Full Name: ________________________________ Block/Period: ____

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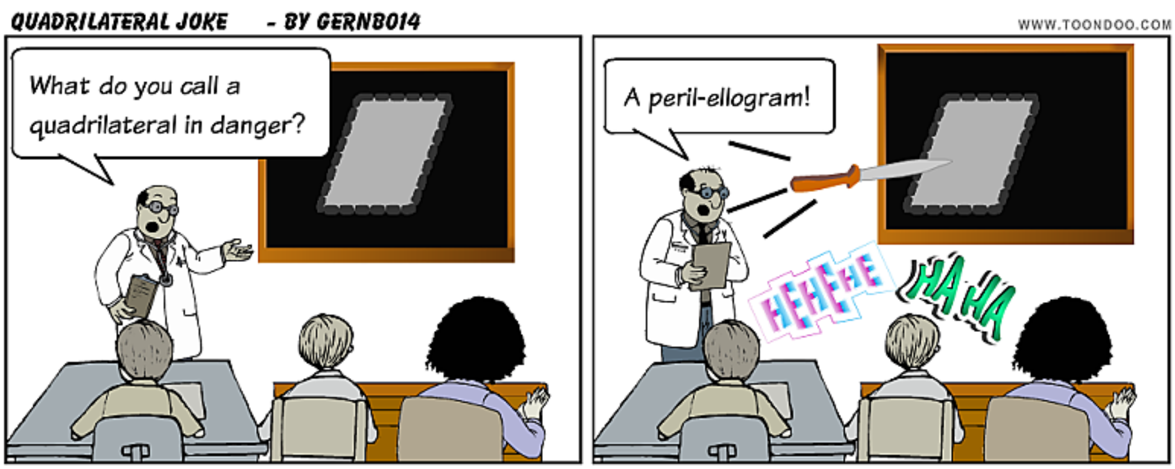
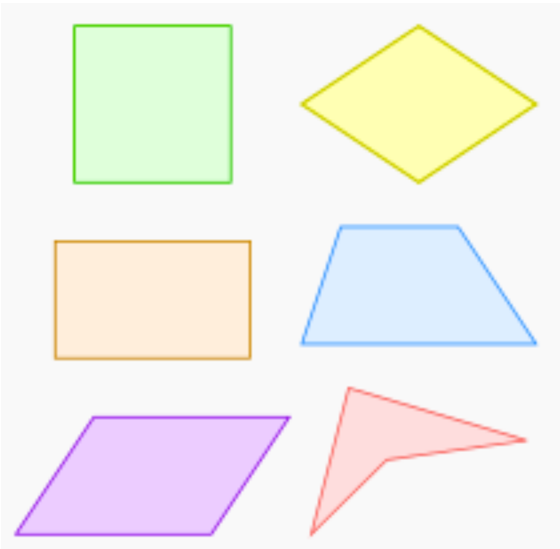
QuadPacket

Chapter 5

Quadrilaterals

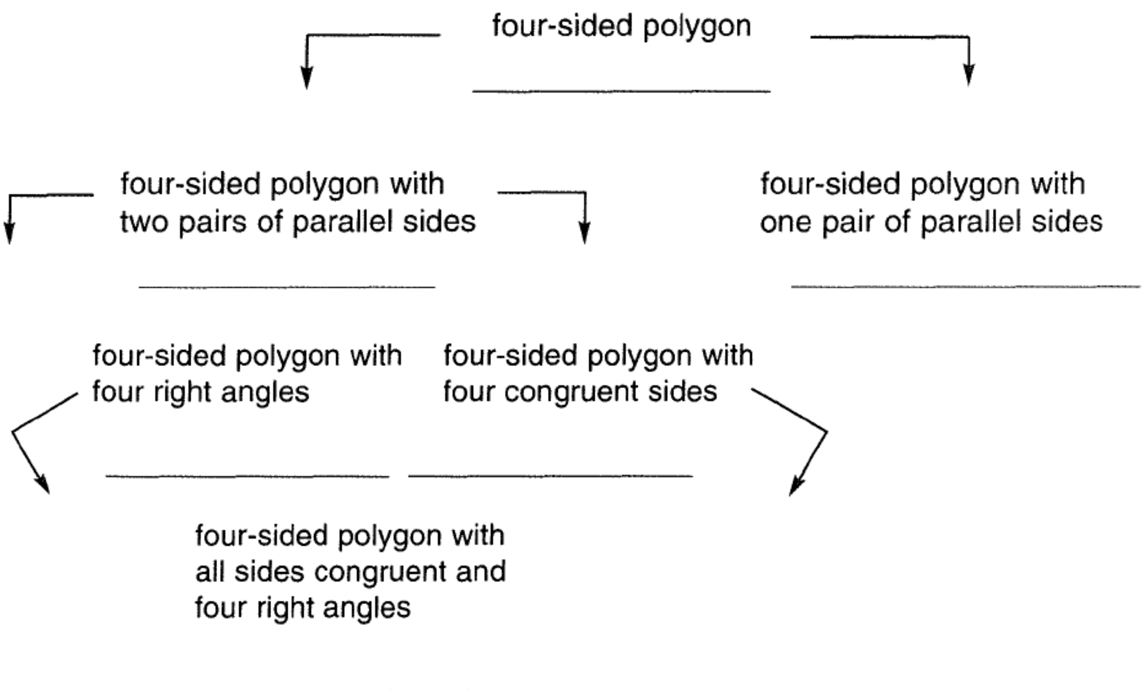




Quadrilaterals (quadrangles, tetragons) are from Latin *quadri* (four) and *latus* (side). They are simple (not self-intersecting) or complex (self-intersecting), also called crossed.

Fill in: The interior angles of a simple, planar quadrilateral add up to ________ degrees.

Place the following terms in the diagram. Use each term only once.
Square, kite, quadrilateral, rhombus, rectangle, parallelogram, trapezoid, and chevron



Which terms were left out?

____

____

Sketch
A crossed (complex) quadrilateral

A concave quadrilateral (no congruent sides)

A convex quadrilateral (no congruent sides)

A kite

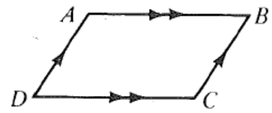
A rhombus

A chevron

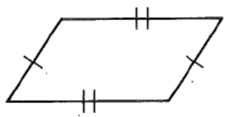
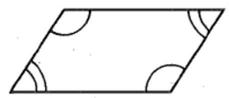
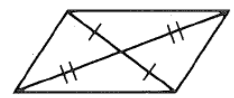
5–1 Properties of Parallelograms

Distinguish between the definition of a parallelogram and its theorems!

parallelogram A quadrilateral with both pairs of opposite sides parallel is a parallelogram.
In ▱$ABCD$, $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$.

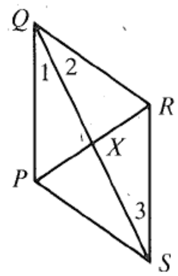


Definition

Opposite sides of a parallelogram are congruent.	**Opposite angles of a parallelogram are congruent.**	**Diagonals of a parallelogram bisect each other.**
		

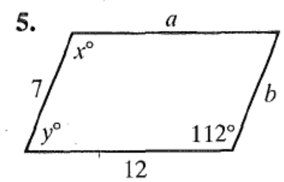
Theorems 5–1, 5–2, 5–3.

