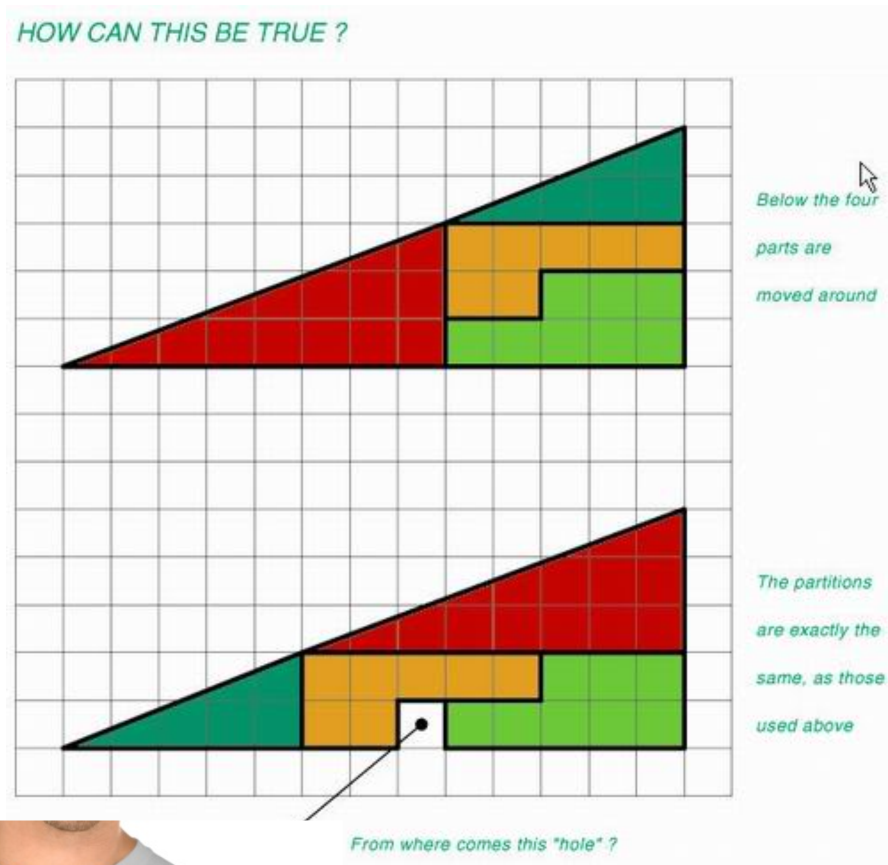
Full Name: ________________________________ Block/Period: ____
v. 1.0

TrianglePacket

Chapter 4

Congruent Triangles



Sheet 424: Triangle Congruency Postulates

Postulate 12 (SSS)
If **three sides** of one triangle
are congruent to
three sides of another triangle,
then the triangles are congruent.

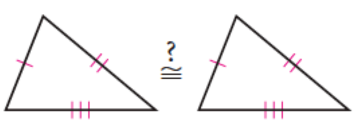
Postulate 13 (SAS)
If **two sides** and the <u>included angle</u> of one triangle
are congruent to
two sides and the included angle of another triangle,
then the triangles are congruent.

Postulate 14 (ASA)
If **two angles** and the <u>included side</u> of one triangle
are congruent to
two angles and the included side of another triangle,
then the two triangles are congruent.

Ways to Prove Two
Triangles Congruent

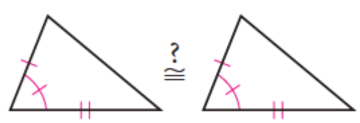
Postulates:	SSS SAS ASA	Similar AA(A)
Theorems:	AAS HL (LL,HA,LA)	No Conclusion SSA

Side-Side-Side (SSS)



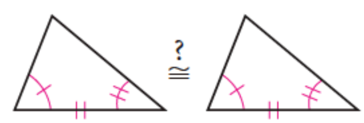
Three pairs of congruent sides

Side-Angle-Side (SAS)



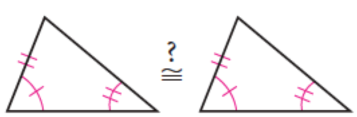
Two pairs of congruent sides and one pair of congruent angles (angles between the pairs of sides)

Angle-Side-Angle (ASA)



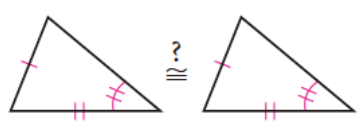
Two pairs of congruent angles and one pair of congruent sides (sides between the pairs of angles)

Angle-Angle-Side (AAS) or SAA



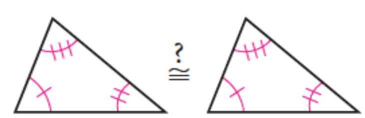
Two pairs of congruent angles and one pair of congruent sides (sides not between the pairs of angles)

Side-Side-Angle (SSA)



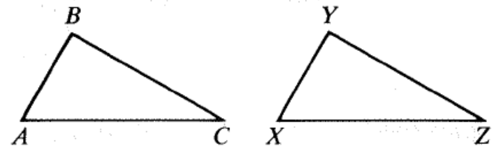
Two pairs of congruent sides and one pair of congruent angles (angles not between the pairs of sides)

Angle-Angle-Angle (AAA)

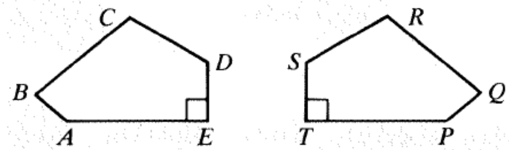


Three pairs of congruent angles

4–1 Congruent Figures



$\triangle ABC \cong \triangle XYZ$ $\triangle CBA \cong \triangle ZYX$
$\triangle BCA \cong \triangle YZX$ $\triangle ACB \cong \triangle XZY$



$ABCDE \cong PQRST$ $AEDCB \cong PTSRQ$
$BCDEA \cong QRSTP$ $CDEAB \cong RSTPQ$

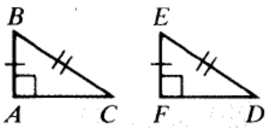
Two objects are **congruent** if they have the same size and shape.

Two polygons/**triangles are congruent** if and only if their vertices can be matched up so that corresponding angles and sides of the polygons/triangles are congruent.

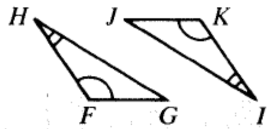
Vertices are listed in order of **correspondence**.

The figures shown are congruent. Write two statements that describe the congruence.

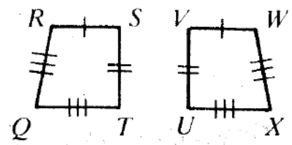
1.



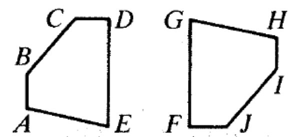
2.



3.



4.



CPCTC = Corresponding Parts of Congruent Triangles are Congruent,
"corr. parts of ≅ ⚠ are ≅." Angles and sides are called "parts" ☹.

