases are **lateral faces**. Lateral faces are parallelograms that intersect each other in parallel segments called **lateral edges**.



Right trapezoidal prism

The lateral area of a right prism equals the perimeter of a base times the height of the prism. L.A. = ph
The total area of a right prism equals the lateral area plus the areas of both bases. T.A. = L.A. + $2B$
The volume of a right prism equals the area of a base times the height of the prism. $V = Bh$

Example 2. **Find the total area and volume of the right triangular prism with equilateral triangle bases.**



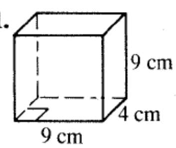
Solution

T.A. = L.A. + $2B$
$p = 3 \cdot 6 = 18; h = 8$
L.A. = $ph = 18 \cdot 8 = 144$
$B = \frac{1}{2}bh = \frac{1}{2} \cdot 6 \cdot 3\sqrt{3} = 9\sqrt{3}$
T.A. = L.A. + $2B = 144 + 18\sqrt{3}$
The total area is $(144 + 18\sqrt{3})$ cm^2.

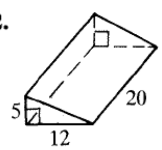
$V = Bh$ and $B = 9\sqrt{3}$ (from Example 2), so
$V = 9\sqrt{3} \cdot 8 = 72\sqrt{3}$. The volume is $72\sqrt{3}$ cm^3.

Set B. Find the (a) lateral area (b) total area and (c) volume of each right prism.

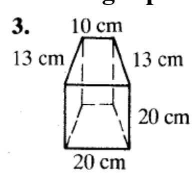
1.



2.



3.



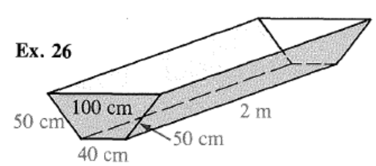
☺

15. If the edge of a cube is doubled, the total area is multiplied by __?__ and the volume is multiplied by __?__.

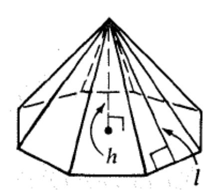
16. If the length, width, and height of a rectangular solid are all tripled, the lateral area is multiplied by __?__, the total area is multiplied by __?__, and the volume is multiplied by __?__.

26. A drinking trough for horses is a right trapezoidal prism with dimensions shown below. If it is filled with water, about how much will the water weigh? (*Hint*: 1 m^3 of water weighs 1 metric ton.)

Ex. 26

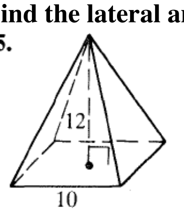


Pyramids have only one base. The **lateral faces** of a pyramid are triangles. The segment from the vertex perpendicular to the plane of the base is the **altitude** of the pyramid.
A **regular pyramid** has a regular polygon as its base. The **slant height**, l, is shown to the right.
For regular pyramids, $L.A. = \frac{1}{2}pl$ and $V = \frac{1}{3}Bh$ where p = perimeter and B = base area.



Find the lateral area, total area, and volume of the regular square pyramid.

5.



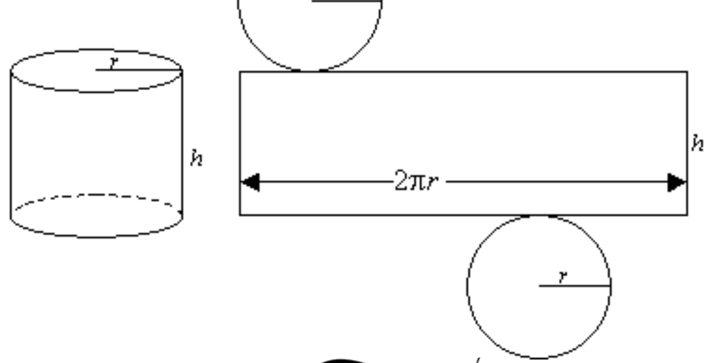
9. A regular square pyramid with base edge 24 and lateral edge 24.

10. A regular square pyramid with height 8 and slant height 17.

12–3 Cylinders and Cones

Remember that the circumference of a circle is $2\pi r$ and the area of a circle is πr^2.

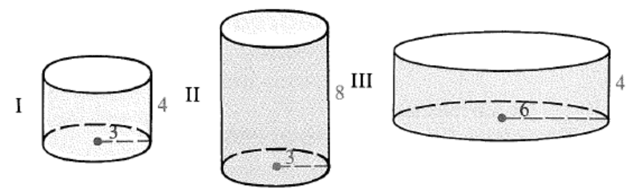
Right cylinder



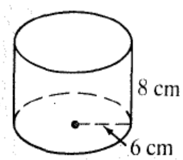
The total area of a cylinder is the lateral area plus twice the area of a base. $T.A. = L.A. + 2B = 2\pi rh + 2\pi r^2$
The volume of a cylinder equals the area of a base times the height of the cylinder. $V = Bh = \pi r^2 h$

Set A.