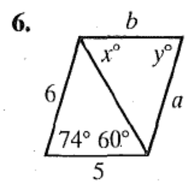
***PQRS* is a parallelogram. Complete.**

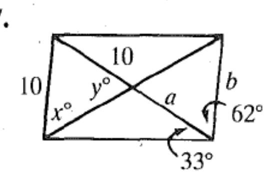


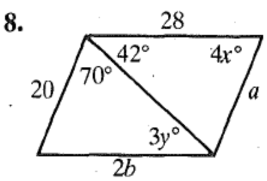
1. If $PS = 5$, then $QR = \underline{\ ?\ }$.
2. If $PR = 20$, then $PX = \underline{\ ?\ }$.
3. If $m\angle QPS = 125$, then $m\angle QRS = \underline{\ ?\ }$, and $m\angle PQR = m\angle PSR = \underline{\ ?\ }$.
4. If $m\angle 1 = 27$ and $m\angle 2 = 30$, then $m\angle 3 = \underline{\ ?\ }$ and $m\angle PSR = \underline{\ ?\ }$.

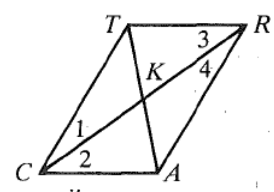
In Exercises 5–8, the quadrilateral is a parallelogram. Find the values of *a*, *b*, *x*, and *y*.

5. 

6. 

7. 

8. 



Given: ▱$TRAC$ **(on the far right). Complete.**

9. If $TC = 3x - 5$ and $RA = 25$, then $x = \underline{\ ?\ }$.

10. If $TK = 2x + 7$ and $KA = 15$, then $x = \underline{\ ?\ }$.

11. If $m\angle 2 = 2y - 5$ and $m\angle 3 = y + 17$, then $y = \underline{\ ?\ }$.

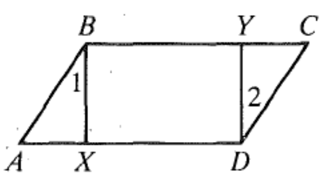
12. If $m\angle RTC = 120$ and $m\angle CAR = 8y$, then $y = \underline{\ ?\ }$.

13. If $m\angle CTR = 2y - 13$ and $m\angle ACT = y + 1$, then $y = \underline{\ ?\ }$.

14. Complete the proof.

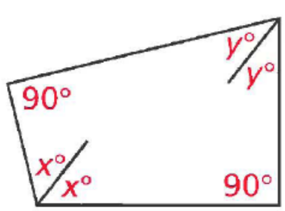
Given: ▱$ABCD$; $\angle 1 \cong \angle 2$

Prove: $\overline{AX} \cong \overline{YC}$



Statements	Reasons
1. ▱$ABCD$; $\angle 1 \cong \angle 2$	1. ________
2. $\angle A \cong \angle C$	2. ________
3. $\overline{AB} \cong \overline{DC}$	3. ________
4. △______ ≅ △______	4. ________
5. $\overline{AX} \cong \overline{YC}$	5. ________

Hint: use SSS, SAS, ASA or AAS.



38. Prove: If a segment whose endpoints lie on opposite sides of a parallelogram passes through the midpoint of a diagonal, that segment is bisected by the diagonal.

What can you conclude about

26. $2x + 2y$?

27. $x + y$?

5–2 Ways to Prove that Quadrilaterals Are Parallelograms

List 5 ways to prove that a quadrilateral is a parallelogram.

1. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

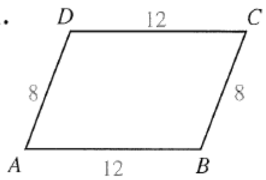
2.

3.

4.

5.

Set A. Decide if each quadrilateral must be a parallelogram. If the answer is yes, state the definition or theorem that applies.

1.


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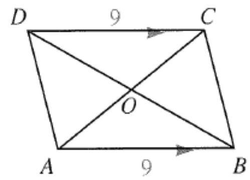

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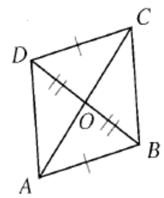

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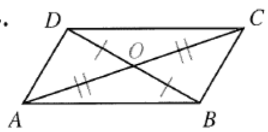

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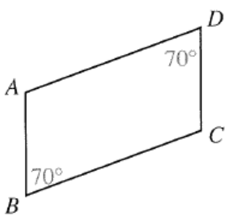

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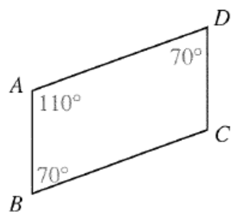

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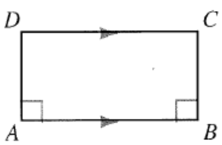

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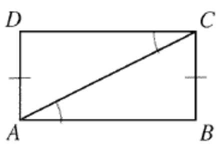

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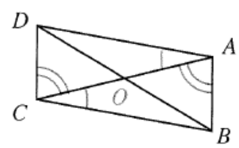
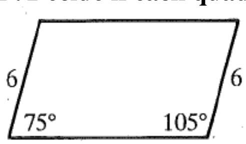


10. Draw a quadrilateral that has two pairs of congruent sides but that is *not* a parallelogram.

11. Draw a quadrilateral that is *not* a parallelogram but that has one pair of congruent sides and one pair of parallel sides.

Set B. Decide if each quadrilateral must be a parallelogram. If the answer is yes, state the definition or theorem that applies.

1.


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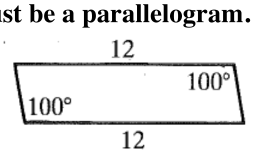
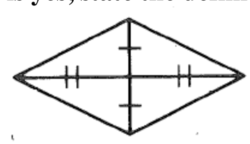


3.


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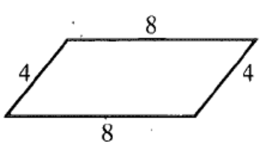

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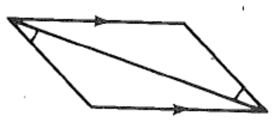

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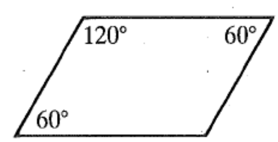
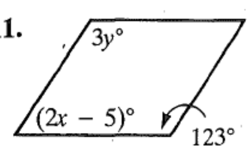
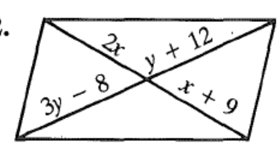
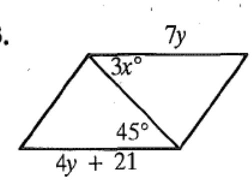


What values must *x* and *y* have to make the quadrilateral a parallelogram?

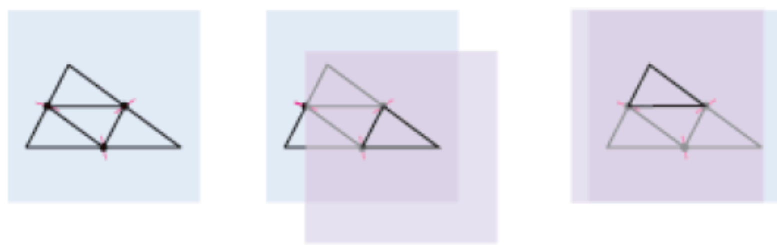
11.


12.


13.


Sheet 531: Investigating Triangle Midsegment Properties

Tools: straightedge, patty paper (2), and regular paper.
In this investigation you will discover two properties of triangle midsegments. Each person can investigate a different triangle.



Step 1 Draw a triangle on a piece of patty paper. Pinch the patty paper to locate midpoints of the sides. Draw the midsegments. You should now have four small triangles.

Step 2 Place a second piece of patty paper over the first and copy one of the four triangles.

