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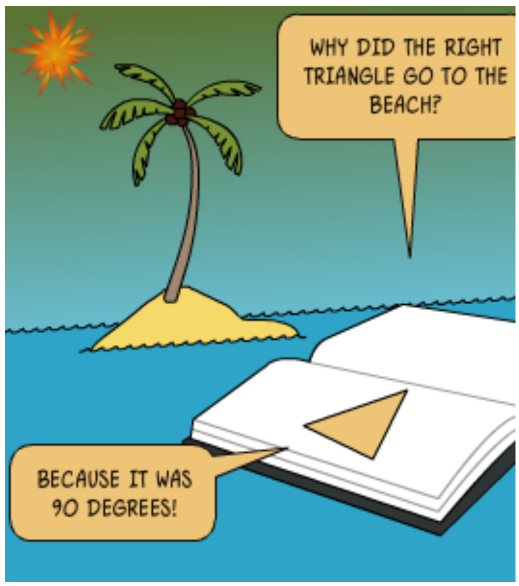
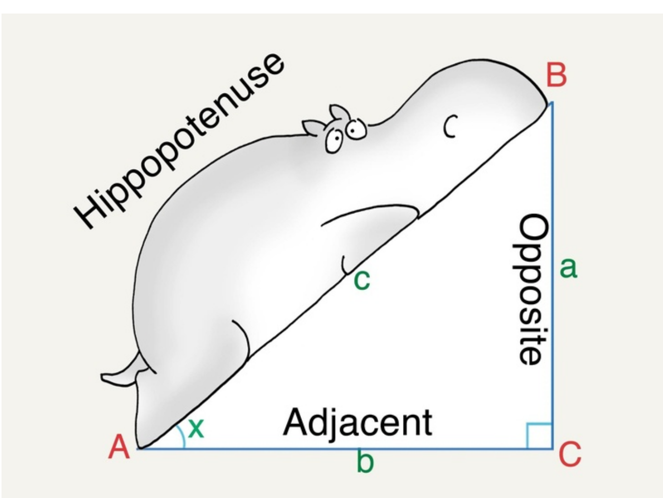
RightPacket

Chapter 8

Right Triangles and Trigonometry





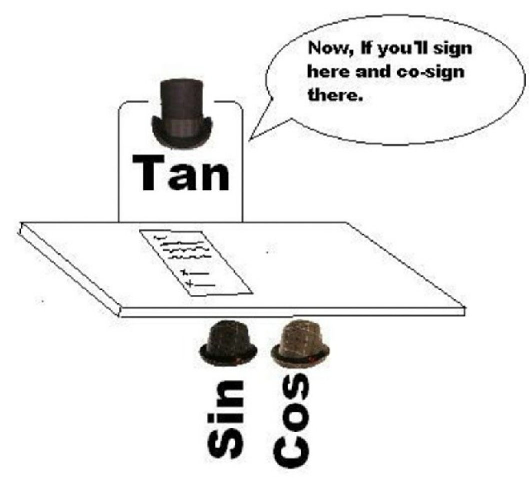


Table of Trigonometric Ratios

Angle	Sine	Cosine	Tangent	Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175	**46°**	.7193	.6947	1.0355
2°	.0349	.9994	.0349	**47°**	.7314	.6820	1.0724
3°	.0523	.9986	.0524	**48°**	.7431	.6691	1.1106
4°	.0698	.9976	.0699	**49°**	.7547	.6561	1.1504
5°	.0872	.9962	.0875	**50°**	.7660	.6428	1.1918
6°	.1045	.9945	.1051	**51°**	.7771	.6293	1.2349
7°	.1219	.9925	.1228	**52°**	.7880	.6157	1.2799
8°	.1392	.9903	.1405	**53°** <	.7986	.6018	1.3270
9°	.1564	.9877	.1584	**54°**	.8090	.5878	1.3764
10°	.1736	.9848	.1763	**55°**	.8192	.5736	1.4281
11°	.1908	.9816	.1944	**56°**	.8290	.5592	1.4826
12°	.2079	.9781	.2126	**57°**	.8387	.5446	1.5399
13°	.2250	.9744	.2309	**58°**	.8480	.5299	1.6003
14°	.2419	.9703	.2493	**59°**	.8572	.5150	1.6643
15°	.2588	.9659	.2679	**60°** <	.8660	.5000	1.7321
16°	.2756	.9613	.2867	**61°**	.8746	.4848	1.8040
17°	.2924	.9563	.3057	**62°**	.8829	.4695	1.8807
18°	.3090	.9511	.3249	**63°**	.8910	.4540	1.9626
19°	.3256	.9455	.3443	**64°**	.8988	.4384	2.0503
20°	.3420	.9397	.3640	**65°**	.9063	.4226	2.1445
21°	.3584	.9336	.3839	**66°**	.9135	.4067	2.2460
22°	.3746	.9272	.4040	**67°**	.9205	.3907	2.3559
23°	.3907	.9205	.4245	**68°**	.9272	.3746	2.4751
24°	.4067	.9135	.4452	**69°**	.9336	.3584	2.6051
25°	.4226	.9063	.4663	**70°**	.9397	.3420	2.7475
26°	.4384	.8988	.4877	**71°**	.9455	.3256	2.9042
27°	.4540	.8910	.5095	**72°**	.9511	.3090	3.0777
28°	.4695	.8829	.5317	**73°**	.9563	.2924	3.2709
29°	.4848	.8746	.5543	**74°**	.9613	.2756	3.4874
30° <	.5000	.8660	.5774	**75°**	.9659	.2588	3.7321
31°	.5150	.8572	.6009	**76°**	.9703	.2419	4.0108
32°	.5299	.8480	.6249	**77°**	.9744	.2250	4.3315
33°	.5446	.8387	.6494	**78°**	.9781	.2079	4.7046
34°	.5592	.8290	.6745	**79°**	.9816	.1908	5.1446
35°	.5736	.8192	.7002	**80°**	.9848	.1736	5.6713
36°	.5878	.8090	.7265	**81°**	.9877	.1564	6.3138
37° <	.6018	.7986	.7536	**82°**	.9903	.1392	7.1154
38°	.6157	.7880	.7813	**83°**	.9925	.1219	8.1443
39°	.6293	.7771	.8098	**84°**	.9945	.1045	9.5144
40°	.6428	.7660	.8391	**85°**	.9962	.0872	11.4301
41°	.6561	.7547	.8693	**86°**	.9976	.0698	14.3007
42°	.6691	.7431	.9004	**87°**	.9986	.0523	19.0811
43°	.6820	.7314	.9325	**88°**	.9994	.0349	28.6363
44°	.6947	.7193	.9657	**89°**	.9998	.0175	57.2900
45° <	.7071	.7071	1.0000				

8–1 Similarity in Right Triangles

Example 1 Simplify.

a. $3\sqrt{98} = 3 \cdot \sqrt{49 \cdot 2}$
$= 3 \cdot \sqrt{49} \cdot \sqrt{2}$
$= 3 \cdot 7 \cdot \sqrt{2}$
$= 21\sqrt{2}$

b. $\sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$
$= \frac{\sqrt{15}}{5}$

c. $\frac{18}{\sqrt{2}} = \frac{18}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{18\sqrt{2}}{2}$
$= 9\sqrt{2}$

> Always put radicals √ in **simplest form**:
> 1. No perfect square factor is under the radical sign.
> 2. No fraction is under the radical sign.
> 3. No fraction has a radical in the denominator.