2. a. Find the lateral areas of cylinders I, II, and III.
   b. Notice that the height of II is twice the height of I. Is the lateral area of II twice the lateral area of I?
   c. Notice that the radius of III is twice the radius of I. Is the lateral area of III twice the lateral area of I?
3. a. Find the total areas of cylinders I, II, and III.
   b. Are the ratios of the total areas the same as those of the lateral areas in Exercise 2?
4. a. Find the volumes of cylinders I, II, and III.
   b. Notice that the height of II is twice the height of I. Is the volume of II twice the volume of I?
   c. Notice that the radius of III is twice the radius of I. Is the volume of III twice the volume of I?



Example 1
Find the lateral area, total area, and volume of the cylinder.



Solution

$L.A. = 2\pi rh = 2\pi \cdot 6 \cdot 8 = 96\pi \text{ (cm}^2)$

$T.A. = L.A. + 2B = 96\pi + 2(\pi \cdot 6^2) = 96\pi + 72\pi = 168\pi \text{ (cm}^2)$

$V = \pi r^2 h = \pi \cdot 6^2 \cdot 8 = 288\pi \text{ (cm}^3)$

Set B.

Find the (a) lateral area, (b) total area, and (c) volume of each cylinder.

1. $r = 5; h = 8$
2. $r = 6; h = 9$
3. $r = 5; h = 4$

5. The lateral area of a cylinder is 96π. If $h = 12$, find r.
6. The volume of a cylinder is 375π. If $h = 15$, find the lateral area.

7. The lateral area of a cylinder is 96π. If $r = 8$, find the volume.
8. The total area of a cylinder is 256π cm^2. If $r = h$, find r.

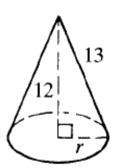
Cones

Use the Pythagorean Theorem to relate dimensions: $h^2 + r^2 = l^2$.

The total area of a cone equals the lateral area plus the area of the base. $T.A. = L.A. + B = \pi rl + \pi r^2$

The volume of a cone equals one third the area of the base times the height of the cone. $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$

Example 2
Find the lateral area, total area, and volume of the cone.



Solution
Use the Pythagorean Theorem to find r.

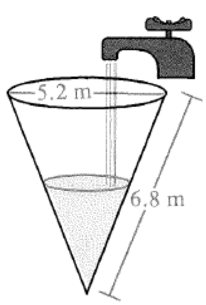
$h^2 + r^2 = l^2$
$12^2 + r^2 = 13^2$
$r = 5$

$L.A. = \pi rl = \pi \cdot 5 \cdot 13 = 65\pi$
$T.A. = L.A. + B = 65\pi + \pi(5^2) = 90\pi$
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(5^2)(12) = 100\pi$

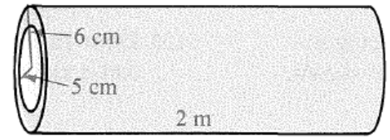
Complete the table for the cone shown.

	r	h	l	L.A.	T.A.	V
9.	3	?	5	?	?	?
10.	10	24	?	?	?	?
11.	8	?	?	136π	?	?
12.	7	?	?	?	?	392π

☺
23. Water is pouring into a conical reservoir at the rate of 1.8 m^3 per minute. Find, to the nearest minute, the number of minutes it will take to fill the reservoir. (The figure reads 5.2m and 6.8m.)



22. A pipe is 2 m long and has inside radius 5 cm and outside radius 6 cm. Find the volume of metal contained in the pipe to the nearest cubic centimeter.



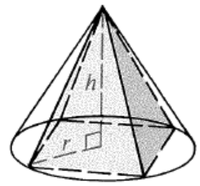
Ex. 22

33. A regular square pyramid with base edge 4 cm is inscribed in a cone with height 6 cm. What is the volume of the cone?

34. A regular square pyramid is inscribed in a cone with radius 4 cm and height 4 cm.

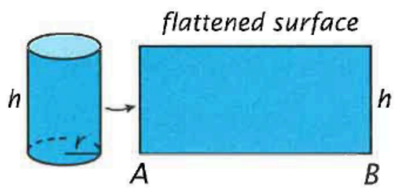
a. What is the volume of the pyramid?

b. Find the slant heights of the cone and the pyramid.



Exs. 33, 34

Cylinder and Cone Areas. The lateral area of a right cylinder or cone is the area of its curved surface.