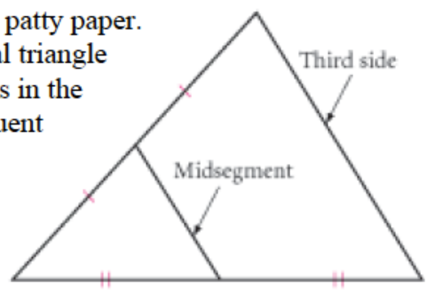
Step 3 Compare all four triangles by sliding the copy of one small triangle over the other three triangles. Compare your results with the results of your group. Copy and complete the conjecture.

Three Midsegments Conjecture C-41

The three midsegments of a triangle divide it into __?__ .

Redraw the original triangle on regular paper with just one midsegment.

Step 4 Mark all the congruent angles on the original patty paper. If you find it too cluttered, redraw the original triangle on regular paper with just one midsegment, as in the diagram at right, and then mark all the congruent angles. Using the Corresponding Angles Conjecture or its converse, what conclusions can you make about a midsegment and the large triangle's third side?



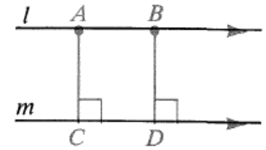
Step 5 Compare the length of the midsegment to the large triangle's third side. How do they relate? Copy and complete the conjecture.

Triangle Midsegment Conjecture C-42

A midsegment of a triangle is __?__ to the third side and __?__ the length of __?__ .

5–3 Theorems Involving Parallel Lines

Theorem 5–8.

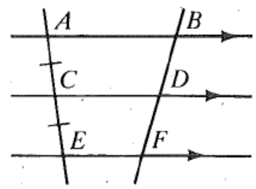


If two lines are parallel, then all points on one line are equidistant from the other line.

Theorem 5–9.

If three parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$ and $\overline{AC} \cong \overline{CE}$,
then $\overline{BD} \cong \overline{DF}$.

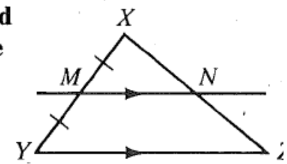


A1. If $BD = x + 6$ and $BF = 3x - 8$, find the value of x, BD, DF, and BF. Use the figure above.

Theorem 5–10.

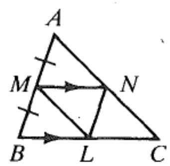
A line that contains the midpoint of one side of a triangle and is parallel to another side passes through the midpoint of the third side.

If M is the midpoint of $\overline{XY}$ and $\overleftrightarrow{MN} \parallel \overline{YZ}$,
then N is the midpoint of $\overline{XZ}$.



Example 2

Name all the points shown that *must* be midpoints of the sides of the large triangle.



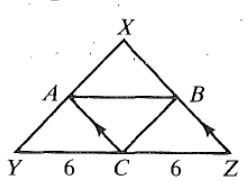
Solution

Since $AM = MB$, M is a midpoint.
Since M is a midpoint and $\overline{MN} \parallel \overline{BC}$, N is a midpoint.

Set A.

Name all the points shown that *must* be midpoints of the sides of the large triangle.

8.



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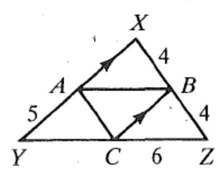
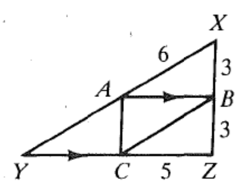


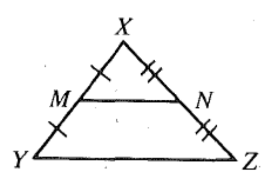
10.



Theorem 5–11. *The Midsegment Theorem.*

The segment that joins the midpoints of two sides of a triangle
(1) is parallel to the third side.
(2) is half as long as the third side.

If M is the midpoint of $\overline{XY}$ and N is the midpoint of $\overline{XZ}$,
then (1) $\overline{MN} \parallel \overline{YZ}$.
(2) $MN = \frac{1}{2}YZ$.

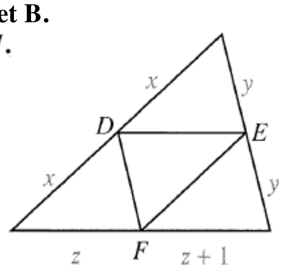


***M* is the midpoint of $\overline{YX}$ and *N* is the midpoint of $\overline{XZ}$.** **Use the figure above.**

11. If $MN = 6$, then $YZ = $ _?_.
12. If $YZ = 20$, then $MN = $ _?_.
13. If $MN = x$ and $YZ = 3x - 25$, then $x = $ _?_, $MN = $ _?_, and $YZ = $ _?_.
14. If $m\angle XMN = 40$, then $m\angle XYZ = $ _?_.

Set B.

7.



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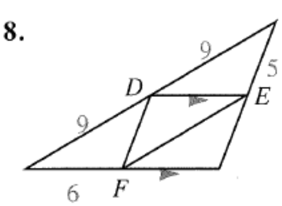

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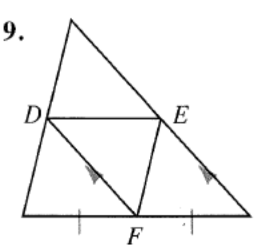
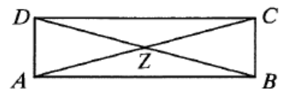


Self-Test 1

The diagonals of $\square ABCD$ intersect at Z. Tell whether each statement *must be*, *may be*, or *cannot be* true.



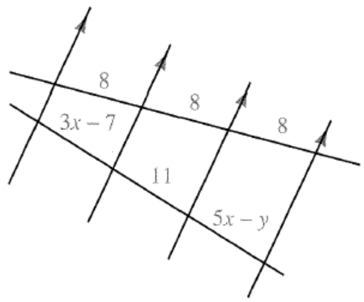
1. $\overline{AC} \cong \overline{BD}$ **2.** $\overline{DZ} \cong \overline{BZ}$

3. $\overline{AD} \parallel \overline{BC}$ **4.** $m\angle DAB = 85$ and $m\angle BCD = 95$

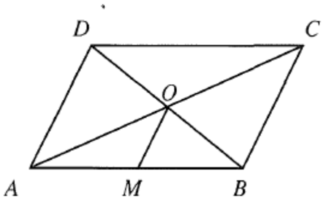
5. List five ways to prove that quad. $ABCD$ is a parallelogram.

6. a. State a theorem that allows you to conclude that $3x - 7 = 11$.

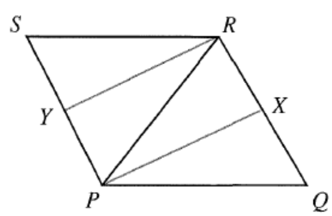
b. Find the values of x and y.



7. Given: $\square ABCD$;
M is the midpoint of $\overline{AB}$.
Prove: $MO = \frac{1}{2}AD$

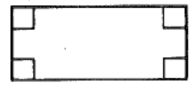


8. Given: $\square PQRS$;
$\overline{PX}$ bisects $\angle QPR$;
$\overline{RY}$ bisects $\angle SRP$.
Prove: $RYPX$ is a $\square$.

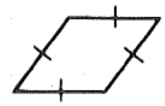


5–4 Special Quadrilaterals

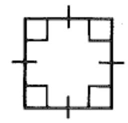
rectangle A quadrilateral with four right angles is a rectangle.
A rectangle is a parallelogram because both pairs of opposite angles are congruent.