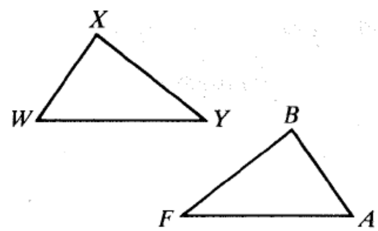
Triangles are Congruent if and only if Corresponding Angles and Sides are Congruent.

TAC
CASC

3 pairs of angles and **3 pairs of sides** are congruent in two congruent triangles.

Suppose $\triangle WXY \cong \triangle ABF$. Complete.

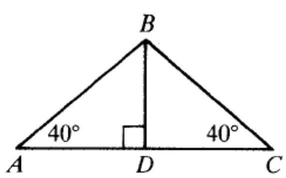
5. $\angle W \cong$ _?_ **6.** $m\angle B =$ _?_
7. $\overline{XY} \cong$ _?_ **8.** $AF =$ _?_
9. $\angle F \cong$ _?_ **10.** $\overline{YX} \cong$ _?_
11. $\triangle YWX \cong$ _?_ **12.** $\triangle BFA \cong$ _?_



The triangles shown are congruent. Complete.

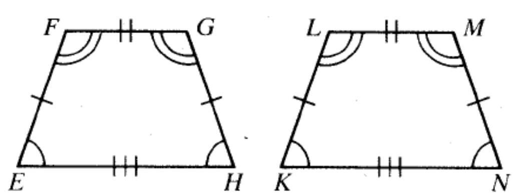
13. $\triangle ABD \cong$ _?_ **14.** $\overline{AB} \cong$ _?_
15. $DC =$ _?_ **16.** $\angle ABD \cong$ _?_

17. What property allows you to conclude that $\overline{BD} \cong \overline{BD}$?
($\overline{BD}$ is called a **common side** of the two triangles.)

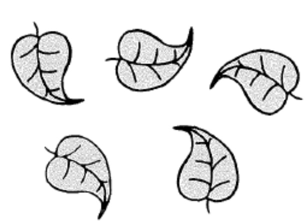


The quadrilaterals shown are congruent. Complete.

20. $\angle E \cong$ _?_ $\cong$ _?_ $\cong$ _?_
21. $\overline{EF} \cong$ _?_ $\cong$ _?_ $\cong$ _?_
22. $\overline{FG} \cong$ _?_
23. $\overline{EH} \cong$ _?_
24. $EFGH \cong$ _?_ and $EFGH \cong$ _?_



17. The five leaves shown are all congruent, but one differs from the others. Which one is different and how?



Sheet 423: Triangle Congruency Investigation

Name: ___________________________

Material: Patty paper (6 in^2), scissors, straightedge, pencil, tape (optional).
Objective: Explore the cases for which two triangles are congruent if given only three corresponding parts.

A whole original triangle is given for each activity. Triangle A covers activities 1 through 3.

For each activity, three parts of another triangle (or triangles) are drawn. The parts are congruent with the corresponding parts of the given original triangle.

Instructions

a) Trace the parts of the triangle onto a patty paper. Trace the lines **and dots**. Use a straightedge.

b) Cut the patty paper into its three separate parts. Only make two cuts (**don't trim**). Leave space to extend angle rays.

c) Arrange the pieces into a triangle. The rays extending from an angle may need to be extended (or overlapped).
Sides have to terminate at a given angle. That is, the dots have to match up.
IMPORTANT: Pay attention which angles or sides are included (between the two other parts). Be sure to put the parts in the correct sequence in activities 4, 5, and 6.

d) Is your triangle congruent to the original triangle?

e) Try to rearrange the parts into another triangle.

f) Can you make ***another*** *triangle from the three parts that is not congruent with the original triangle?*

g) If you can make a triangle that is not congruent with the original triangle, you have a counter example. To keep it, trace it onto a new piece of patty paper, or tape it together.

h) Fill in the conjectures below.

Triangle Congruence Shortcut Conjectures

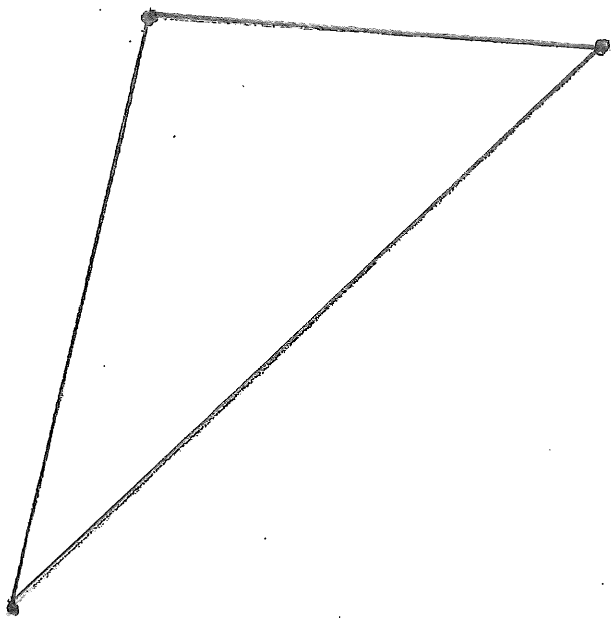
Triangles in the following cases *must be congruent*:

SSS? AAA? SAS? ASA? AAS? (SAA) SSA?

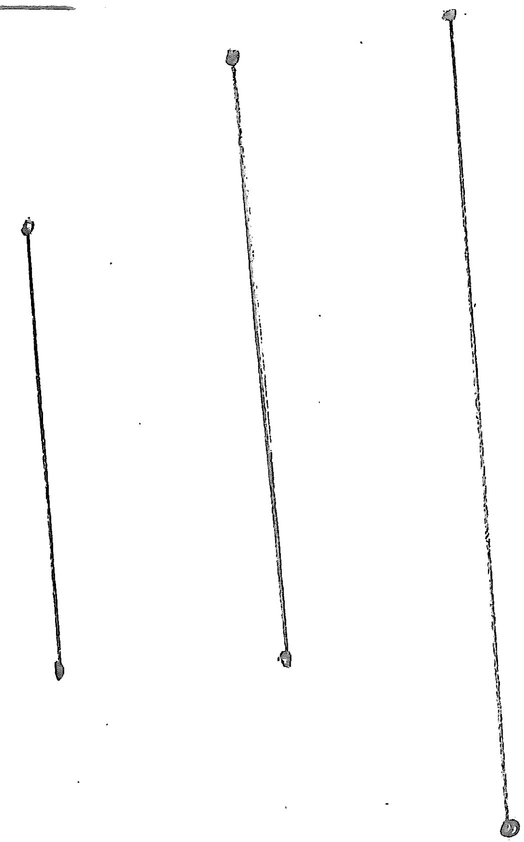
Triangles in the following cases *may or may not be congruent*:

SSS? AAA? SAS? ASA? AAS? (SAA) SSA?