Simplify.

1. $\sqrt{18}$ **2.** $\sqrt{60}$ **3.** $\sqrt{50}$ **4.** $\sqrt{48}$ **5.** $\sqrt{600}$

6. $3\sqrt{80}$ **7.** $5\sqrt{24}$ **8.** $\sqrt{20} \cdot \sqrt{2}$ **9.** $\sqrt{3} \cdot \sqrt{27}$ **10.** $8\sqrt{2} \cdot 3\sqrt{6}$

11. $\sqrt{\frac{2}{7}}$ **12.** $\sqrt{\frac{16}{5}}$ **13.** $\frac{12}{\sqrt{3}}$ **14.** $\frac{3\sqrt{2}}{5\sqrt{3}}$ **15.** $\frac{\sqrt{10}}{3\sqrt{30}}$

If a, b, and x are positive numbers and $\frac{a}{x} = \frac{x}{b}$, then x is the **geometric mean** between a and b. Notice that x is both means in the proportion. →

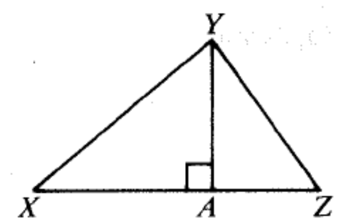
Find the geometric mean between the two numbers.

16. 5 and 20 **17.** 64 and 49 **18.** 1 and 3

> If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

Theorem 8–1

∠XYZ is a right angle. Complete.



21. $\overline{YA}$ is the __?__ to the __?__ of right △XYZ.

22. △XYZ ~ △__?__ ~ △__?__

23. If $m\angle YXZ = 40$, then $m\angle ZYA =$ __?__, $m\angle YZA =$ __?__, and $m\angle XYA =$ __?__.

Corollary 1. "Get Out Of Jail"

$\frac{XA}{YA} = \frac{YA}{AZ}$ $\frac{\text{piece of hypotenuse}}{\text{altitude}} = \frac{\text{altitude}}{\text{other piece of hypotenuse}}$

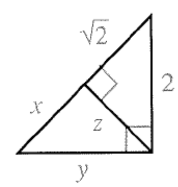
Corollary 2. "Boomerang"

For leg $\overline{XY}$: $\frac{XZ}{XY} = \frac{XY}{XA}$

For leg $\overline{YZ}$: $\frac{XZ}{YZ} = \frac{YZ}{AZ}$

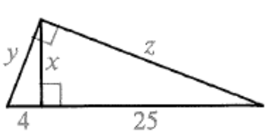
$\frac{\text{hypotenuse}}{\text{leg}} = \frac{\text{leg}}{\text{piece of hyp. adj. to leg}}$

37.



Find the values of x, y, and z.

31.



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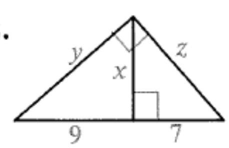

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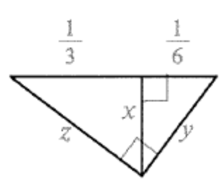

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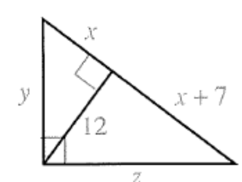

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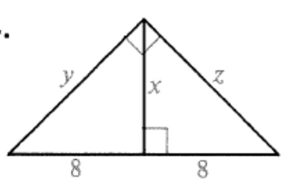

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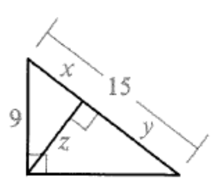

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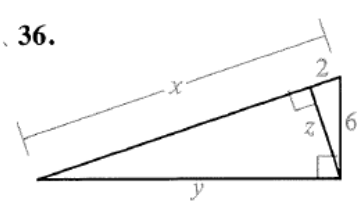

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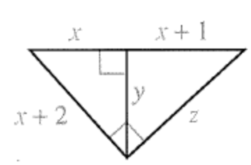


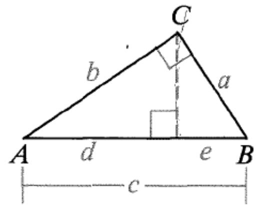
8–2 The Pythagorean Theorem

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

Theorem 8–2

Given: $\triangle ABC$; $\angle ACB$ is a rt. $\angle$.

Prove: $c^2 = a^2 + b^2$



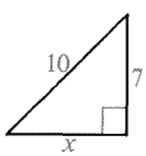
Proof:

Statements	Reasons
1. Draw a perpendicular from C to $\overline{AB}$.	1. Through a point outside a line, there is exactly one line __?__.
2. $\frac{c}{a} = \frac{a}{e}$; $\frac{c}{b} = \frac{b}{d}$	2. When the altitude is drawn to the hypotenuse of a rt. $\triangle$, each leg is the geometric mean between __?__.
3. $ce = a^2$; $cd = b^2$	3.
4. $ce + cd = a^2 + b^2$	4.
5. $c(e + d) = a^2 + b^2$	5.
6. $c^2 = a^2 + b^2$	6.

Set A.

State an equation you could use to find the value of x. Then find the value of x in simplest radical form.

13.



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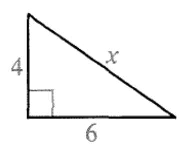

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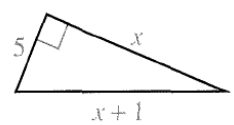

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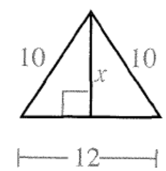

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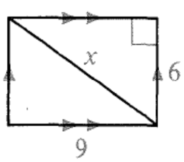

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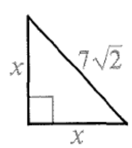
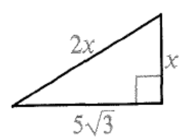


18.