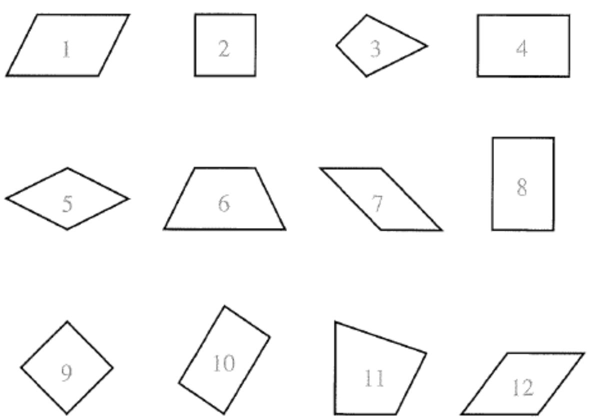


rhombus A quadrilateral with four congruent sides is a rhombus.
A rhombus is a parallelogram because both pairs of opposite sides are congruent.



square A quadrilateral with four right angles and four congruent sides is a square.
A square is a parallelogram, and a rectangle, and a rhombus.





1. Write the numbers.

Parallelograms:

Rectangles:

Rhombuses:

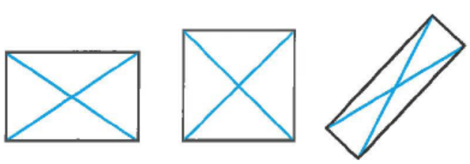
Squares:

Both rectangles and rhombuses:

Rectangles but not squares:

Rhombuses but not squares:

Rectangles. Each of the figures below is a rectangle. The diagonals are shown in blue.



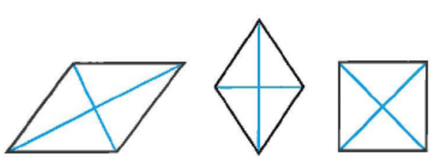
16. What is always true about the angles of a rectangle?

What seems to be true about

17. the opposite sides of a rectangle?

18. the diagonals of a rectangle?

Rhombuses. Each of the figures below is a *rhombus.* The diagonals are shown in blue.

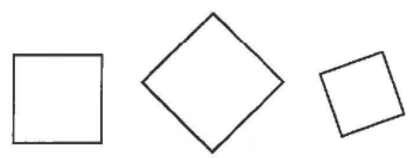


What seems to be true about

19. the sides of a rhombus?

20. the diagonals of a rhombus?

Squares. Each of the figures below is a *square.*

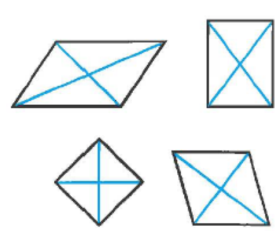


What property do you think squares have in common with

21. rectangles?

22. rhombuses?

Parallelograms. Each of the figures below is a parallelogram. The diagonals are shown in blue.



What seems to be true about

23. the opposite sides of a parallelogram?

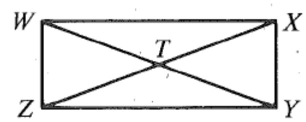
24. the opposite angles of a parallelogram?

25. the diagonals of a parallelogram?

Set A.

A rectangle has congruent diagonals (Thm 5–12).
If one angle of a parallelogram is a right angle, then the parallelogram is a rectangle (Thm 5–16).

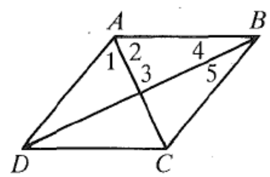
Given: Quad. *WXYZ* is a rectangle. Complete.



1. If $WY = 19$, then $ZX =$ _?_.
2. If $WY = 19$, then $WT =$ _?_.
3. If $TX = 4.5$, then $WY =$ _?_.
4. If $WY = 3a + 16$ and $ZX = 5a - 18$, then $a =$ _?_.
5. If $m\angle TWZ = 70$, then $m\angle TZW =$ _?_ and $m\angle WTZ =$ _?_.

A rhombus has diagonals that are perpendicular (Thm 5–13) and has diagonals that bisect its angles (Thm 5–14).
If two consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus (Thm 5–17).

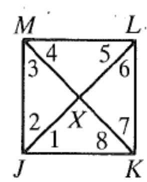
Given: Quad. *ABCD* is a rhombus. Complete.



6. If $AD = 13$, then $AB =$ _?_.
7. If $m\angle 4 = 25$, then $m\angle 5 =$ _?_.
8. If $m\angle DAB = 130$, then $m\angle ADC =$ _?_.
9. If $m\angle 4 = 3x - 2$ and $m\angle 5 = 2x + 7$, then $x =$ _?_.
10. If $m\angle 1 = 5x + 18$ and $m\angle 5 = 3x - 8$, then $x =$ _?_.
11. If $m\angle 2 = 3y + 9$ and $m\angle 4 = 2y - 4$, then $y =$ _?_.

A square has all the properties of parallelograms, rectangles, and rhombuses.

Given: Quad. *JKLM* is a square. Complete.



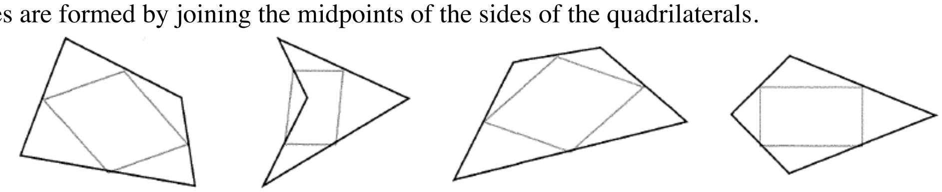
12. If $MJ = 12$, then $ML =$ _?_ and $LK =$ _?_.
13. If $MX = 8$, then $XJ =$ _?_.
14. If $JL = 18$, then $MK =$ _?_, $JX =$ _?_, and $XK =$ _?_.
15. $m\angle MJK =$ _?_ and $m\angle MXJ =$ _?_
16. The numbered angles are all congruent, and each angle has measure _?_.

Set B.

	Property	Parallelogram	Rectangle	Rhombus	Square
1.	Opp. sides are ∥.				
2.	Opp. sides are ≅.				
3.	Opp. ∠s are ≅.				
4.	A diag. forms two ≅ △s.				
5.	Diags. bisect each other.				
6.	Diags. are ≅.				
7.	Diags. are ⊥.				
8.	A diag. bisects two ∠s.				
9.	All ∠s are rt. ∠s.				
10.	All sides are ≅.				

B11. In the diagrams below, the smaller figures are formed by joining the midpoints of the sides of the quadrilaterals.

a. What seems to be the common property of the smaller figures?



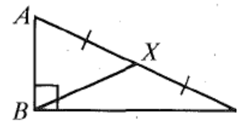
b. Describe how you would prove your answer to part a.

The circumcircle passes through each vertex of a triangle; its radius is the distance between the circumcenter and each vertex. The circumcenter is the intersection of the perpendicular bisectors. For a right triangle, the circumcenter is the midpoint of the hypoteneuse.

Theorem 5–15.

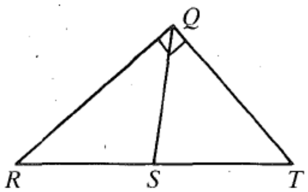
The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices.

If $\triangle ABC$ has right $\angle ABC$ and X is the midpoint of $\overline{AC}$, then $XA = XC = XB$.



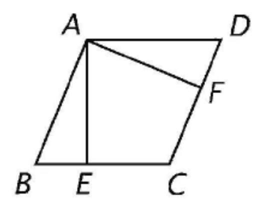
In right $\triangle RQT$, $\overline{QS}$ is a median. Complete.