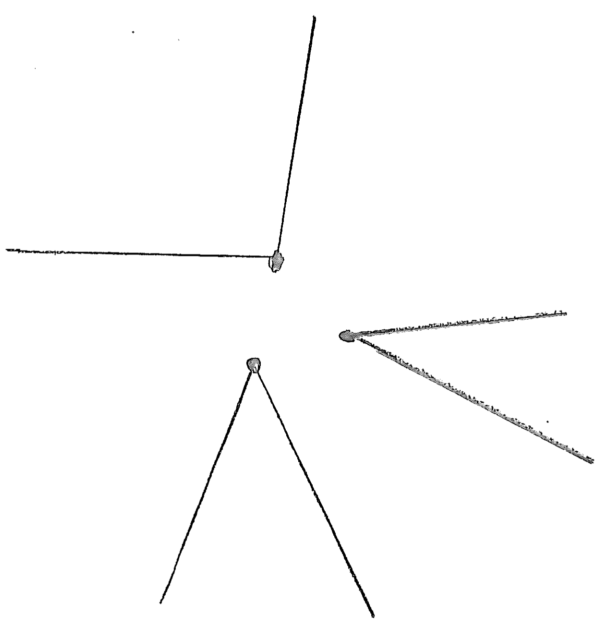
TRIANGLE A.



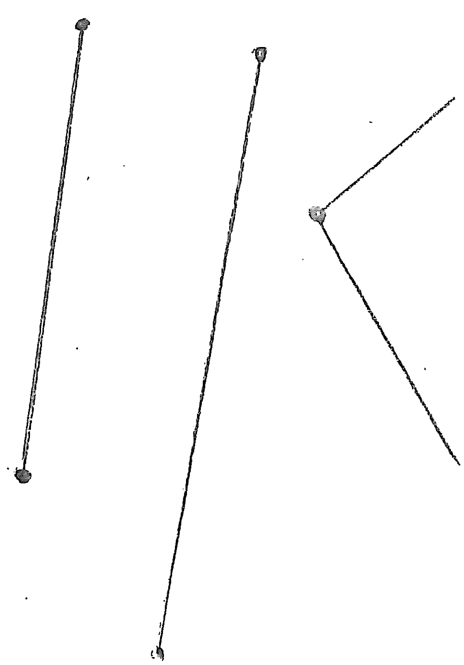
1. SSS



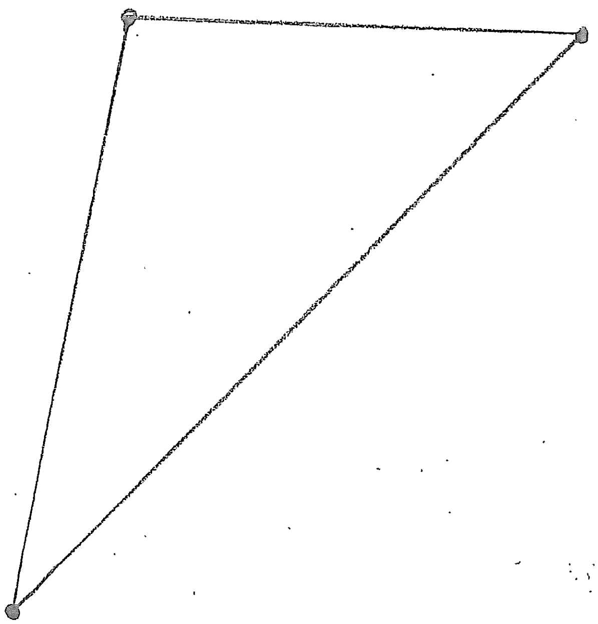
2. AAA



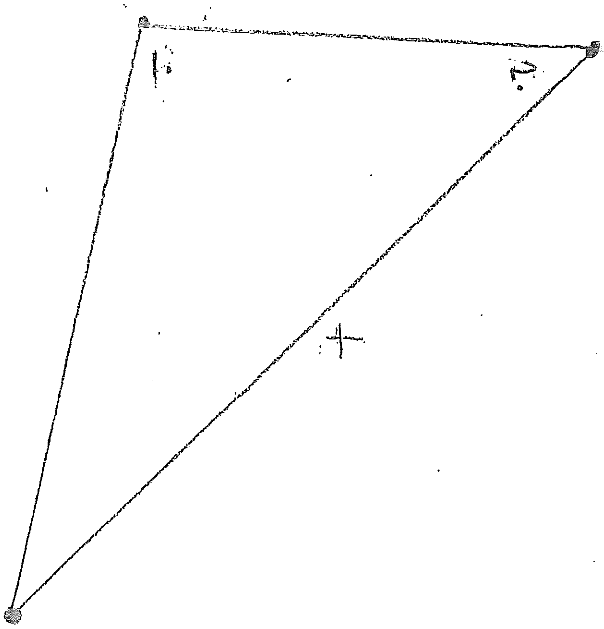
3. SAS



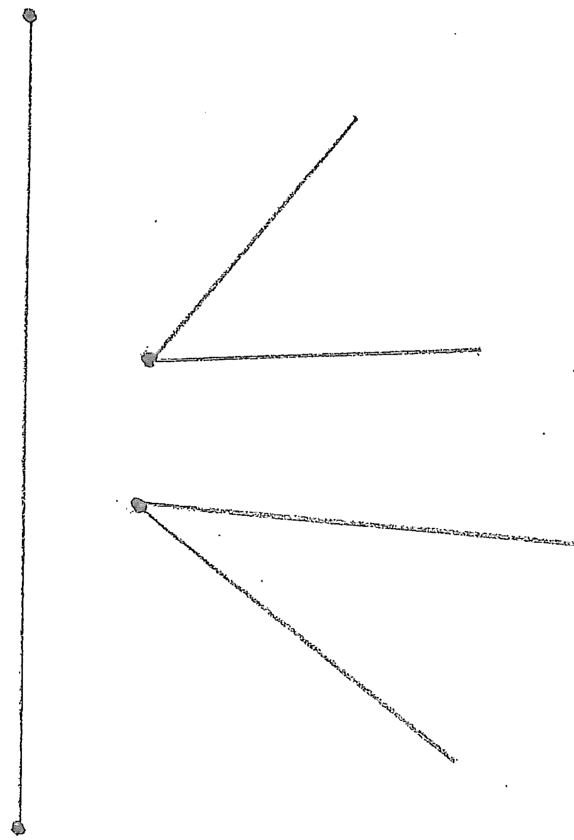
TRIANGLE B.