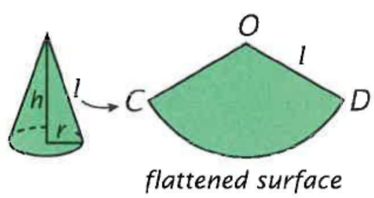


50. What does the curved surface of a right cylinder look like if the cylinder is slit down one side and flattened out?

51. Write an expression for AB in terms of r.

52. Write an expression for the lateral area of a right cylinder in terms of r and h.

53. What does the expression $2\pi r(r + h)$ represent for a right cylinder?



54. What does the curved surface of a right cone look like if the cone is flattened out?

55. Write an expression for the length of arc CD in terms of r.

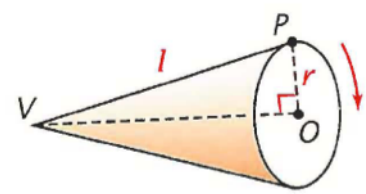
56. What fraction of the area of circle O is the lateral area? Express your answer in terms of r and l.

57. Write an expression for the lateral area of a right cone in terms of r and l.

58. Write an expression for the lateral area in terms of r and h.

59. What does the expression $\pi r(r + l)$ represent for a right cone?

Rolling Cones. A right circular cone rolls on a flat horizontal surface.*



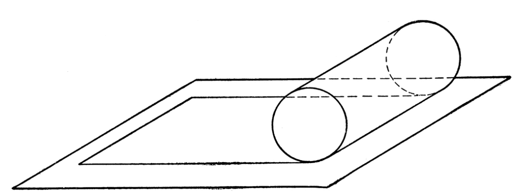
63. Describe what happens to the cone.

Suppose the cone returns to the same spot after rotating twice about its axis.

64. What can you conclude about $\angle PVO$? Explain.

* *Mathematics Meets Technology*, by Brian Bolt (Cambridge University Press, 1991).

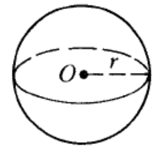
Below: cool drawing with nowhere else to go.



$1^3 = 1$ $2^3 = 8$ $3^3 = 27$ $4^3 = 64$ $5^3 = 125$ $6^3 = 216$ $7^3 = 343$

$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$ $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$ $8^3 = 512$ $9^3 = 729$ $10^3 = 1000$

12–4 Spheres



The area of a sphere equals 4π times the square of the radius. $A = 4\pi r^2$

The volume of a sphere equals $\frac{4}{3}\pi$ times the cube of the radius. $V = \frac{4}{3}\pi r^3$

Example 1

a. Find the area and volume of a sphere with radius 5.

Solution

a. $A = 4\pi r^2$
$= 4\pi \cdot 5^2 = 4\pi \cdot 25 = 100\pi$

$V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi \cdot 5^3 = \frac{4}{3}\pi \cdot 125 = \frac{500}{3}\pi$

b. The volume of a sphere is 972π. Find its area.

b. $V = \frac{4}{3}\pi r^3$ $972\pi = \frac{4}{3}\pi r^3$

$\left(\frac{3}{4}\right)(972) = r^3$

$729 = r^3$

$9 = r$

$A = 4\pi r^2 = 4\pi \cdot 9^2 = 324\pi$

Complete the table for spheres.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
radius	3	6	2	$\frac{1}{3}$	$\sqrt{3}$	?	?
area	?	?	?	?	?	576π	?
volume	?	?	?	?	?	?	$\frac{1372}{3}\pi$

B10. Jupiter has a radius of 69,911 km. Earth has a radius of 6,371 km. What is the ratio of volumes?

10. Refer to Exercises 2 and 3 above. If the radius of a sphere is multiplied by 3, then the area is multiplied by __?__ and the volume is multiplied by __?__.

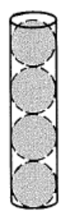
18. A scoop of ice cream with radius 4 cm is placed on an ice-cream cone with radius 3 cm and height 15 cm. Is the cone big enough to hold the ice cream if it melts?

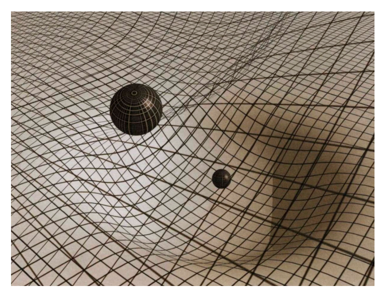
Spheres in different dimensions

Dimensions	*Volume*	*Dimensions*	*Volume*
1	$2r$	5	$\frac{8}{15}\pi^2 r^5$
2	πr^2		
3	$\frac{4}{3}\pi r^3$	6	$\frac{1}{6}\pi^3 r^6$
4	$\frac{1}{2}\pi^2 r^4$	7	$\frac{16}{105}\pi^3 r^7$

☺

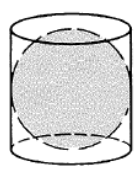
24. Four solid metal balls fit snugly inside a cylindrical can. A geometry student claims that two extra balls of the same size can be put into the can, provided all six balls can be melted and the molten liquid poured into the can. Is the student correct? (*Hint*: Let the radius of each ball be r.)