17. If $ST = 8$, then $QS =$ ___?___.
18. If $QS = 12$, then $RT =$ ___?___.
19. If $m\angle T = 55$, then $m\angle SQT =$ ___?___.
20. If $m\angle T = 65$, then $m\angle RQS =$ ___?___.
21. If $m\angle R = 35$, then $m\angle RQS =$ ___?___.



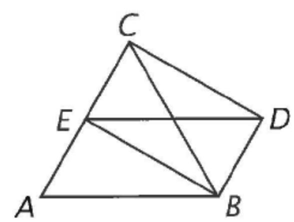
47. a. Mark the figure with the given information.
b. It looks as if $AE = AF$. Is this necessarily true? Explain why or why not.

Rhombus Problem. ABCD is a rhombus, AE ⊥ BC, and AF ⊥ CD.

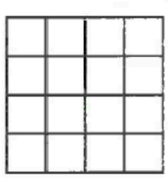


49. a. Mark the figure with the given information.
b. It looks as if $\triangle ABC$ is equilateral. What can be proved about $\triangle ABC$?

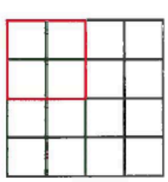
Triangle Problem. ABDE is a parallelogram and BDCE is a rectangle.



Counting Squares. Acute Alice drew this figure and asked Obtuse Ollie how many squares it contains. Ollie said, "16."

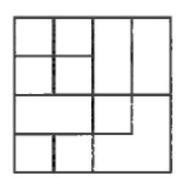


Alice then shaded the one outlined in red in the figure below and asked Ollie, "Did you count this one?" To which Ollie replied, "17!"



1. a. How many squares are there? Explain.
b. What if it were a 5 by 5 square? N by N?

For revenge, Ollie then drew the figure below and asked Alice how many squares it contains.* Alice said, "14."



2. Do you agree? If not, what is your answer? Explain.

*From *Are You as Smart as You Think?* by Terry Stickels (Thomas Dunne Books, 2000).

Sheet 552: Investigating Trapezoid Midsegment Properties

Tools: straightedge, and patty paper (2).
In this investigation you will discover properties of trapezoid midsegments. Each person can investigate a different trapezoid. Make sure you draw the two bases perfectly parallel. How can you do that? What can you use?

Step 1 Draw a small trapezoid on the left side of a piece of patty paper. Pinch the paper to locate the midpoints of the nonparallel sides. Draw the midsegment.

Step 2 Label the angles as shown. Place a second piece of patty paper over the first and copy the trapezoid and its midsegment.

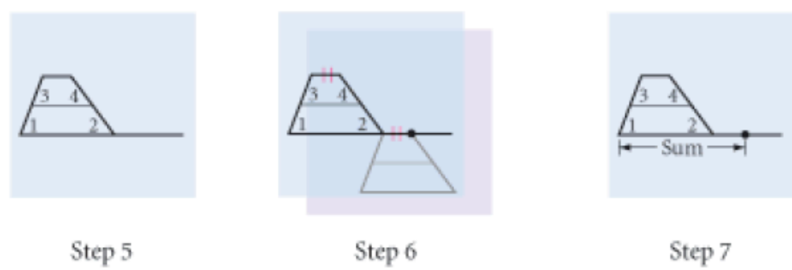
Step 3 Compare the trapezoid's base angles with the corresponding angles at the midsegment by sliding the copy up over the original.

Step 4 Are the corresponding angles congruent? What can you conclude about the midsegment and the bases? Compare your results with the results of other students.

The midsegment of a triangle is half the length of the third side. How does the length of the midsegment of a trapezoid compare to the lengths of the two bases? Let's investigate.

Step 5 On the original trapezoid, extend the longer base to the right by at least the length of the shorter base.

Step 6 Slide the second patty paper under the first. Show the sum of the lengths of the two bases by marking a point on the extension of the longer base.



Step 7 How many times does the midsegment fit onto the segment representing the sum of the lengths of the two bases? What do you notice about the length of the midsegment and the sum of the lengths of the two bases?

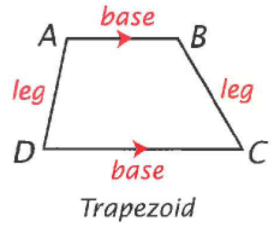
Step 8 Combine your conclusions from Steps 4 and 7 and complete this conjecture.

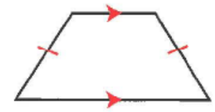
Trapezoid Midsegment Conjecture C-43

The midsegment of a trapezoid is _?_ to the bases and is equal in length to _?_.

What happens if one base of the trapezoid shrinks to a point? Then the trapezoid collapses into a triangle, the midsegment of the trapezoid becomes a midsegment of the triangle, and the Trapezoid Midsegment Conjecture becomes the Triangle Midsegment Conjecture. **How** does the Trapezoid Midsegment Conjecture turn into the Triangle Midsegment Conjecture? (It must, right?)

5–5 Trapezoids



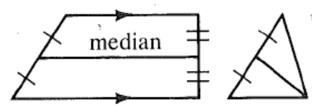


Isosceles trapezoid

trapezoid A quadrilateral with exactly one pair of parallel sides is a trapezoid.
The parallel sides are called **bases.**
The other sides are called **legs.**

isosceles trapezoid A trapezoid with congruent legs is isosceles.

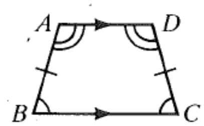
median of a trapezoid The segment that joins the midpoints of the legs of the trapezoid is the median. Notice the difference between the median of a trapezoid and a median of a triangle.



A1. Do this: Draw a right-angle trapezoid in the right margin; they are used in integral calculus.

Base angles of an isosceles trapezoid are congruent.

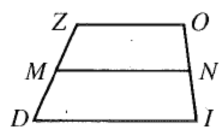
If trapezoid $ABCD$ has $\overline{AB} \cong \overline{DC}$, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.



Thm 5–18.

The median of a trapezoid
(1) is parallel to the bases.
(2) has a length equal to the average of the base lengths.

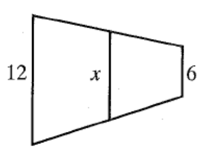
If trapezoid $ZOID$ has median $\overline{MN}$,
then (1) $\overline{ZO} \parallel \overline{MN} \parallel \overline{DI}$.
(2) $MN = \frac{1}{2}(ZO + DI)$.



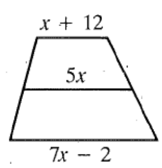
Thm 5–19.

Example 1 In parts (a) and (b), a trapezoid and its median are shown. Find the value of x.

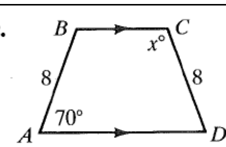
a.



b.



c.



Solution

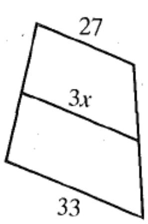
a. $x = \frac{1}{2}(6 + 12)$
$x = 9$

b. $5x = \frac{1}{2}[(x + 12) + (7x - 2)]$
$2(5x) = 8x + 10$
$2x = 10$
$x = 5$

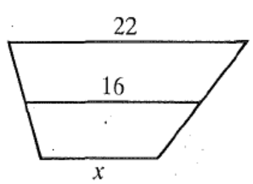
c. $m\angle A = m\angle D = 70$
$m\angle A + m\angle B = 180$
$70 + m\angle B = 180$
$m\angle B = 110$
$= m\angle C$
$= x$

Each diagram shows a trapezoid and its median. Find the value of *x*.

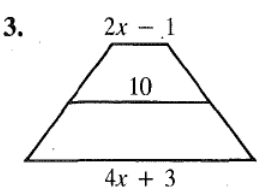
1.