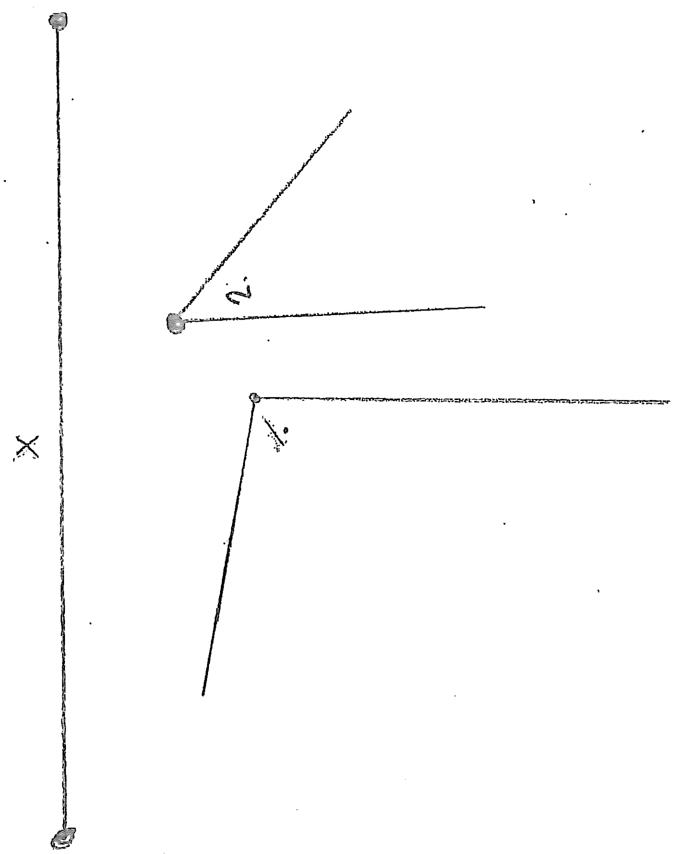
TRIANGLE C.



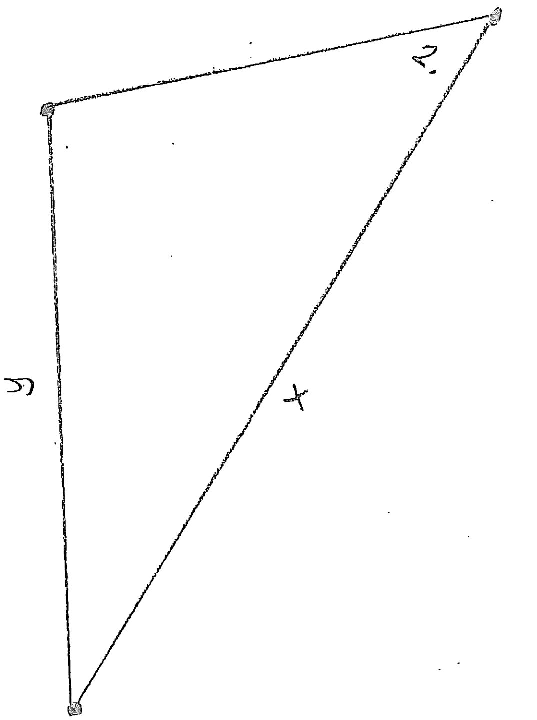
4. ASA



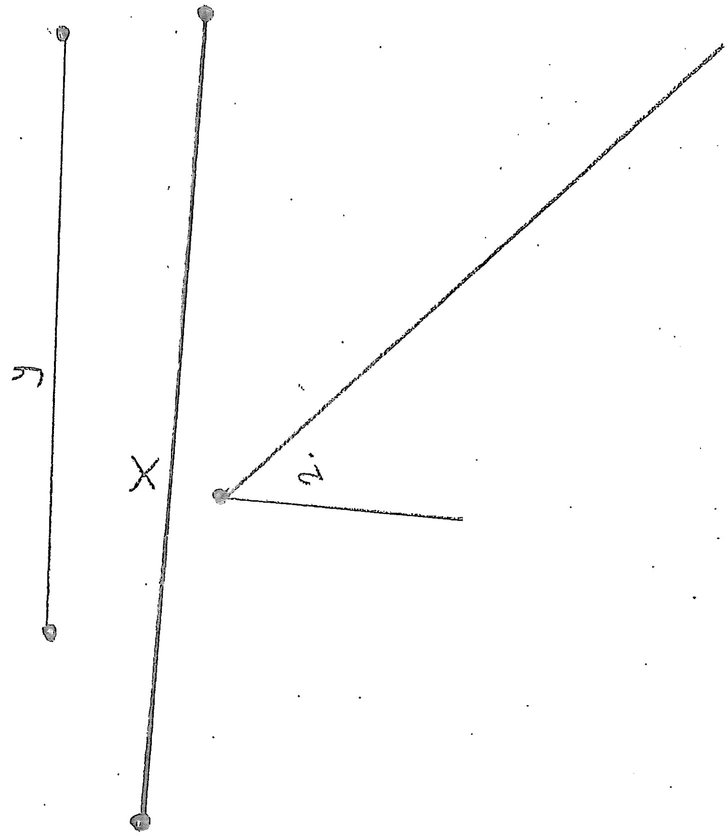
5. AAS (SAA)



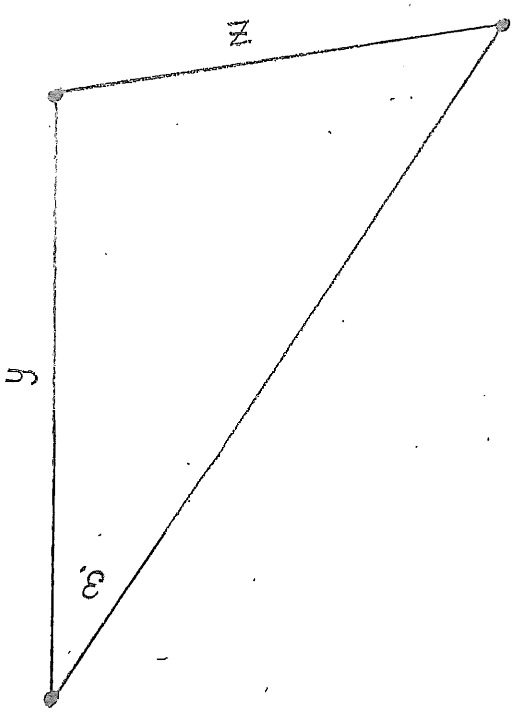
TRIANGLE D.



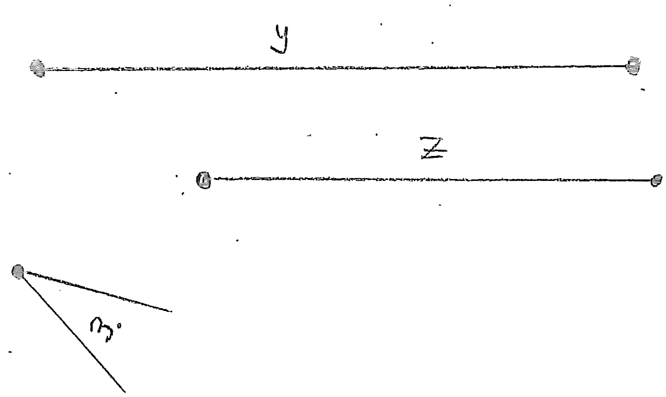
6. SSA



TRIANGLE E.



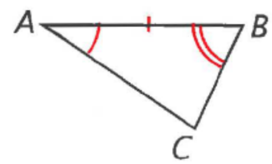
7. SSA

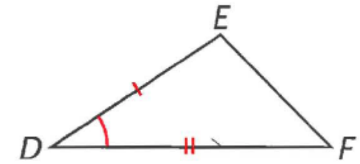


4–2 Some Ways to Prove Triangles Congruent

Three postulates and one theorem serve as shortcuts to proving that two triangles are congruent. For many triangles, we only need to know about **three** of the **six** "parts," for example that pairs of 2 angles and 1 side are congruent. The postulates are given here.