25. A sphere with radius r is inscribed in a cylinder. Find the volume of the cylinder in terms of r.

26. A sphere is inscribed in a cylinder. Show that the area of the sphere equals the lateral area of the cylinder.



28. A hollow rubber ball has outer radius 11 cm and inner radius 10 cm.

a. Find the exact volume of the rubber. Then evaluate the volume to the nearest cubic centimeter.

b. The volume of the rubber can be approximated by the formula:

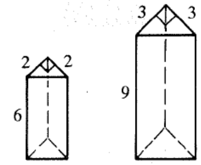
$V \approx$ inner surface area $\cdot$ thickness of rubber

Use this formula to approximate V. Compare your answer with the answer in part (a).

c. Is the approximation method used in part (b) better for a ball with a thick layer of rubber or a ball with a thin layer?

12–5 Areas and Volumes of Similar Solids

All spheres are similar.
Given two spheres with radii r and s, respectively.
The scale factor (ratio of lengths) is $f = r/s$.
The ratio of surface areas is $f^2 = (r/s)^2$.
The ratio of volumes is $f^3 = (r/s)^3$.



Two solids are similar if and only if their bases are similar and their corresponding lengths are proportional.

If the scale factor of two similar solids is $a:b$, then
(1) the ratio of corresponding perimeters is $a:b$.
(2) the ratio of the base areas, of the lateral areas, and of the total areas is $a^2:b^2$.
(3) the ratio of the volumes is $a^3:b^3$.

10. Two spheres have radii 5 cm and 7 cm. Find the ratio of:
a. the areas **b.** the volumes

13. Two spheres have a ratio of surface areas of 25:16. Find the ratio of:
a. the radii **b.** the volumes

Example 2

The ratio of the volumes of two similar pyramids is 27:8. Find the scale factor, the ratio of the base perimeters, and the ratio of the lateral areas.

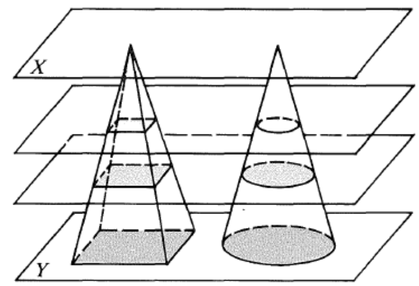
b. $\frac{a^3}{b^3} = \frac{27}{8} = \frac{3^3}{2^3}$, so the scale factor is $\frac{3}{2}$.
The ratio of the base perimeters equals the scale factor, $\frac{3}{2}$.
The ratio of the lateral areas is $\frac{3^2}{2^2} = \frac{9}{4}$.

6. Two similar cylinders have lateral areas 81π and 144π. Find the ratios of:
a. the heights **b.** the total areas **c.** the volumes

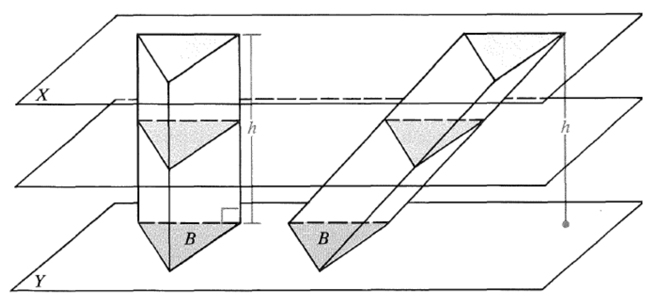
14. A snow man is made using three balls of snow with diameters 30 cm, 40 cm, and 50 cm. If the head weighs about 6 kg, find the total weight of the snow man. (Ignore the arms, eyes, nose and mouth.)

17. Two similar pyramids have lateral areas 8 ft^2 and 18 ft^2. If the volume of the smaller pyramid is 32 ft^3, what is the volume of the larger pyramid?

Cavalieri's Principle
If two solids lying between parallel planes have equal heights and all cross sections at equal distances from their bases have equal areas, then the solids have equal volumes.



B1. The volume of the right prism (to the left below) is Bh. What is the volume of the *oblique* prism?



Teddy Bear Trouble. Obtuse Ollie's grandmother makes 20 identical teddy bears each month for a local toy shop. To do so, she uses 6 square meters of material for the fur, 5 kilograms of kapok for the stuffing, 4 meters of ribbon for bows around their necks, and 40 buttons for their eyes.