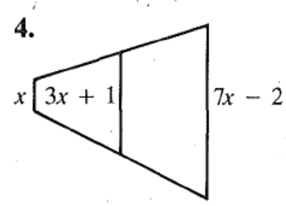
2.



3.

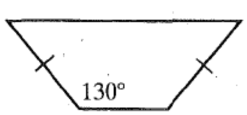


4.

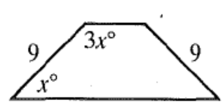


Find the measure of each angle in the isosceles trapezoids.

5.



6.



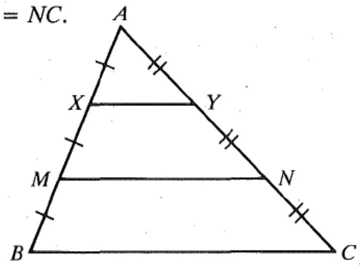
7. One angle of an isosceles trapezoid has measure 48. Find the measures of the other angles.

8. Two congruent angles of an isosceles trapezoid have measures $5x - 17$ and $2x + 13$. Find the value of x and then give the measure of each angle of the trapezoid.

Example 2 In $\triangle ABC$, $AX = XM = MB$ and $AY = YN = NC$.

a. If $XY = 6$, and $BC = 18$, find MN.

b. If $XY = 12$, find MN.



Solution

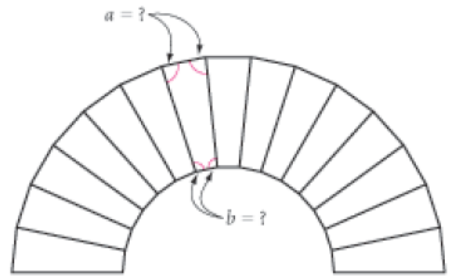
a. $\overline{MN}$ is the median of trap. $BXYC$.

$MN = \frac{1}{2}(XY + BC) = \frac{1}{2}(6 + 18) = 12$

b. $\overline{XY}$ joins the midpoints of two sides of $\triangle AMN$.

$XY = \frac{1}{2}MN \qquad 12 = \frac{1}{2}MN \qquad 24 = MN$

18. What is the measure of each angle in the isosceles trapezoid face of a voussoir in this 15-stone arch?



Set A.

Use the diagram in Example 2. Complete.

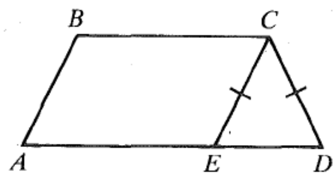
9. If $XY = 9$, then $MN =$ _?_ and $BC =$ _?_.

10. If $MN = 32$, then $XY =$ _?_ and $BC =$ _?_.

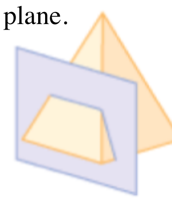
11. If $XY = 8$ and $MN = x + 12$, then $x =$ _?_ and $BC =$ _?_.

12. Given: Isosceles trap. $ABCD$; $\overline{CD} \cong \overline{CE}$

Prove: $ABCE$ is a ▱.



20. Sketch the section formed when this pyramid is sliced by the plane.



Set B.

Draw a quadrilateral of the type named. Join, in order, the midpoints of the sides. What special kind of quadrilateral do you appear to get?

21. rhombus

22. rectangle

23. isosceles trapezoid

24. non-isosceles trapezoid

25. quadrilateral with no congruent sides

A *kite* is a quadrilateral that has two pairs of congruent sides, but opposite sides are not congruent.

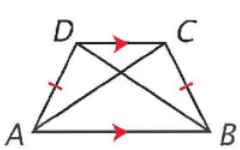
28. Draw a convex kite. Discover, state, and prove whatever you can about the diagonals and angles of a kite.

29. **a.** Draw a convex kite. Join, in order, the midpoints of the sides. What special kind of quadrilateral do you appear to get?

b. Repeat part (a), but draw a nonconvex kite.

Set C.

Theorem 36. The diagonals of an isosceles trapezoid are equal.

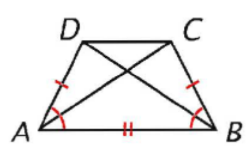


Given: ABCD is an isosceles trapezoid with bases AB and DC.

Prove: DB = CA.

Proof

29. Because ABCD is an isosceles trapezoid, DA = CB. Why?



30. Also, ∠DAB = ∠CBA. Why?

31. AB = AB. Why?

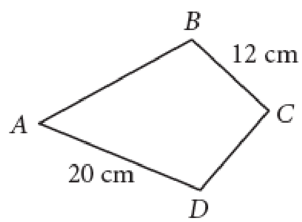
32. So △DAB ≅ △CBA. Why?

33. Therefore, DB = CA. Why?

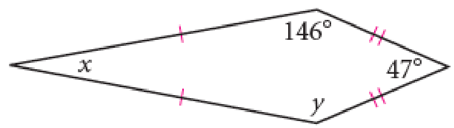
Sheet #551: Quadrilateral Properties Problems.

DG 4th
p271

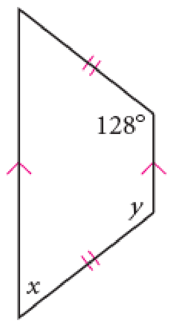
1. *ABCD* is a kite.
perimeter = ?



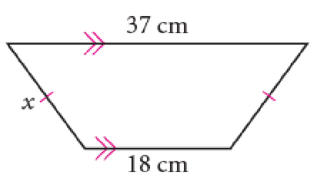
2. x = ?
y = ?



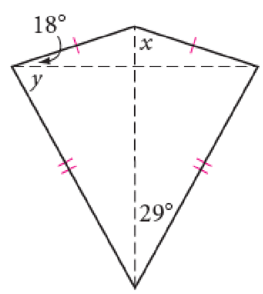
3. x = ?
y = ?



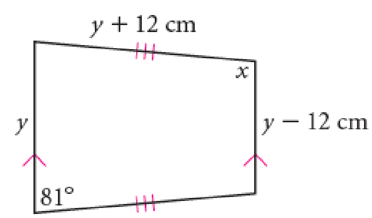
4. x = ?
perimeter = 85 cm



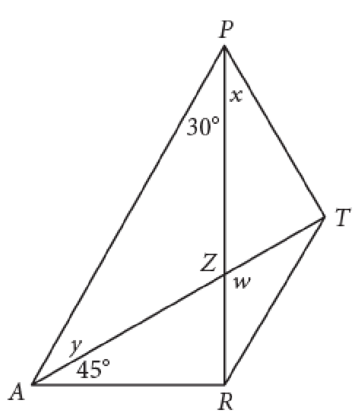
5. x = ?
y = ?



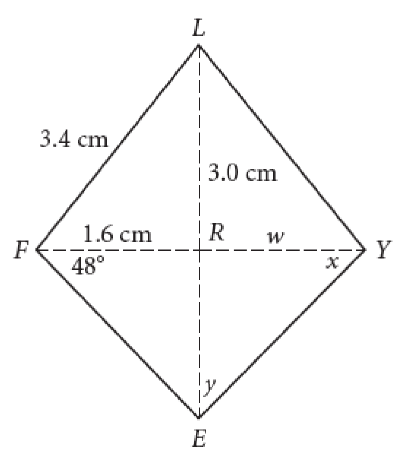
6. x = ?
y = ?
perimeter = 164 cm



7. *ARTP* is an isosceles trapezoid with $RA = PT$. Find w, x, and y. ⓗ



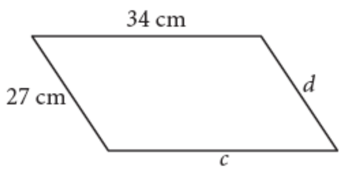
8. *FLYE* is a kite with $FL = LY$. Find w, x, and y.