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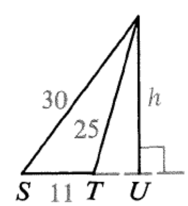


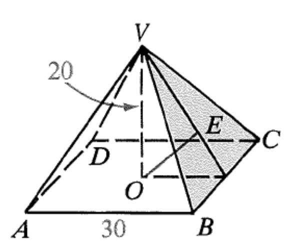
(*Hint*: Let $TU = x$; $SU = x + 11$.)

Set B.

18. The perimeter of a rhombus is 40 cm, and one diagonal is 12 cm long. How long is the other diagonal?

32. A rectangle is 2 cm longer than it is wide. The diagonal of the rectangle is 10 cm long. Find the perimeter of the rectangle.

39. O is the *center* of square $ABCD$ (the point of intersection of the diagonals) and $\overline{VO}$ is perpendicular to the plane of the square. Find OE, the distance from O to the plane of $\triangle VBC$.



Pythagorean Triples. The set of numbers "3-4-5" is well known as the simplest *Pythagorean triple;* that is, a set of three integers that can be the lengths of the sides of a right triangle.

There are infinitely many Pythagorean triples; those in which no number is more than 50 are listed at the right below.

18. Show why the set "7-24-25" is a Pythagorean triple.

19. Can all three numbers in a Pythagorean triple be even? Explain.

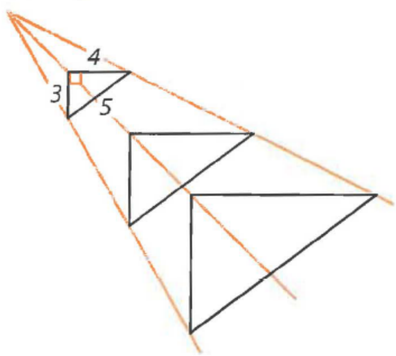
20. Can all three numbers in a Pythagorean triple be odd? Explain.

3-4-5
5-12-13
6-8-10
7-24-25
8-15-17
9-12-15
9-40-41
10-24-26
12-16-20
12-35-37
14-48-50
15-20-25
15-36-39
16-30-34
18-24-30
20-21-29
21-28-35
24-32-40
27-36-45
30-40-50

Some of the Pythagorean triples in the list are related to one another.

21. How is the triple 14-48-50 related to the triple 7-24-25?

The figure below suggests why Pythagorean triples are related in the way that they are.



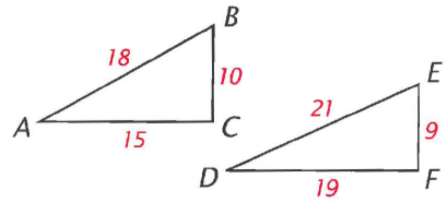
22. Under what transformation are the larger triangles images of the 3-4-5 right triangle?

23. What Pythagorean triples do they illustrate?

24. Name the other triples in the yellow list that also are part of this set.

25. Find two triples in the list that are related to 5-12-13.

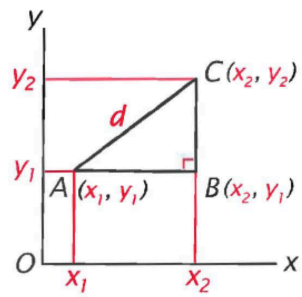
Not Quite Right. The triangles below look like right triangles, but they actually are not.



26. Use the lengths of their sides to show why each figure looks so much like a right triangle.

27. What kind of angles do you think ∠C and ∠F actually are? Explain your reasoning.

Distance Formula. The Distance Formula comes from the Pythagorean Theorem.



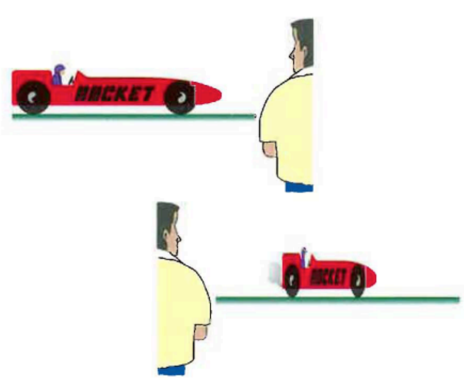
28. Which side of △ABC has length $x_2 - x_1$?

29. What is the length of side BC?

30. Why does it follow that $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$?

30b. Find the distance between (1, 2) and (–1, 5).

Rocket Car. According to the theory of relativity, the length of an object appears to get shorter as it moves at speeds near the speed of light.



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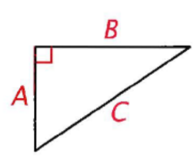
The Pythagorean Theorem is used in developing the formula for this change.

56. Show for the figure at the right why

$$\frac{A}{C} = \sqrt{1 - \left(\frac{B}{C}\right)^2}.$$



$\frac{A}{C}$ represents the ratio by which length shortens when $\frac{B}{C}$ is the ratio of the moving object's speed to the speed of light.

→

57. Use your calculator to find the ratio by which the length of a rocket car shortens when it is moving at 90% of the speed of light (in other words, when $\frac{B}{C} = 0.90$).

8–3 The Converse of the Pythagorean Theorem

Pythagorean Triples

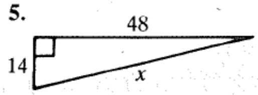
A triangle with sides of 3, 4, and 5 is a right triangle because $3^2 + 4^2 = 5^2$. Any triangle with sides $3n$, $4n$, and $5n$, $n > 0$, is also a right triangle because $(3n)^2 + (4n)^2 = (5n)^2$. Multiples of any three lengths that form a right triangle will also form right triangles.

These groups of three lengths are called **Pythagorean triples**. If you use them, you can save yourself time and effort.

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
⋮	⋮	⋮	⋮

Set A.

Use Pythagorean triples to quickly find the value of x.

5.


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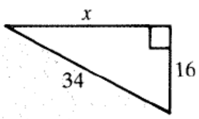

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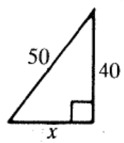

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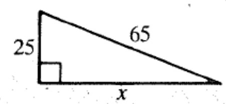
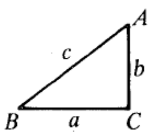


Converse of Pythagorean Theorem

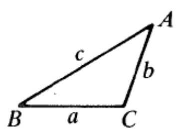
Theorem 8–3

If $c^2 = a^2 + b^2$, then $\angle C$ is a right angle and $\triangle ABC$ is a right triangle.



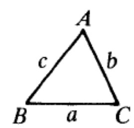
Theorem 8–4

If $c^2 > a^2 + b^2$, then $\angle C$ is obtuse and $\triangle ABC$ is an obtuse triangle.



Theorem 8–5

If $c^2 < a^2 + b^2$, then $\angle C$ is acute and $\triangle ABC$ is an acute triangle.