The owners of the toy shop asked her to make 20 bears twice as large; so she doubled the order for all her materials.†

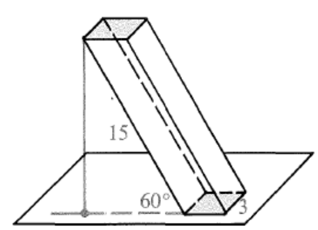
57. Do you think doubling the amount of materials makes sense? If not, what do you think she should have ordered?

58. Which material did Ollie's grandmother use up first?

59. How many large bears did she make?

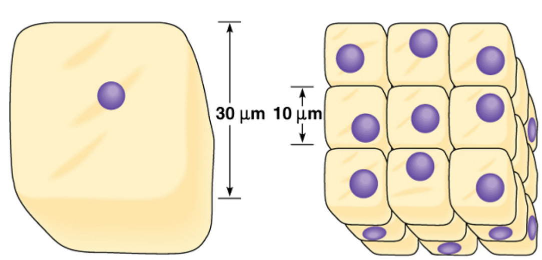
60. Of which material(s) would she have had exactly enough to make 20 of the large bears?

B2. Find the exact volume.



B3. Compare the surface-to-volume ratio for the left cell and the total of all the small cells in the right cube. What is the factor?

Why are cells small?



†From Brian Bolt *Mathematical Cavalcade*, 1992.

Sheet 1161: Find Areas

Name: ______________

For each statement, (a) Draw the shape. (b). Find the Area of the shape described. In some question(s) there might not be enough information.

1. A square with side 5 cm

2. A rectangle with sides 4 cm and 5cm

3. A circle with radius 5 in

4. A trapezoid with bases 5 m and 7 m and a height 9 m.

5. A kite with diagonal 6 m

6. A parallelogram with sides 5 cm and 10 cm

7. A slice of pizza with a 30 degree central angle and radius of 6 inc

8. An equilateral triangle with side 7 m

9. The surface of a cube with side 5 lightyears

10. The surface of a sphere with radius 2 mm

Name: ____________________

Sheet 1251: Find Volumes and Areas of Solids

1. Find the total surface area and volume of the following:

a. A cube with side 2 mm

b. A rectangular box (right prism) with length 5 m,
width 3 m, and height 2 m.

c. A cylinder with radius 3 in and height 5 in.

d. A sphere with radius 6 km.

2. What is the radius of a sphere with volume 540 π cubic inches?

Sheet 1181: Review Areas

1. Find the area of a square with perimeter 32. 11–1

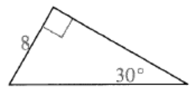
2. Find the area of a rectangle with length 4 and diagonal 6.

3. Find the area of a square with side $3\sqrt{2}$ cm.

4. Find the area of a rhombus with side 17 and longer diagonal 30. 11–2

5. A parallelogram has sides 8 and 12. The shorter altitude is 6. Find the length of the other altitude.

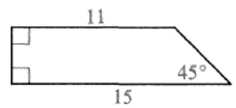
6. Find the perimeter and the area of the triangle shown.



7. Find the height of a trapezoid with median 12 and area 84. 11–3

8. Find the area of an isosceles trapezoid with legs 5 and bases 4 and 12.

9. Find the perimeter and the area of the figure shown.



10. Find the area of a square with apothem 3 m. 11–4

11. Find the area of an equilateral triangle with radius $2\sqrt{3}$.

12. Find the area of a regular hexagon with perimeter 12 cm.

13. Find the circumference and area of a circle with radius 30. Use $\pi \approx 3.14$. 11–5

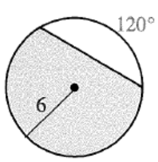
14. The area of a circle is 121π cm^2. Find the diameter.

15. A square with side 8 is inscribed in a circle. Find the circumference and the area of the circle.

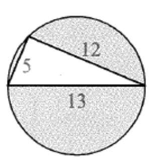
16. Find the length of a 135° arc in a circle with radius 24. 11–6

Find the area of each shaded region.

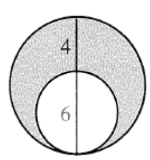
17.



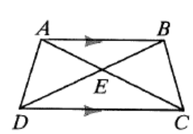
18.



19.



20. If $AB = 9$ and $CD = 12$, find the ratio of the areas of: 11–7
a. $\triangle AEB$ and $\triangle DEC$ **b.** $\triangle AED$ and $\triangle DEC$



21. Two regular octagons have perimeters 16 cm and 32 cm, respectively. What is the ratio of their areas?

22. Two similar polygons have the scale factor 7:5. The area of the large polygon is 147. Find the area of the smaller polygon.