Use your new conjectures in the following exercises.
In Exercises 1–6, each figure is a parallelogram.

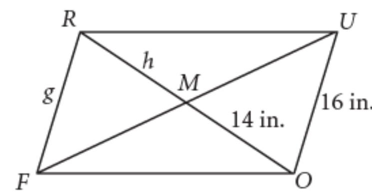
for Exercises 7 and 8
DG 4th
p283

1. $c = \underline{?}$
$d = \underline{?}$

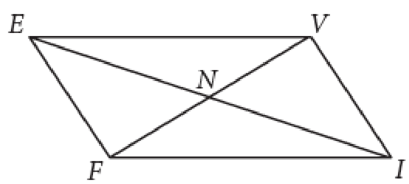


2. $a = \underline{?}$
$b = \underline{?}$

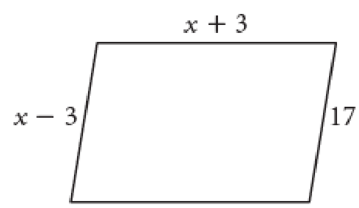
3. $g = \underline{?}$
$h = \underline{?}$



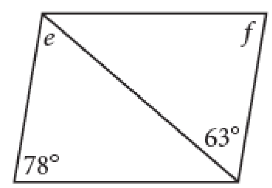
4. $VF = 36$ m
$EF = 24$ m
$EI = 42$ m
What is the perimeter of $\triangle NVI$? ⓗ



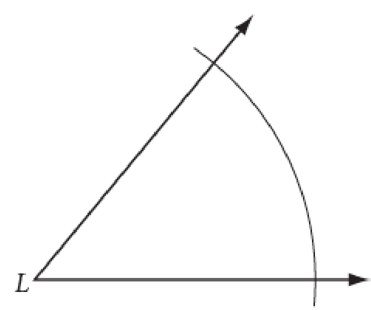
5. What is the perimeter?



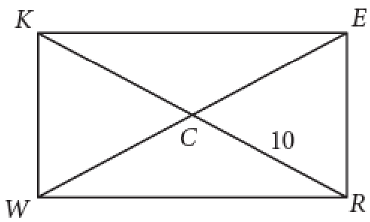
6. $e = \underline{?}$
$f = \underline{?}$



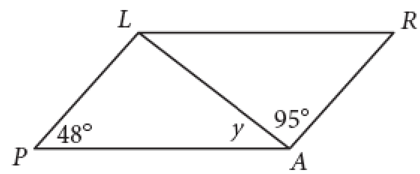
7. ***Construction*** Given side $\overline{LA}$, side $\overline{AS}$, and $\angle L$, construct parallelogram $LAST$.



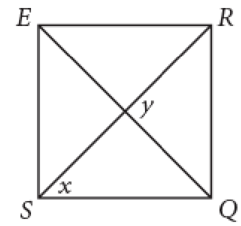
11. *WREK* is a rectangle.
$CR = 10$
$WE = \underline{?}$



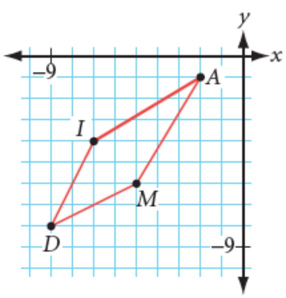
12. *PARL* is a parallelogram.
$y = \underline{?}$ ⓗ



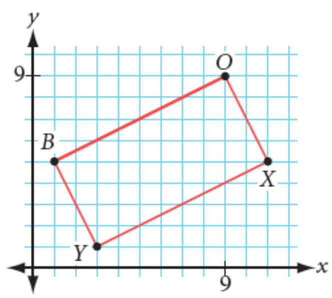
13. *SQRE* is a square.
$x = \underline{?}$
$y = \underline{?}$



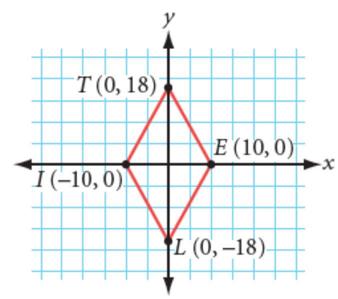
14. Is *DIAM* a rhombus? Why?



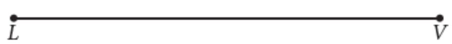
15. Is *BOXY* a rectangle? Why?



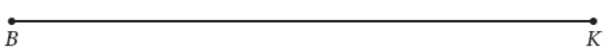
16. Is *TILE* a parallelogram? Why?

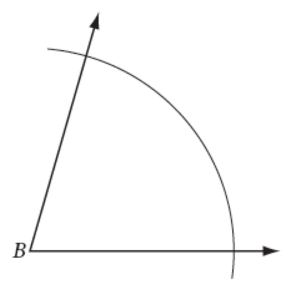


17. ***Construction*** Given the diagonal $\overline{LV}$, construct square *LOVE*. ⓗ

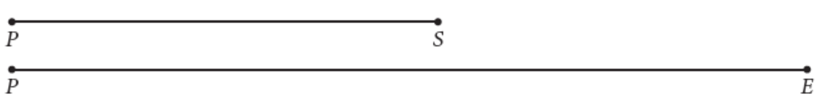


18. ***Construction*** Given diagonal $\overline{BK}$ and $\angle B$, construct rhombus *BAKE*. ⓗ





19. ***Construction*** Given side $\overline{PS}$ and diagonal $\overline{PE}$, construct rectangle *PIES*.



Self-Test 2

Quad. *WXYZ* must be a special figure to meet the conditions stated. Write the best name for that special quadrilateral.

1. $\overline{WX} \cong \overline{YZ}$ and $\overline{WX} \parallel \overline{YZ}$

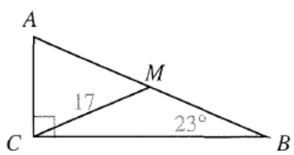
2. $\overline{WX} \parallel \overline{YZ}$ and $\overline{WX} \not\cong \overline{YZ}$

3. $\overline{WX} \cong \overline{YZ}$, $\overline{XY} \cong \overline{ZW}$, and diag. $\overline{WY} \cong$ diag. $\overline{XZ}$

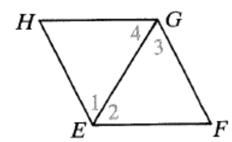
4. Diagonals $\overline{WY}$ and $\overline{XZ}$ are congruent and are perpendicular bisectors of each other.

5. An isosceles trapezoid has sides of lengths 5, 8, 5, and 14. Find the length of the median.

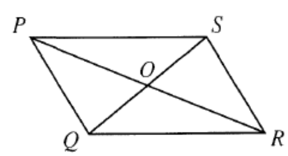
6. M is the midpoint of hypotenuse $\overline{AB}$.
   Find AM and $m\angle ACM$.



7. Given: $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$
   Prove: $EFGH$ is a rhombus.

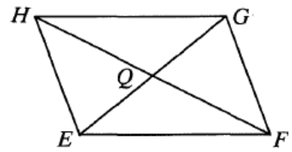


8. $PQRS$ is a ▱.
   a. If X is the midpoint of $\overline{PQ}$ and Y is the midpoint of $\overline{SR}$, what special kind of quadrilateral is $XQRY$?
   b. Prove your answer to part (a).
   c. Draw a line through O intersecting $\overline{PQ}$ at J and $\overline{SR}$ at K. If J and K are not midpoints, what special kind of quadrilateral is $JQRK$?