Example 2 If a triangle is formed with the given lengths, is it acute, right, or obtuse?

a. 8, 12, 13 **b.** 4, 4, 7

Solution

a. $13^2 \ ? \ 8^2 + 12^2$
$169 \ ? \ 64 + 144$
$169 < 208$
The lengths form an acute triangle.

b. $7^2 \ ? \ 4^2 + 4^2$
$49 \ ? \ 16 + 16$
$49 > 32$
The lengths form an obtuse triangle.

Set B.

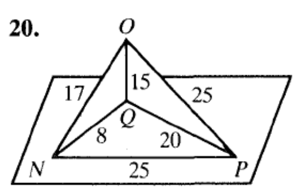
If a triangle is formed with sides having the lengths given, is it acute, right, or obtuse? If a triangle can't be formed, say *not possible*.

1. 6, 8, 10 **2.** 4, 6, 8 **3.** 1, 4, 6

4. 8, 10, 12 **5.** $\sqrt{7}, \sqrt{7}, \sqrt{14}$ **6.** $4, 4\sqrt{3}, 8$

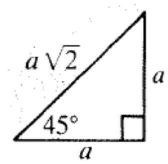
18. If x and y are positive numbers with $x > y$, show that a triangle with sides of lengths $2xy$, $x^2 - y^2$, and $x^2 + y^2$ is always a right triangle.

If the diagram were drawn to scale, which angles would be right angles?

20.


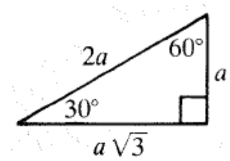
8–4 Special Right Triangles

Special triangles are important to trigonometry since they provide **exact values** (not numerical approximations obtained by calculators) for ratios between the sides of **similar** right triangles.



Theorem 8–6

The sides of a 45°-45°-90° triangle and the sides of a 30°-60°-90° triangle are related as shown.



Theorem 8–7

The ratio of the leg opposite a 45° angle to the hypotenuse is $1/\sqrt{2}$ = **$\sqrt{2}/2$ exactly** and 0.707 approximately. The same ratio for a 30° angle is **1/2** and for a 60° angle it is **$\sqrt{3}/2$ exactly** and 0.866 approximately. These ratios hold for all similar triangles with these special angles.

Example 1 Given the length of the legs, find the length of the hypotenuse of each 45°-45°-90° triangle.

a. 5 **b.** $3\sqrt{2}$ **c.** $5\sqrt{6}$

Solution

a. $5\sqrt{2}$ **b.** $3\sqrt{2}\cdot\sqrt{2} = 6$ **c.** $5\sqrt{6}\cdot\sqrt{2} = 5\sqrt{12} = 10\sqrt{3}$

Example 2 Given the length of the hypotenuse, find the length of the legs of each 45°-45°-90° triangle.

a. $8\sqrt{2}$ **b.** 10 **c.** $4\sqrt{3}$

Solution

a. $\frac{8\sqrt{2}}{\sqrt{2}} = 8$ **b.** $\frac{10}{\sqrt{2}} = \frac{10\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$ **c.** $\frac{4\sqrt{3}}{\sqrt{2}} = \frac{4\sqrt{3}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{4\sqrt{6}}{2} = 2\sqrt{6}$

Example 3 Using the side given, find the other two sides of each 30°-60°-90° triangle.

a. shorter leg: $8\sqrt{3}$ **b.** hypotenuse: 12 **c.** longer leg: $\sqrt{6}$

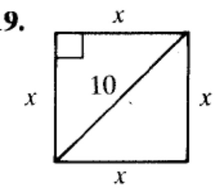
Solution

a. hyp.: $= (\text{shorter leg})\cdot 2$
$= 8\sqrt{3}\cdot 2$
$= 16\sqrt{3}$
longer leg $= (\text{shorter leg})\cdot\sqrt{3}$
$= 8\sqrt{3}\cdot\sqrt{3}$
$= 24$

b. shorter leg $= \frac{\text{hyp.}}{2}$
$= \frac{12}{2}$
$= 6$
longer leg $= (\text{shorter leg})\cdot\sqrt{3}$
$= 6\sqrt{3}$

c. shorter leg $= \frac{\text{longer leg}}{\sqrt{3}}$
$= \frac{\sqrt{6}}{\sqrt{3}}$
$= \frac{\sqrt{2}\cdot\sqrt{3}}{\sqrt{3}}$
$= \sqrt{2}$
hyp. $= (\text{shorter leg})\cdot 2$
$= 2\sqrt{2}$

19.



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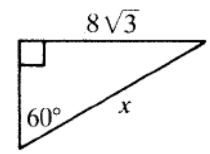
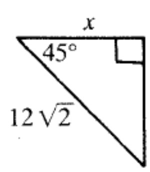


Find the value of *x*.

17.



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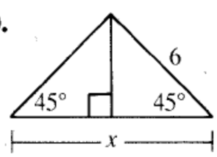

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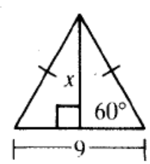

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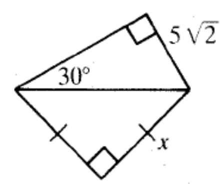
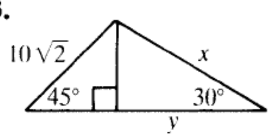


Find the values of *x* and *y*.

23.



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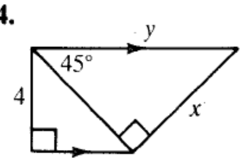

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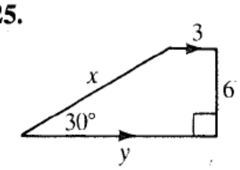
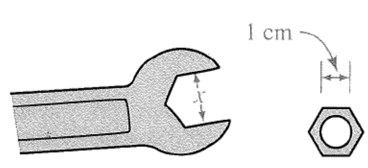
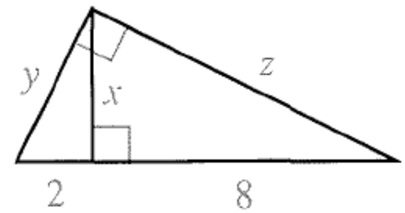


38. If the wrench just fits the hexagonal nut, what is the value of *x*?



Self-Test 1

1. Find the geometric mean between 3 and 15.
2. The diagram shows the altitude drawn to the hypotenuse of a right triangle.
   a. $x = \underline{\ ?\ }$
   b. $y = \underline{\ ?\ }$
   c. $z = \underline{\ ?\ }$



3. The sides of a triangle are given. Is the triangle acute, right, or obtuse?
   a. 11, 60, 61 **b.** 7, 9, 11 **c.** 0.2, 0.3, 0.4
4. A rectangle has length 8 and width 4. Find the lengths of the diagonals.
5. Find the perimeter of a square that has diagonals 10 cm long.
6. The sides of an equilateral triangle are 12 cm long. Find the length of an altitude of the triangle.
7. How long is the altitude to the base of an isosceles triangle if the sides of the triangle are 13, 13, and 10?