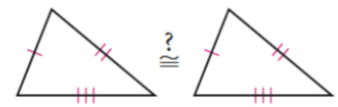
The words "included" and "opposite" describe the relative positions of the sides and angles of a triangle. In △ABC, for example, side AB is said to be *included* by ∠A and ∠B and to be *opposite* ∠C. In △DEF, ∠D is said to be *included* by sides DE and DF and to be *opposite* side EF.





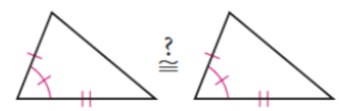
Postulate 12 (SSS)
If **three sides** of one triangle are congruent to three sides of another triangle, then the triangles are congruent.
Postulate 13 (SAS)
If **two sides** and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.
Postulate 14 (ASA)
If **two angles** and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

Side-Side-Side (SSS)



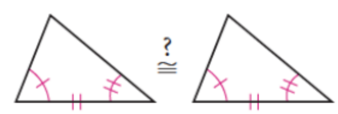
Three pairs of congruent sides

Side-Angle-Side (SAS)



Two pairs of congruent sides and one pair of congruent angles (angles between the pairs of sides)

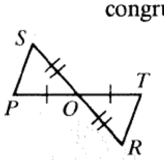
Angle-Side-Angle (ASA)



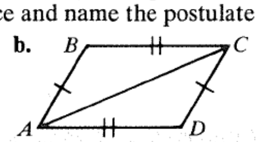
Two pairs of congruent angles and one pair of congruent sides (sides between the pairs of angles)

Example Can the two triangles be proved congruent? If so, write the congruence and name the postulate used.

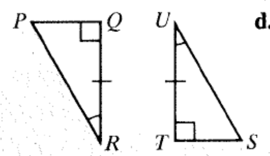
a.



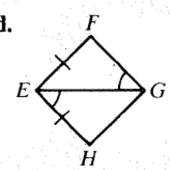
b.



c.



d.

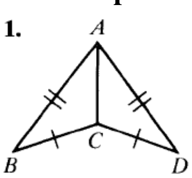


Solution

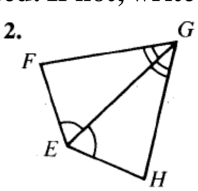
a. Since the vertical angles *POS* and *TOR* are congruent, △*POS* ≅ △*TOR* by the SAS Postulate.
b. Since $\overline{AC}$ is a common side, △*ABC* ≅ △*CDA* by the SSS Postulate.
c. △*PQR* ≅ △*STU* by the ASA Postulate.
d. Comparing △*EFG* to △*EHG*, you find that two sides and the non-included angle of △*EFG* are congruent to two sides and the included angle of △*EHG*. No congruence can be deduced.

Can the two triangles be proved congruent? If so, write the congruence and name the postulate used. If not, write *no congruence can be deduced.*

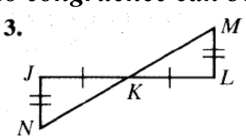
1.



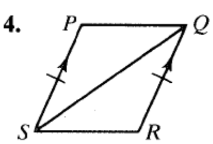
2.



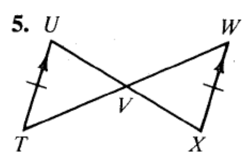
3.



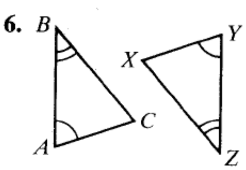
4.



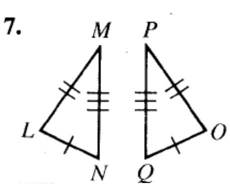
5.



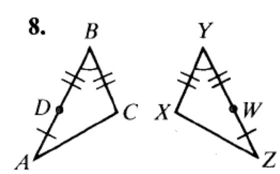
6.



7.

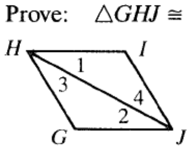


8.



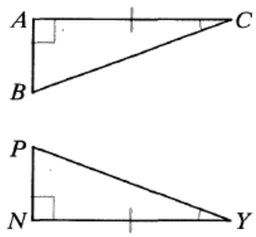
Complete.

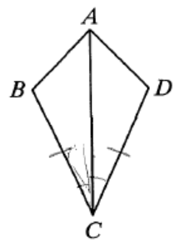
9. Given: $\overline{HI} \parallel \overline{GJ}$; $\overline{HG} \parallel \overline{IJ}$
Prove: △*GHJ* ≅ △*IJH*

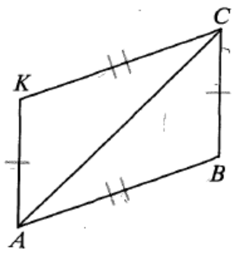


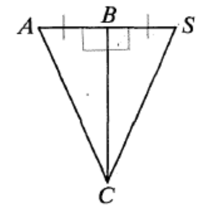
Statements	Reasons
1. ________	1. Given
2. ∠1 ≅ ∠2; ∠3 ≅ ∠4	2. ________
3. ________	3. Reflexive Prop.
4. △*GHJ* ≅ △*IJH*	4. ________

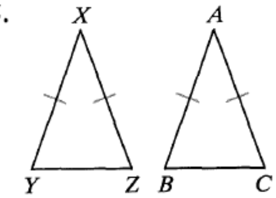
Decide whether you can deduce by the SSS, SAS, or ASA Postulate that another triangle is congruent to triangle *ABC*. If so, write the congruence and name the postulate used. If not, write *no congruence can be deduced.*

1.


2.


3.


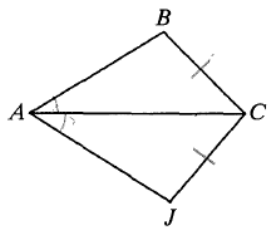
4.


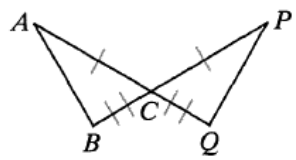
5.


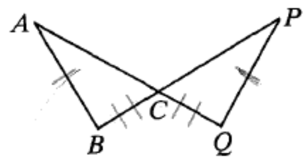
Solution

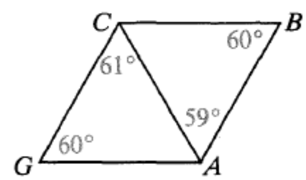
Congruence: $\triangle ABC \cong \triangle NPY$

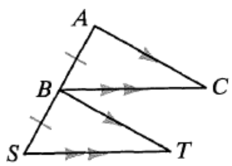
Postulate: Angle-Side-Angle

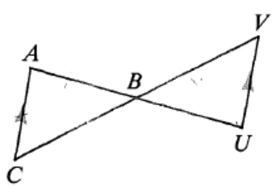
6.


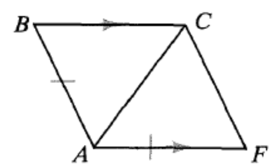
7.


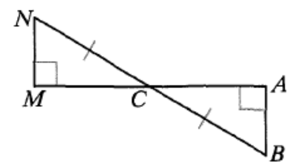
8.