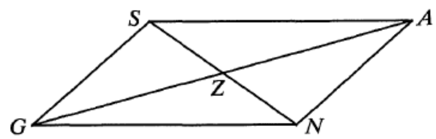
Chapter Review

In parallelogram $EFGH$, $m\angle EFG = 70$.



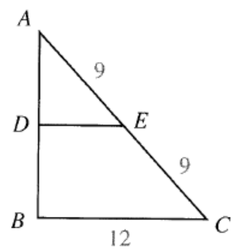
1. $m\angle HEF =$ _?_ 5–1
2. If $m\angle EFH = 32$, then $m\angle EHF =$ _?_.
3. If $HQ = 14$, then $HF =$ _?_.
4. If $EH = 8x - 7$ and $FG = 5x + 11$, then $x =$ _?_.

In each exercise you could prove that quad. $SANG$ is a parallelogram if one more fact, in addition to those stated, were given. State that fact.



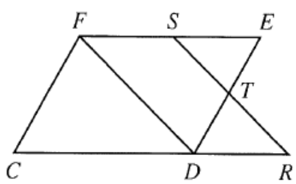
5. $GN = 9$; $NA = 5$; $SA = 9$ 5–2
6. $\angle ASG \cong \angle GNA$
7. $\overline{SZ} \cong \overline{NZ}$
8. $\overline{SA} \parallel \overline{GN}$; $SA = 17$

State the principal theorem that justifies the statement about the diagram.



9. If $\overline{DE} \parallel \overline{BC}$, then D is the midpoint of $\overline{AB}$. 5–3
10. If D is the midpoint of $\overline{AB}$, then $\overline{DE} \parallel \overline{BC}$.
11. If D is the midpoint of $\overline{AB}$, then $DE = 6$.

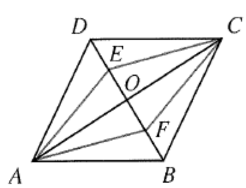
12. Given: $\square CDEF$; S and T are the midpoints of $\overline{EF}$ and $\overline{ED}$.
    Prove: $\overline{SR} \cong \overline{FD}$



Give the most descriptive name for quad. $MNOP$.

13. $\overline{MN} \cong \overline{PO}$; $\overline{MN} \parallel \overline{PO}$ 5–4
14. $\overline{MN} \parallel \overline{PO}$; $\overline{NO} \parallel \overline{MP}$; $\overline{MO} \perp \overline{NP}$
15. $\angle M \cong \angle N \cong \angle O \cong \angle P$
16. $MNOP$ is a rectangle with $MN = NO$.

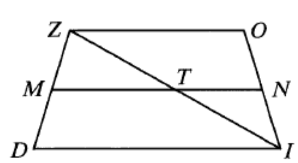
17. Given: $ABCD$ is a rhombus; $DE = BF$
    Prove: $AECF$ is a rhombus.



Draw and label a diagram. List, in terms of the diagram, what is given and what is to be proved. Then write a proof.

18. $\overline{PX}$ and $\overline{QY}$ are altitudes of acute $\triangle PQR$, and Z is the midpoint of $\overline{PQ}$. Prove that $\triangle XYZ$ is isosceles.

$\overline{MN}$ is the median of trapezoid $ZOID$.



19. The bases of trap. $ZOID$ are _?_ and _?_. 5–5
20. If $ZO = 8$ and $MN = 11$, then $DI =$ _?_.
21. If $ZO = 8$, then $TN =$ _?_.
22. If trap. $ZOID$ is isosceles and $m\angle D = 80$, then $m\angle O =$ _?_.

Answers

Self-Test 1.

1. may be **2.** must be **3.** must be **4.** cannot be **5.** See page 172. **6. a.** If 3 ∥ lines cut off ≅ seg. on one trans., then they cut off ≅ seg. on every trans. **b.** $x = 6$, $y = 19$ **7.** 1. $ABCD$ is a ▱. (Given) 2. $\overline{AC}$ and $\overline{BD}$ bis. each other. (Diags. ▱ bis. each other.) 3. O is the midpt. of $\overline{BD}$. (Def. of bis.) 4. M is the midpt. of $\overline{AB}$. (Given) 5. $MO = \frac{1}{2}AD$ (Thm. 5-11) **8.** 1. $PQRS$ is a ▱. (Given) 2. $\overline{SR} \parallel \overline{PQ}$; $\overline{SP} \parallel \overline{RQ}$ (Def. of ▱) 3. $m\angle QPR = m\angle SRP$ (If lines ∥, alt. int. ∠s ≅.) 4. $\overrightarrow{PX}$ bis. $\angle QPR$; $\overrightarrow{RY}$ bis. $\angle SRP$. (Given) 5. $m\angle RPX = \frac{1}{2}m\angle QPR$; $m\angle PRY = \frac{1}{2}m\angle SRP$ (∠ Bis. Thm.) 6. $\frac{1}{2}m\angle QPR = \frac{1}{2}m\angle SRP$ (Mult. Prop. of =) 7. $m\angle RPX = m\angle PRY$ (Subst.) 8. $\overline{YR} \parallel \overline{PX}$ (If alt. int. ∠s ≅, lines ∥.) 9. $RYPX$ is a ▱. (Def. of ▱)

Self-Test 2.

1. ▱ **2.** trap. **3.** rect. **4.** sq. **5.** 11 **6.** 17, 67 **7.** 1. $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$ (Given) 2. $\overline{HG} \parallel \overline{EF}$; $\overline{HE} \parallel \overline{GF}$ (If alt. int. ∠s ≅, lines ∥.) 3. $EFGH$ is a ▱. (Def. of ▱) 4. $\overline{HG} \cong \overline{HE}$ (If 2 ∠s of a △ are ≅, sides opp. the ∠s are ≅.) 5. $HGFE$ is a rhom. (Thm. 5-17) **8. a.** ▱ **b.** 1. $PQRS$ is a ▱. (Given) 2. $\overline{PQ} \parallel \overline{SR}$ (Def. of ▱) 3. X is the midpt. of $\overline{PQ}$; Y is the midpt. of $\overline{SR}$. (Given) 4. $XQ = \frac{1}{2}PQ$; $YR = \frac{1}{2}SR$ (Midpt. Thm.) 5. $PQ = SR$ (Thm. 5-1) 6. $\frac{1}{2}PQ = \frac{1}{2}SR$ (Mult. Prop. of =) 7. $XQ = YR$ (Subst.) 8. $XQRY$ is a ▱. (Thm. 5-5) **c.** trap.

Chapter 5 Review.

1. 110 **3.** 28 **5.** $GS = 5$ or $\overline{SA} \parallel \overline{GN}$ **7.** $\overline{AZ} \cong \overline{GZ}$ **9.** Thm. 5-10 **11.** Thm. 5-11, part (2) **13.** ▱ **15.** rect. **17.** Key steps of proof: 1. $DO = BO$; $AO = CO$ (Diags. ▱ bis. each other.) 2. $EO = FO$ (Subtr. Prop. =) 3. $AECF$ is a ▱. (Thm. 5-7) 4. $\overline{BD} \perp \overline{AC}$ (Diags. of rhom. ⊥.) 5. $\triangle COE \cong \triangle COF$ (SAS) 6. $\overline{CE} \cong \overline{CF}$ (CPCT) 7. $AECF$ is a rhom. (Thm. 5-17) **19.** $\overline{ZO}$, $\overrightarrow{DI}$ **21.** 4