Sheet 1252: Review Solids

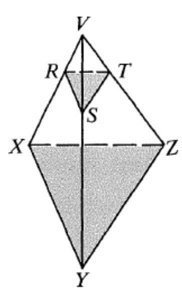
1. In a right prism, each _?_ is also an altitude. 12–1
2. Find the lateral area of a right octagonal prism with height 12 and base edge 7.
3. Find the total area and volume of a rectangular solid with dimensions 8, 6, and 5.
4. A right square prism has base edge 9 and volume 891. Find the total area.
5. Find the volume of a regular triangular pyramid with base edge 8 and height 10. 12–2
6. A regular pentagonal pyramid has base edge 6 and lateral edge 5. Find the slant height and the lateral area.

A regular square pyramid has base edge 30 and total area 1920.

7. Find the area of the base, the lateral area, and the slant height.
8. Find the height and the volume of the pyramid.
9. Find the lateral area and the total area of a cylinder with radius 4 and height 3. 12–3
10. Find the lateral area, total area, and volume of a cone with radius 6 cm and slant height 10 cm.
11. A cone has volume 8π cm^3 and height 6 cm. Find its slant height.
12. The radius of a cylinder is doubled and its height is halved. How does the volume change?

13. A sphere has radius 7 m. Use $\pi \approx \frac{22}{7}$ to find the approximate area of the sphere. 12–4
14. Find, in terms of π, the volume of a sphere with diameter 12 ft.
15. Find the volume of a sphere with area 484π cm^2.

Plane $RST \parallel$ plane XYZ and $VS:VY = 1:3$.



12–5

16. $\dfrac{\text{perimeter of } \triangle RST}{\text{perimeter of } \triangle XYZ} = \underline{\ ?\ }$
17. $\dfrac{\text{total area of small pyramid}}{\text{total area of large pyramid}} = \underline{\ ?\ }$
18. $\dfrac{\text{volume of small pyramid}}{\text{volume of bottom part}} = \underline{\ ?\ }$
19. Two similar cylinders have lateral areas 48π and 27π. Find the ratio of their volumes.

Sheet 1253 Plane Figures and Solids, Areas and Volumes

Name: ______________________

Trig: $\sin A = \frac{opp}{hyp}$, $\cos A = \frac{adj}{hyp}$, $\tan A = \frac{opp}{adj}$. **Special Triangles:** 45°–45°–90° $1:1:\sqrt{2}$. 30°–60°–90° $1:\sqrt{3}:2$.

Circle: $c = 2\pi r$, $A = \pi r^2$. **Sphere:** $A = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$. **Right Solids.** Prism: $L.A. = ph$, $V = Bh$.

Pyramid: $L.A. = \frac{1}{2}pl$, $V = \frac{1}{3}Bh$. Cylinder: $L.A. = 2\pi rh$, $V = \pi r^2 h$. Cone: $r^2 + h^2 = l^2$, $L.A. = \pi rl$, $V = \frac{1}{3}\pi r^2 h$.

1. Write the formula for the area of a rectangle in words:

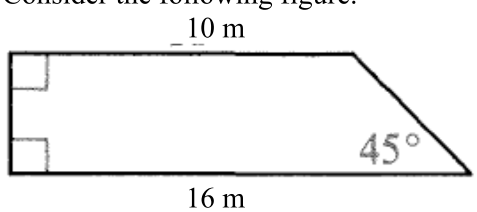
   ____

2. Find the area of a square with perimeter 8 m (meters). Write *units*, always!

3. A parallelogram has sides of lengths 4 m and 9 m. One of the altitudes has a length of 6 m and is perpendicular to the sides of length 4 m. Find the area of the parallelogram.

4. Find the area of a trapezoid with bases 6 m and 10 m and height 4 m.

5. Consider the following figure.



   a) What is the name of the shape?

   b) Find the area of the figure.

   c) Find the perimeter of the figure. You can leave any square roots (no need to use the calculator).

6. An equilateral triangle has a side length of 8 m.

   a) Make a big drawing.

   b) Find the area of the triangle.
   You can leave any square roots (no need to use the calculator). *Note: The "radius" has not been given and is not needed.*

7. Consider a circle with a radius of 5 m.

   a) Find the circumference in terms of π. (Don't use the calculator to approximate π.)

   b) Find the area in terms of π. (Don't use the calculator to approximate π.)

8. A circle is inscribed in a square with side 1 m. Find the area of the circle in terms of π. (Don't use the calculator to approximate π.) *The square is outside the circle.*

9. A square with side 1 m is inscribed in a circle. *The circle is outside the square.*

a) Draw the shapes.

a) Find the area of the square.

b) Find the area of the circle in terms of π. (Don't use the calculator to approximate π.)

c) Find the perimeter of the square.

d) Find the circumference of the circle in terms of π. (Don't use calculator to approximate π or square roots.)