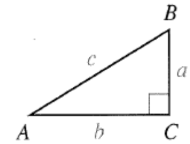
8–5 The Tangent Ratio

In TRIGONOMETRY there are three common ratios for the sides of all similar triangles with given angle but any one of the ratios would suffice together with the Pythagorean theorem. A mnemonic for the trigonometric ratios is SOH CAH TOA.

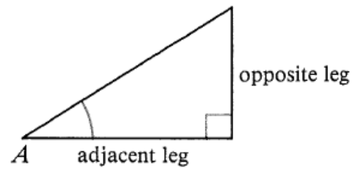
In the right triangle shown:

$\tan A = \frac{a}{b}$ $\sin A = \frac{a}{c}$ $\cos A = \frac{b}{c}$

The tangent, sine, and cosine ratios are useful in solving problems involving right triangles.



$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$



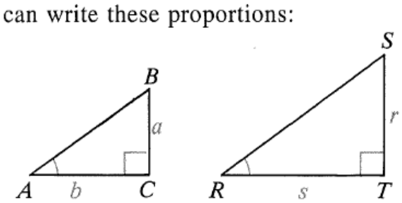
Trigonometric ratios simplify solving triangles. If we know angle A, then we know the ratio tan A.
The ratio of opposite/adjacent is the same for all similar right triangles with angle 33°, tan 33° ≈ 0.649.
The value of the tangent of an angle depends on the size of the angle, not the size of the right triangle.

In the right triangles shown below, $m\angle A = m\angle R$. Then by the AA Similarity Postulate, the triangles are similar. We can write these proportions:

$\frac{a}{r} = \frac{b}{s}$ (Why?)

$\frac{a}{b} = \frac{r}{s}$ (A property of proportions)

$\tan A = \tan R$ (Def. of tangent ratio)



In Exercises 1–3 express tan R as a ratio.

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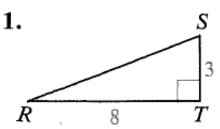

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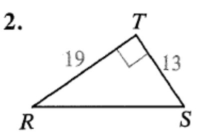

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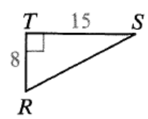


4–6. Express tan S as a ratio for each triangle above.
How are the answers in 1–3 and 4–6 related?

7. Use the table at the beginning of the packet. Find the tangent ratios to 4 decimals, reading the tangent column from left to right. Estimate angles to the nearest degree, reading the table from right to left and finding the closest value.

a. $\tan 24° \approx$ _?_ **b.** $\tan 41° \approx$ _?_ **c.** $\tan 88° \approx$ _?_
d. tan _?_ ≈ 2.4751 **e.** tan _?_ ≈ 0.3057 **f.** tan _?_ ≈ 0.8098

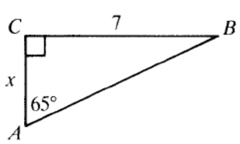
Table of Trigonometric Ratios

Angle	Sine	Cosine	Tangent	Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175	**46°**	.7193	.6947	1.0355
2°	.0349	.9994	.0349	**47°**	.7314	.6820	1.0724
3°	.0523	.9986	.0524	**48°**	.7431	.6691	1.1106
4°	.0698	.9976	.0699	**49°**	.7547	.6561	1.1504
5°	.0872	.9962	.0875	**50°**	.7660	.6428	1.1918
6°	.1045	.9945	.1051	**51°**	.7771	.6293	1.2349

☺

Example 3

Find the value of x to the nearest tenth.



Solution

$\tan 65° = \frac{7}{x}$. You can solve this equation, or you can find $m\angle B$ and write an equation that is easier to solve.

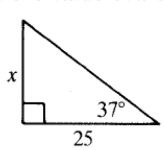
Method 1

$\tan 65° = \frac{7}{x}$
$x(\tan 65°) = 7$
$x = \frac{7}{\tan 65°}$
$x \approx \frac{7}{2.1445}$
$x \approx 3.2641641$, or 3.3

Method 2

$m\angle B = 90 - m\angle A$
$= 90 - 65 = 25$
$\tan 25° = \frac{x}{7}$
$x = 7(\tan 25°)$
$x \approx 7(0.4663)$
$x \approx 3.2641$, or 3.3

Find the value of x to the nearest tenth.

11.


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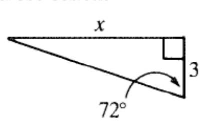

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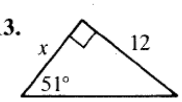

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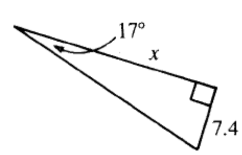
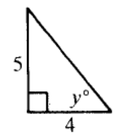
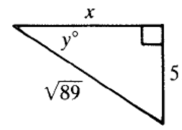


Example 4 Find $y°$ correct to the nearest degree.

a.



b.



Solution

a. $\tan y° = \frac{5}{4} = 1.25$

Both tan 51° and tan 52° are close to 1.25. Which is closer?

$\tan 51° = 1.2349$
$1.25 - 1.2349 = 0.0151$

$\tan 52° = 1.2799$
$1.2799 - 1.25 = 0.0299$

1.25 is closer to 1.2349 than to 1.2799, so $y° \approx 51°$.

b. $\tan y° = \frac{5}{x}$, so you need to find the value of x first.

$x^2 + 5^2 = (\sqrt{89})^2$
$x = 8$

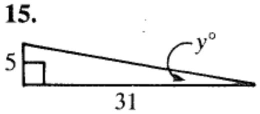
$\tan y° = \frac{5}{8} = 0.625$

Using the table, the value closest to 0.625 is 0.6249, so $y° \approx 32°$.

Set A.

Find $y°$ correct to the nearest degree.

15.



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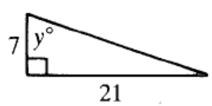

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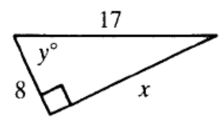

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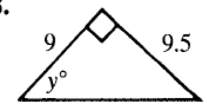
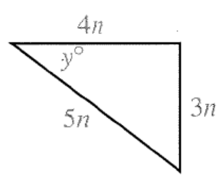


Set B. ☺

10.



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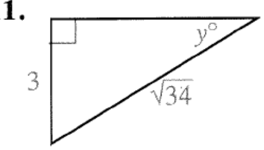

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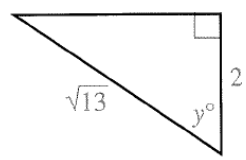

Find w, then z, correct to the nearest integer.