9.


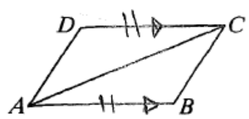
11.


12.


13.


15.


16. Supply the missing reasons.
Given: $\overline{AB} \parallel \overline{DC}$; $\overline{AB} \cong \overline{DC}$
Prove: $\triangle ABC \cong \triangle CDA$

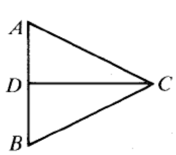


Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{DC}$	1. ?
2. $\overline{AC} \cong \overline{AC}$	2. ?
3. $\overline{AB} \parallel \overline{DC}$	3. ?
4. $\angle BAC \cong \angle DCA$	4. ?
5. $\triangle ABC \cong \triangle CDA$	5. ?

All of the statements and reasons for the following proof have been provided. Number the statements and the reasons in an appropriate order. (There may be more than one correct order.)

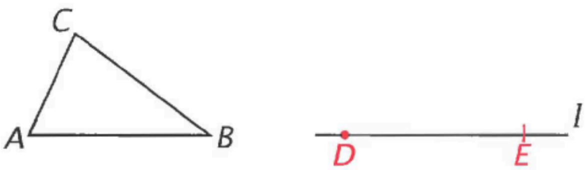
10. Given: $\overline{DC}$ bisects $\overline{AB}$; $\overline{AC} \cong \overline{BC}$
Prove: $\triangle ADC \cong \triangle BDC$



Statements	Reasons
(___) $\triangle ADC \cong \triangle BDC$	(___) Def. of midpoint
(___) $\overline{DC}$ bisects $\overline{AB}$; $\overline{AC} \cong \overline{BC}$	(___) Def. of segment bisector
(___) $\overline{DC} \cong \overline{DC}$	(___) SSS Post.
(___) D is the midpoint of $\overline{AB}$.	(___) Reflexive Prop.
(___) $\overline{AD} \cong \overline{DB}$	(___) Given

Construction 5: To copy a triangle

The compass-and-straightedge constructions for copying an angle and copying a triangle are almost the same.

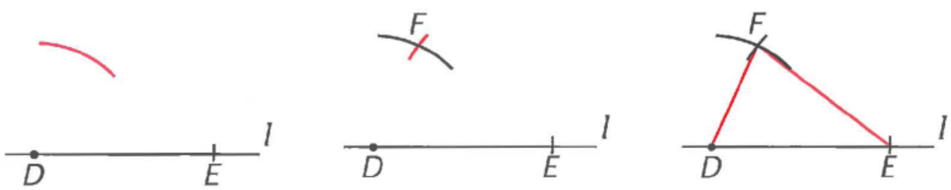


Let △ABC be the triangle to be copied. Draw a line (l) and copy AB on it (DE in the figure at the right above).

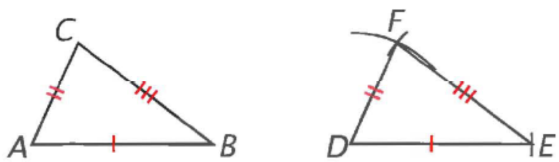
Return to the triangle and set the radius of the compass to the length of AC. With D as center, draw an arc having this radius, as shown in the figure at the left below.

Reset the radius of the compass to the length of BC. With E as center, draw an arc having this radius so that it intersects the first arc at point F, as shown in the middle figure below.

Draw DF and EF to finish the construction, as shown in the figure at the right below.

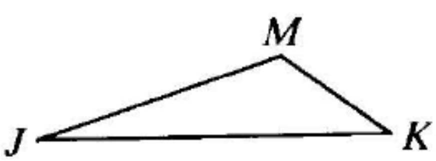


To see why △ABC ≅ △DEF, mark the equal lengths on the sides of the two triangles. They are congruent by SSS.



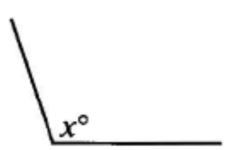
Exercise

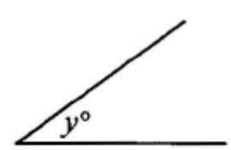
1. Using compass and straightedge, construct a triangle congruent to $\triangle JKM$.



Given two angles with measures *x*° and *y*°.

11. Construct an angle with measure $x + y$.





12. Construct an angle with measure $x – y$.

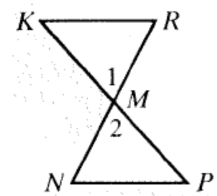
4–3 Using Congruent Triangles

A Way to Prove Two Segments or Two Angles Congruent

1. Identify two triangles in which the two segments or angles are corresponding parts.
2. Prove that the triangles are congruent.
3. State that the two parts are congruent, using the reason
   Corr. parts of ≅ ▵ are ≅.

Example

Given: $\angle 1 \cong \angle R$; $\angle 2 \cong \angle N$; $\overline{MR} \cong \overline{MN}$
Prove: $\overline{KR} \cong \overline{PN}$

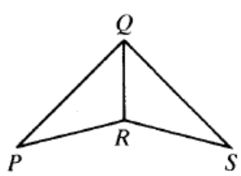


Solution

Plan for Proof: You can prove $\overline{KR} \cong \overline{PN}$ if you can show they are corresponding parts of congruent triangles. Use the given and apply ASA to show $\triangle RKM \cong \triangle NPM$.
(Two-column or paragraph proof should be provided next.)

All of the statements and reasons for the following proofs have been provided. Number the statements and the reasons in an appropriate order. (There may be more than one correct order.)

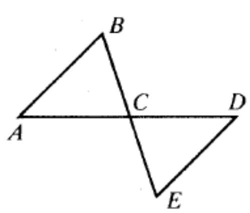
2. Given: $\overline{PR} \cong \overline{SR}$; $\overline{PQ} \cong \overline{SQ}$
Prove: $\angle P \cong \angle S$



Statements
(___) $\overline{QR} \cong \overline{QR}$
(___) $\triangle PQR \cong \triangle SQR$
(___) $\overline{PR} \cong \overline{SR}$; $\overline{PQ} \cong \overline{SQ}$
(___) $\angle P \cong \angle S$

Reasons
(___) Reflexive Prop.
(___) SSS Post.
(___) Corr. parts of ≅ ▵ are ≅.
(___) Given

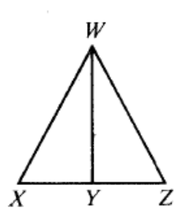
3. Given: C is the midpoint of $\overline{AD}$; $\angle A \cong \angle D$
Prove: $\overline{BC} \cong \overline{EC}$



Statements
(___) $\overline{AC} \cong \overline{CD}$
(___) $\overline{BC} \cong \overline{EC}$
(___) C is the midpt. of $\overline{AD}$; $\angle A \cong \angle D$
(___) $\angle ACB \cong \angle DCE$
(___) $\triangle ABC \cong \triangle DEC$

Reasons
(___) Corr. parts of ≅ ▵ are ≅.
(___) Vertical ∠s are ≅.
(___) ASA Post.
(___) Def. of midpoint
(___) Given

4. Given: $\overline{WY} \perp \overline{XZ}$; $\overline{XY} \cong \overline{YZ}$
Prove: $\angle X \cong \angle Z$



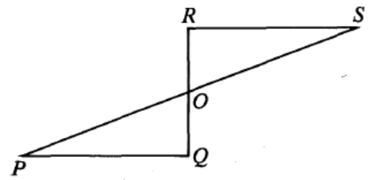
Statements
(___) $\overline{WY} \cong \overline{WY}$
(___) $\angle XYW \cong \angle ZYW$
(___) $\angle X \cong \angle Z$
(___) $\triangle XYW \cong \triangle ZYW$
(___) $\overline{WY} \perp \overline{XZ}$; $\overline{XY} \cong \overline{YZ}$

Reasons
(___) If 2 lines are ⊥, then they form ≅ adj. ∠s.
(___) Corr. parts of ≅ ▵ are ≅.
(___) SAS Post.
(___) Given
(___) Reflexive Prop.