10. Write the formula for the lateral area of a right cone in words:

____

____

11. In a right prism, each ________________________ is also an altitude.

12. Find the volume of a cube with side 3 m (meters). Write *units*, always!

13. Find the total area of a rectangular solid with dimensions 3 m, 5 m, and 7 m.

14. Consider a cylinder with radius 3 m and height 10 m. Find the following in terms of π. (Don't use the calculator to approximate π.) Write *units*!

a) Lateral area.

b) Total area.

c) Volume.

15. A cone has radius 2m and height 5m.

(a) Draw a large sketch of the cone. Label the radius, height, and slant height.

(b) Find the volume.

(c) Find the slant height. Leave any square roots as they are. No decimal approximations.

(d) Find the lateral area, base area, and total area.

16. Consider a sphere with radius 2 m. Answer in terms of π, with improper fractions, and with units. No decimals!

a) Find the surface area of the sphere.

b) Find the volume of the sphere.

17. Square 1 has side *s*. What is its area? What is the area of Square 2 with side 3*s*? What is the ratio of the two areas?

Cube 1 has side *s*. What is its volume? What is the volume of Cube 2 with side 3*s*? What is the ratio of the two volumes?

Sheet 1253

~~Sheet 1243~~ Plane Figures and Solids, Areas and Volumes

Name: KEY

Trig: $\sin A = \frac{opp}{hyp}$, $\cos A = \frac{adj}{hyp}$, $\tan A = \frac{opp}{adj}$. **Special Triangles:** 45°–45°–90° $1:1:\sqrt{2}$. 30°–60°–90° $1:\sqrt{3}:2$.

Circle: $c = 2\pi r$, $A = \pi r^2$. **Sphere:** $A = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$. **Right Solids.** Prism: $L.A. = ph$, $V = Bh$.

Pyramid: $L.A. = \frac{1}{2}pl$, $V = \frac{1}{3}Bh$. Cylinder: $L.A. = 2\pi rh$, $V = \pi r^2 h$. Cone: $r^2 + h^2 = l^2$, $L.A. = \pi rl$, $V = \frac{1}{3}\pi r^2 h$.

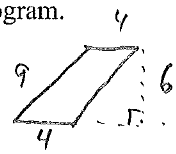
1. Write the formula for the area of a rectangle in words:

   base times height

2. Find the area of a square with perimeter 8 m (meters). Write *units*, always!

   $s = 2m$ $\boxed{A = 4m^2}$

3. A parallelogram has sides of lengths 4 m and 9 m. One of the altitudes has a length of 6 m and is perpendicular to the sides of length 4 m. Find the area of the parallelogram.



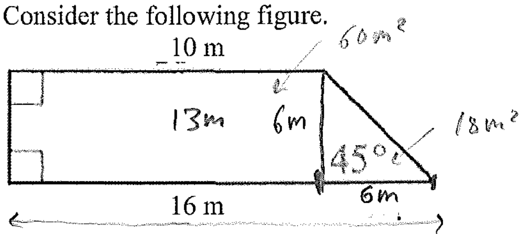
   $\boxed{A = 24m^2}$

4. Find the area of a trapezoid with bases 6 m and 10 m and height 4 m.

   Median = 8m

   $\boxed{A = 32m^2}$

5. Consider the following figure.



   a) What is the name of the shape?

   Right trapezoid

   b) Find the area of the figure.

   $A = 13m \cdot 6m = \boxed{78m^2}$

   MEDIAN · HEIGHT

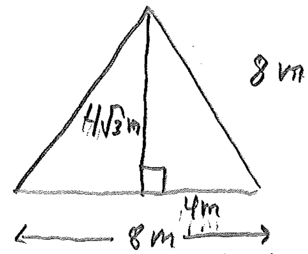
   c) Find the perimeter of the figure. You can leave any square roots (no need to use the calculator).

   $p = 6m + 10m + 6\sqrt{2}\,m + 16m$

   $\boxed{p = (32 + 6\sqrt{2})m}$

6. An equilateral triangle has a side length of 8 m.

   a) Make a big drawing.



   b) Find the area of the triangle. You can leave any square roots (no need to use the calculator). *Note: The "radius" has not been given and is not needed.*

   $A = \frac{1}{2}(8m)(4\sqrt{3}\,m) = \boxed{16\sqrt{3}\,m^2}$

7. Consider a circle with a radius of 5 m.

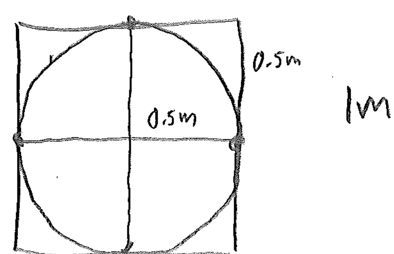
   a) Find the circumference in terms of π. (Don't use the calculator to approximate π.)