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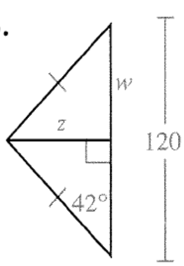

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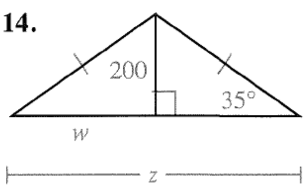

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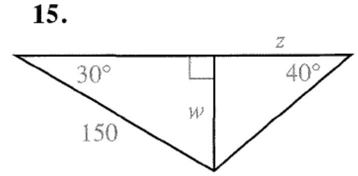


21. The base of an isosceles triangle is 70 cm long. The altitude to the base is 75 cm long. Find, to the nearest degree, the base angles of the triangle.

22. A rhombus has diagonals of length 4 and 10. Find the angles of the rhombus to the nearest degree.

25. A natural question to consider is the following:

Does $\tan A + \tan B = \tan(A + B)$?

Try substituting 35° for A and 25° for B.

a. $\tan 35° + \tan 25° \approx$ _?_ + _?_ = _?_

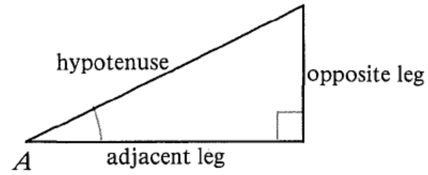
b. $\tan(35° + 25°) = \tan$ _?_ ° ≈ _?_

c. What is your answer to the general question raised in this exercise, *yes* or *no*?

d. Do you think $\tan A - \tan B = \tan(A - B)$? Explain.

8–6 The Sine and Cosine Ratios

$$\sin = \frac{\text{opposite}}{\text{hypotenuse}} \text{ and } \cos = \frac{\text{adjacent}}{\text{hypotenuse}}.$$



Express the sine and cosine of ∠*R* as ratios.

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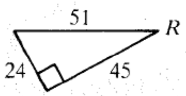

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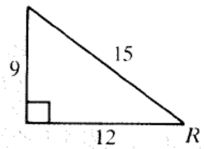

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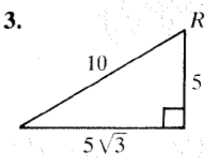

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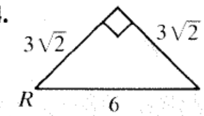


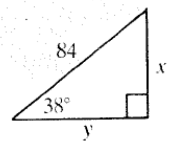
Complete.

5. $\cos 22° \approx$ _?_ **6.** $\sin 79° \approx$ _?_ **7.** cos _?_ ≈ 0.7771

8. sin _?_ ≈ 0.8746 **9.** cos _?_ ≈ 0.3891 **10.** sin _?_ ≈ 0.5321

Example 2

Find the values of x and y to the nearest integer.



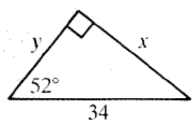
Solution

$\sin 38° = \frac{x}{84}$ $\cos 38° = \frac{y}{84}$

$x = 84(\sin 38°)$ $y = 84(\cos 38°)$

$x \approx 84(0.6157) = 51.7188$, or 52 $y \approx 84(0.7880) = 66.1920$, or 66

Find the values of *x* and *y* to the nearest integer.

11.


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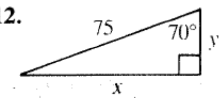

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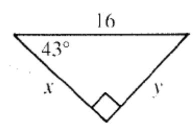

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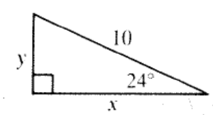
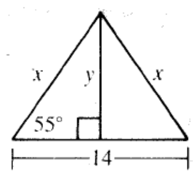


Example 3

Find the values of x and y to the nearest integer.



Solution

In an isosceles triangle, the altitude to the base is the perpendicular bisector of the base.

To find x:

$\cos 55° = \frac{7}{x}$

$x(\cos 55°) = 7$

$x = \frac{7}{\cos 55°} \approx \frac{7}{0.5736}$

$x \approx 12.2036$, or 12

To find y:

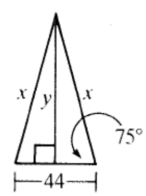
$\sin 55° = \frac{y}{x}$ *or* $\tan 55° = \frac{y}{7}$

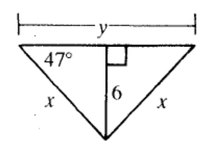
$y = x(\sin 55°)$ $y = 7(\tan 55°)$

$y \approx (12.2036)(0.8192)$ $y \approx 7(1.4281)$

$y \approx 9.9972$, or 10 $y \approx 9.9967$, or 10

Find the values of *x* and *y* to the nearest integer.

15.


16.


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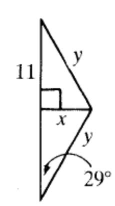

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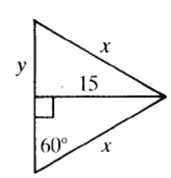
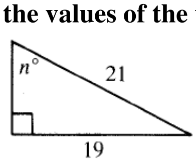


Find the values of the variables to the nearest integer.

19.


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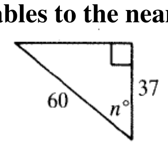

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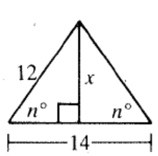

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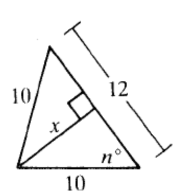
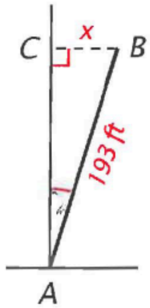


Leaning Tower. Ever since it was built, the Leaning Tower of Pisa has been leaning more and more.



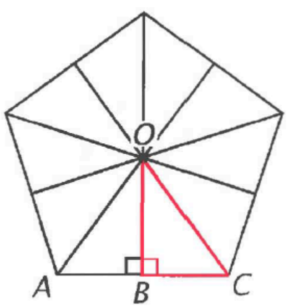
In the figure at the right above, AB represents the tower and CB represents the distance that the top leans over the base.