A1. Complete the following proof.

Given: $\angle P \cong \angle S$;
O is the midpoint of $\overline{PS}$.

Prove: O is the midpoint of $\overline{RQ}$.



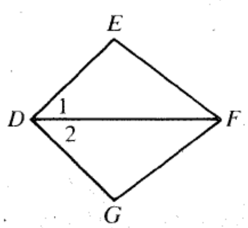
Proof:

Statements	Reasons
1. $\angle P \cong \angle S$	1. ?
2. O is the midpoint of $\overline{PS}$.	2. ?
3. $\overline{PO} \cong \overline{SO}$	3. ?
4. $\angle POQ \cong \angle SOR$	4. ?
5. $\triangle POQ \cong \triangle SOR$	5. ?
6. $\overline{QO} \cong \overline{RO}$	6. ?
7. O is the midpoint of $\overline{RQ}$.	7. ?

1. Complete the following proof.

Given: $\overrightarrow{DF}$ bisects $\angle EDG$;
$\overline{DE} \cong \overline{DG}$

Prove: $\angle E \cong \angle G$



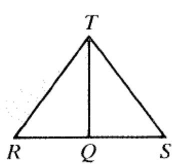
Statements	Reasons
1. $\overrightarrow{DF}$ bisects $\angle EDG$; $\overline{DE} \cong \overline{DG}$	1. ______
2. $\angle 1 \cong \angle 2$	2. ______
3. $\overline{DF} \cong \overline{DF}$	3. ______
4. $\triangle$______ $\cong \triangle$______	4. ______
5. ______	5. ______

In some proofs, it is helpful to plan the proofs by reasoning backward.

Example 2

Given: $\overline{RQ} \cong \overline{QS}$;
$\overline{RT} \cong \overline{TS}$

Prove: $\overline{TQ} \perp \overline{RS}$



Solution

Plan for Proof:
$\overline{TQ} \perp \overline{RS}$ if $\angle RQT \cong \angle SQT$.
$\angle RQT \cong \angle SQT$ if $\triangle RQT \cong \triangle SQT$.
$\triangle RQT \cong \triangle SQT$ by the SSS Postulate.

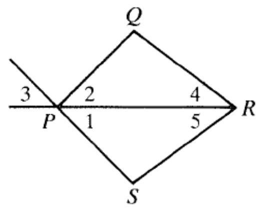
3. Complete the proof of Example 2.

Statements	Reasons
1. ______	1. Given
2. $\overline{TQ} \cong \overline{TQ}$	2. ______
3. $\triangle RQT \cong \triangle SQT$	3. ______
4. $\angle RQT \cong \angle SQT$	4. ______
5. ______	5. ______

Example 3

Given: $\angle 4 \cong \angle 5$; $\overline{QR} \cong \overline{SR}$

Prove: $\angle 2 \cong \angle 3$



Solution

Plan for Proof:

$\angle 1 \cong \angle 3$

$\angle 1 \cong \angle 2$ if $\triangle PQR \cong \triangle PSR$.

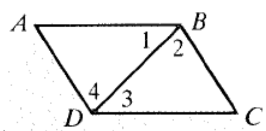
$\triangle PQR \cong \triangle PSR$ by the SAS Postulate.

5. Write a two-column proof of Example 3.

2. Write a two-column proof.

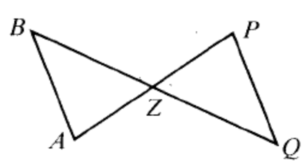
Given: $\overline{AB} \cong \overline{DC}$; $\overline{AD} \cong \overline{BC}$

Prove: $\overline{AB} \parallel \overline{DC}$



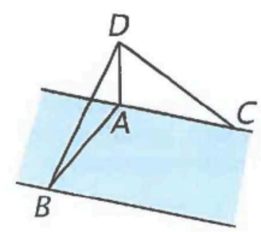
6. Given: $\overline{AB} \parallel \overline{PQ}$; $\overline{AB} \cong \overline{PQ}$

Prove: $\overline{BZ} \cong \overline{ZQ}$



River Problem. Acute Alice and Obtuse Ollie are standing on the bank of Muddy Boggy Creek. Ollie asks Alice how wide she thinks the river is. Alice adjusts the rim of her hat as she looks across the river, then walks along the river bank and gives Ollie an answer.*

The "three-dimensional" figure below illustrates the situation.



47. Copy it and mark the parts that Alice used to figure out her answer.

48. Tell how she did it.

*Adapted from a problem in *You Are a Mathematician*, by David Wells (Wiley, 1995).

Self-Test 1

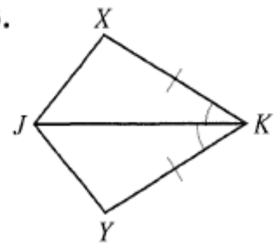
Given: $\triangle KOP \cong \triangle MAT$

1. What can you conclude about $\angle P$? Why?

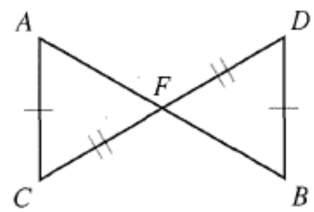
2. Name three pairs of corresponding sides.

Decide whether the two triangles must be congruent. If so, write the congruence and name the postulate used. If not, write *no congruence can be deduced*.

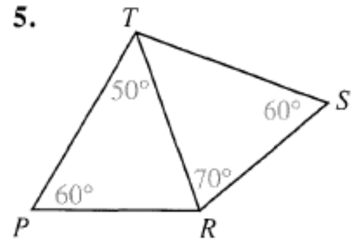
3.



4.



5.

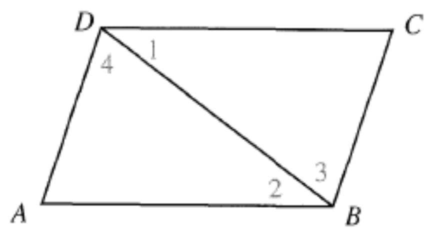


Write proofs in two-column form.