   $C = 2\pi r = 2\pi \cdot 5 = \boxed{10\pi\ m}$

   b) Find the area in terms of π. (Don't use the calculator to approximate π.)

   $A = \pi r^2 = \pi (5m)^2 = \boxed{25\pi\ m^2}$

8. A circle is inscribed in a square with side 1 m. Find the area of the circle in terms of π. (Don't use the calculator to approximate π.) *The square is outside the circle.*

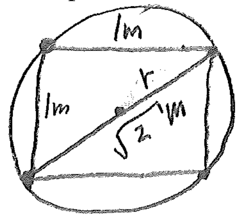


   $A = \pi (0.5m)^2 = \boxed{\frac{\pi}{4}\ m^2}$

KEY

9. A square with side 1 m is inscribed in a circle. *The circle is outside the square.*

a) Draw the shapes.



$r = \frac{\sqrt{2}}{2}$ m

a) Find the area of the square.

$A = 1 m^2$

b) Find the area of the circle in terms of π. (Don't use the calculator to approximate π.)

$r = \frac{\sqrt{2}}{2} m \quad r^2 = \frac{2m^2}{4} = \frac{1}{2} m^2$

$A = \pi r^2 = \pi \left(\frac{\sqrt{2}}{2} m\right)^2 = \frac{\pi}{2} m^2$

c) Find the perimeter of the square.

$P = 4m$

d) Find the circumference of the circle in terms of π. (Don't use calculator to approximate π or square roots.)

$C = \pi d = \pi (\sqrt{2} m) = \sqrt{2} \pi m$

10. Write the formula for the lateral area of a right cone in words:

pi times radius times slant height.

11. In a right prism, each lateral edge is also an altitude.

12. Find the volume of a cube with side 3 m (meters). Write *units*, always!

$V = (3m)^3 = 27 m^3$

13. Find the total area of a rectangular solid with dimensions 3 m, 5 m, and 7 m.

$A = 2(3 \cdot 5 + 3 \cdot 7 + 5 \cdot 7) m^2 = 2(71) m^2$

$A = 142 m^2$

14. Consider a cylinder with radius 3 m and height 10 m. Find the following in terms of π. (Don't use the calculator to approximate π.) Write *units*!

a) Lateral area.

$L.A. = 2\pi r h = 2\pi \cdot 3m \cdot 10m$

$L.A. = 60\pi m^2$

b) Total area.

Base area $= \pi r^2 = \pi (3m)^2 = 9\pi m^2$

$T.A. = 60\pi m^2 + 9\pi m^2 =$ ~~XXXXX~~

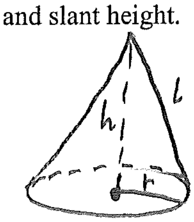
T.A. = (60π+9π+9π)m^2 = 78π m^2

c) Volume.

$V = \pi r^2 h = \pi (3m)^2 \cdot 10m = 90\pi m^2$

15. A cone has radius 2m and height 5m.

(a) Draw a large sketch of the cone. Label the radius, height, and slant height.



r = radius
h = height
L = slant height

$r^2 + h^2 = L^2$

(b) Find the volume.

$V = \frac{1}{3} B \cdot h = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \cdot (2m)^2 (5m)$

$V = \frac{20}{3} \pi m^3$

(c) Find the slant height. Leave any square roots as they are. No decimal approximations.

$L^2 = r^2 + h^2$

$L^2 = 2m^2 + 5m^2$

$L^2 = 29 \; m^2$

$L = \sqrt{29} m$

(d) Find the lateral area, base area, and total area.

$L.A. = \frac{1}{2} pL = \frac{1}{2} \cdot 2\pi r \cdot L = \pi r L = \pi \cdot 2m \cdot \sqrt{29} m = 2\sqrt{29} \pi m$

$B = \pi r^2 = \pi (2m)^2 = 4\pi m^2$

$T.A. = (4 + 2\sqrt{29}) \pi \; m^2$

16. Consider a sphere with radius 2 m. Answer in terms of π, with improper fractions, and with units. No decimals!

a) Find the surface area of the sphere.

$S.A. = 4\pi r^2 = 4\pi (2m)^2 = 16\pi m^2$

b) Find the volume of the sphere.

$V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (2m)^3 = \frac{32\pi}{3} m^3$

17. Square 1 has side *s*. What is its area? s^2 What is the area of Square 2 with side 3*s*? $9s^2$ What is the ratio of the two areas? 1 : 9

Cube 1 has side *s*. What is its volume? s^3 What is the volume of Cube 2 with side 3*s*? $27s^3$ What is the ratio of the two volumes? 1 : 27