45. In 1400, the tower leaned 3° to one side. How long was CB at that time?

46. In 1500, the distance CB had increased to 13.5 feet. At what angle did the tower lean then?

47. Now the tower leans about 5.5°. How long is CB now?

Pentagon. The pentagon has long been a popular choice for the shape of a fortress. An old method for laying out a pentagon on the ground was to stretch a rope into the shape of a right triangle.‡



29. If all ten right triangles surrounding point O are congruent, what is the measure of ∠BOC?

30. If the perimeter of the pentagon is supposed to be 1,000 feet, how long is BC?

31. Use the tangent ratio in △OBC to find OB to the nearest 0.1 foot.

32. Use the sine ratio in △OBC to find OC to the nearest 0.1 foot.

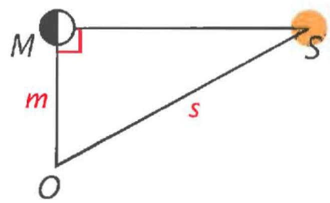
33. Use the Pythagorean Theorem to see if your results for OB and OC seem reasonable.

34. What length of rope is needed to lay out △OBC on the ground?

‡*The Invention of Infinity: Mathematics and Art in the Renaissance,* by J. V. Field (Oxford University Press, 1997).

Sun and Moon. Aristarchus, a Greek astronomer who lived at about the time of Euclid, figured out a way to compare the distances of the sun and the moon from Earth.†

The figure below (not to scale) shows the moon precisely at its first quarter so that ∠M = 90°. The centers of the moon and sun are M and S, and O represents the observer on Earth.



Aristarchus estimated the measure of ∠O to be 87°.

51. Show why it follows that $\frac{m}{s} = \frac{1}{19}$ (approximately).

52. Show why this equality gives $s = 19m$ (that is, that the sun is 19 times as far away as the moon).

The correct measure of ∠O is closer to 89.85°.

53. About how many times as far as the moon is the sun from Earth?

†*Constructing the Universe,* by David Layzer (Scientific American Library, 1984).

8–7 Applications of Right Triangle Trigonometry

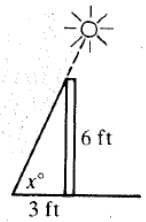
Example 1

At a certain time, a post 6 ft tall casts a 3 ft shadow. What is the angle of elevation of the sun?

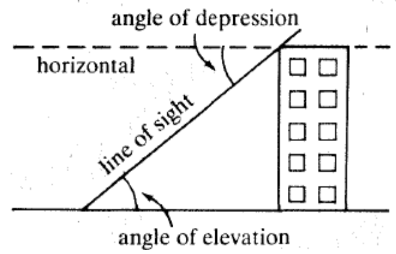
Solution

$\tan x° = \frac{6}{3} = 2$

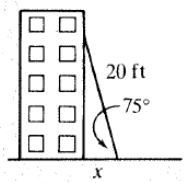
$x \approx 63$



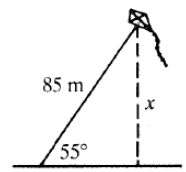
Vocabulary →



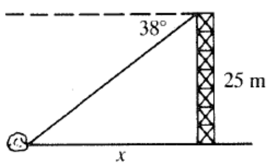
2. A ladder that is 20 ft long is leaning against the side of a building. If the angle formed between the ladder and the ground is 75°, how far is the bottom of the ladder from the base of the building?



4. A kite is flying at an angle of elevation of about 55°. Ignoring the sag in the string, find the height of the kite if 85 m of string have been let out.



6. The angle of depression from the top of a tower to a boulder on the ground is 38°. If the tower is 25 m high, how far from the base of the tower is the boulder?

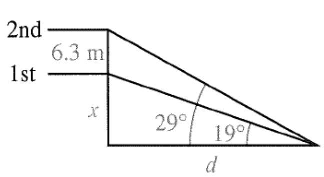


12. The force of gravity pulling an object down a hill is its weight multiplied by the sine of the angle of elevation of the hill.

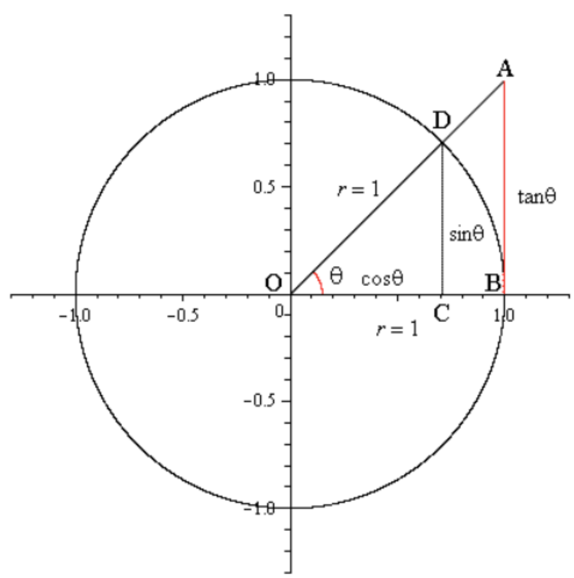
a. With how many pounds of force is gravity pulling on a 3000 lb car on a hill with a 3° angle of elevation?

b. Could you push against the car and keep it from rolling down the hill?

14. From the stage of a theater, the angle of elevation of the first balcony is 19°. The angle of elevation of the second balcony, 6.3 m directly above the first, is 29°. How high above stage level is the first balcony? (*Hint*: Use tan 19° and tan 29° to write two equations involving x and d. Solve for d, then find x.)



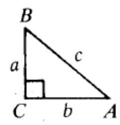
B15. Prove that $AB = \tan\theta$ where θ is the angle COD. Note that AB is tangent to the circle O with radius = 1.



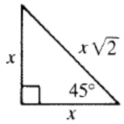
Summary and Mixed Right Triangle Exercises

Right Triangle Relationships

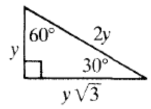
In any right triangle, $a^2 + b^2 = c^2$.



45°-45°-90° △

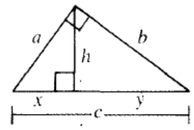


30°-60°-90° △



If the altitude is drawn to the hypotenuse of a right triangle, then:

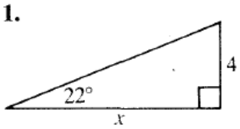
$\frac{x}{h} = \frac{h}{y}$ $\qquad \frac{c}{a} = \frac{a}{x}$ $\qquad \frac{c}{b} = \frac{b}{y}$



$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$ $\qquad \cos = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\qquad \tan = \frac{\text{opposite}}{\text{adjacent}}$

Find the value of *x*.

1.



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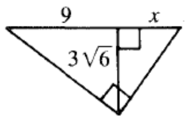

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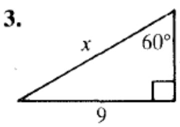

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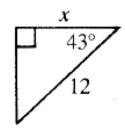

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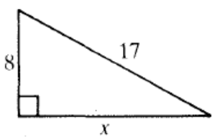
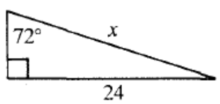
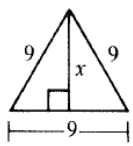


6.



7.



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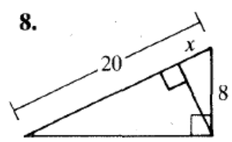
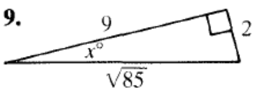
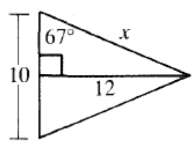


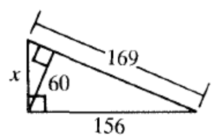
9.



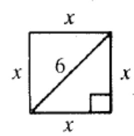
10.



11.

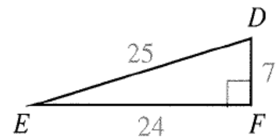


12.



Self-Test 2

Exercises 1–5 refer to the diagram at the right.



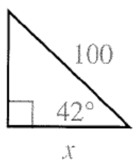
1. $\tan E = \frac{?}{?}$ **2.** $\cos E = \frac{?}{?}$

3. $\sin E = \frac{?}{?}$ **4.** $\tan D = \frac{?}{?}$

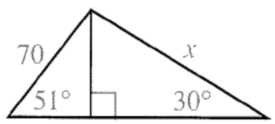
5. To the nearest integer, $m\angle D$ = __?__.

Find the value of *x* to the nearest integer.

6.



7.



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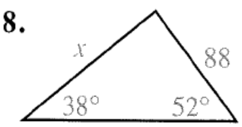
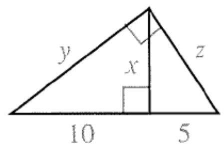


9. From a point on the ground 100 m from the foot of a cliff, the angle of elevation of the top of the cliff is 24°. How high is the cliff?

Chapter Review

1. Find the geometric mean between 12 and 3. **8–1**

2. x = __?__
3. y = __?__
4. z = __?__



5. The legs of a right triangle are 3 and 6. Find the length of the hypotenuse. **8–2**
6. A rectangle has sides 10 and 8. Find the length of a diagonal.

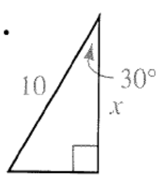
7. The diagonal of a square has length 14. Find the length of a side.
8. The legs of an isosceles triangle are 10 units long and the altitude to the base is 8 units long. Find the length of the base.

Tell whether a triangle formed with sides having the lengths named is acute, right, or obtuse. If a triangle can't be formed, write *not possible*.

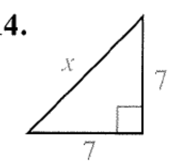
9. 4, 5, 6 **10.** 8, 8, 17 **8–3**
11. 11, 60, 61 **12.** $2\sqrt{3}$, $3\sqrt{2}$, 6

Find the value of *x*. **8–4**

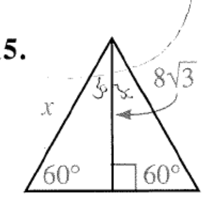
13.



14.



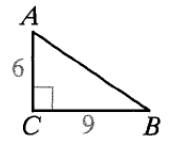
15.



16. The legs of an isosceles right triangle have length 12. Find the lengths of the hypotenuse and the altitude to the hypotenuse.

Complete. Find angle measures and lengths correct to the nearest integer.

17.

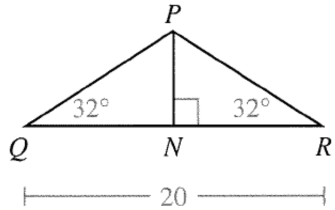


a. $\tan A = \underline{\ ?\ }$

b. $\tan B = \underline{\ ?\ }$

c. $m\angle B \approx \underline{\ ?\ }$

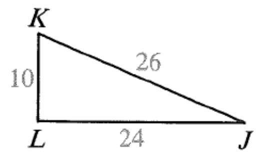
18. 8–5



a. $QN = \underline{\ ?\ }$

b. $PN \approx \underline{\ ?\ }$

19.

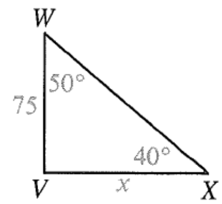


a. $\cos J = \underline{\ ?\ }$

b. $\sin K = \underline{\ ?\ }$

c. $m\angle K \approx \underline{\ ?\ }$

20. 8–6

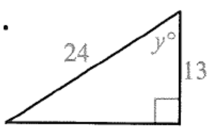


a. $WX \approx \underline{\ ?\ }$

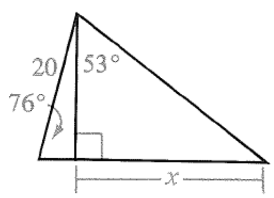
b. $VX \approx \underline{\ ?\ }$

Find the values of *x* and *y* correct to the nearest integer.

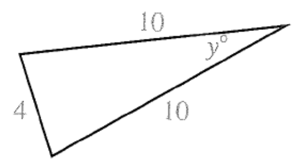
21.



22.



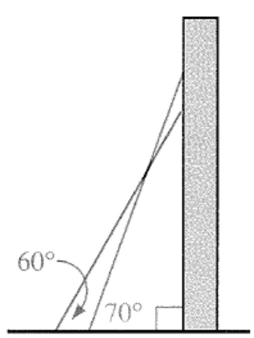
23.



24. Lee, on the ground, looks up at Chong Ye in a hot air balloon at a 35° angle of elevation. If Lee and Chong Ye are 500 ft apart, about how far off the ground is Chong Ye? 8–7

B17.

A 6 m ladder reaches higher up a wall when placed at a 70° angle than when placed at a 60° angle. How much higher, to the nearest tenth of a meter?



Answers

Self-Test 1

1. $3\sqrt{5}$ **2. a.** 4 **b.** $2\sqrt{5}$ **c.** $4\sqrt{5}$ **3. a.** rt. **b.** acute **c.** obtuse **4.** $4\sqrt{5}$ **5.** $20\sqrt{2}$ cm
6. $6\sqrt{3}$ cm **7.** 12

Self-Test 2

1. $\frac{7}{24}$ **2.** $\frac{24}{25}$ **3.** $\frac{7}{25}$ **4.** $\frac{24}{7}$ **5.** 74 **6.** 74 **7.** 109 **8.** 113 **9.** about 45 m

Chapter Review

1. 6 **3.** $5\sqrt{6}$ **5.** $3\sqrt{5}$ **7.** $7\sqrt{2}$ **9.** acute **11.** rt. **13.** $5\sqrt{3}$ **15.** 16 **17. a.** 1.5 **b.** $\frac{2}{3}$
c. 34 **19. a.** $\frac{12}{13}$ **b.** $\frac{12}{13}$ **c.** 67 **21.** 57 **23.** 23