6. Given: $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$
Prove: $\triangle ADB \cong \triangle CBD$

7. Given: $\overline{CD} \cong \overline{AB}$; $\overline{CB} \cong \overline{AD}$
Prove: $\angle 1 \cong \angle 2$

8. Given: $\overline{AD} \parallel \overline{BC}$; $\overline{AD} \cong \overline{CB}$
Prove: $\overline{DC} \parallel \overline{AB}$



4–4 The Isosceles Triangle Theorems

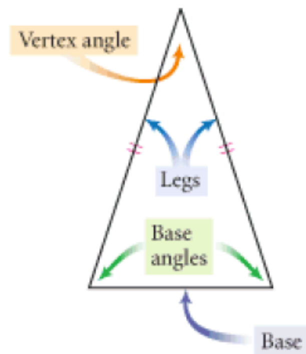
Isosceles triangles are defined as having at least two sides congruent. Isosceles triangles have special names for their parts.

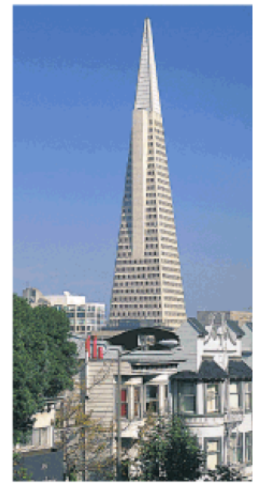
The congruent sides are called **legs**.

The third side is called the **base**.

The angle opposite the base is called the **vertex angle**.

The angles adjacent to the base are called **base angles**.



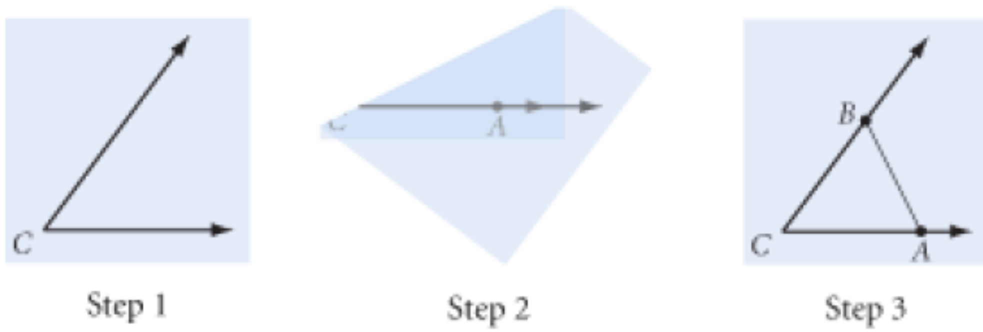


The famous Transamerica Building in San Francisco contains many isosceles triangles.

Base Angles in an Isosceles Triangle

Materials: patty paper, a straightedge, a protractor

Let's examine the angles of an isosceles triangle. Each person in your group should draw a different angle for this investigation. Your group should have at least one acute angle and one obtuse angle.



Step 1 Draw an angle on patty paper. Label it $\angle C$. This angle will be the vertex angle of your isosceles triangle.

Step 2 Place a point A on one ray. Fold your patty paper so that the two rays match up. Trace point A onto the other ray.

Step 3 Label the point on the other ray point B. Draw $\overline{AB}$. You have constructed an isosceles triangle. Explain how you know it is isosceles. Name the base and the base angles.

Step 4 Use your protractor to compare the measures of the base angles. What relationship do you notice? How can you fold the paper to confirm your conclusion?

Step 5 Compare results in your group. Was the relationship you noticed the same for each isosceles triangle? State your observations as your next conjecture.

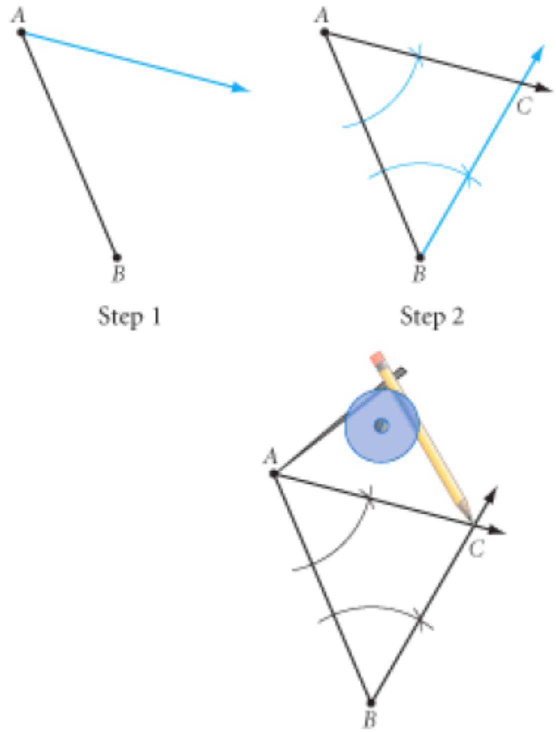
Isosceles Triangle Conjecture

If a triangle is isosceles, then ?. ____

(Theorem 4–1: The Isosceles Triangle Theorem.)

Is the Converse True?

Suppose a triangle has two congruent angles. Must the triangle be isosceles?



Step 1 — Draw a segment and label it $\overline{AB}$. Draw an acute angle at point A. This angle will be a base angle. (Why can't you draw an obtuse angle as a base angle?)

Step 2 — Copy $\angle A$ at point B on the same side of $\overline{AB}$ Label the intersection of the two rays point C.

Step 3 — Use your compass to compare the lengths of sides $\overline{AC}$ and BC. What relationship do you notice? How can you use patty paper to confirm your conclusion?

Step 4 — Compare results in your group. State your observation as your next conjecture.

Converse of the Isosceles Triangle Conjecture

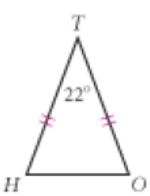
If a triangle has two congruent angles, then _?_ ..

____

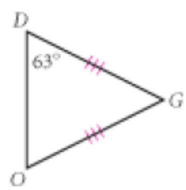
(Theorem 4–2: The Isosceles Triangle Theorem Converse.)

Find the missing measures.

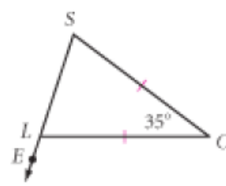
1. $m\angle H =$ _?_



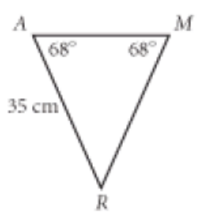
2. $m\angle G =$ _?_



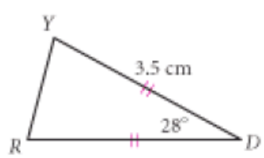
3. $m\angle OLE =$ _?_



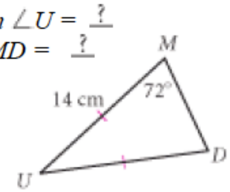
4. $m\angle R =$ _?_
$RM =$ _?_



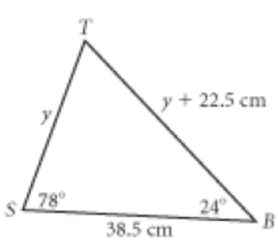
5. $m\angle Y =$ _?_
$RD =$ _?_



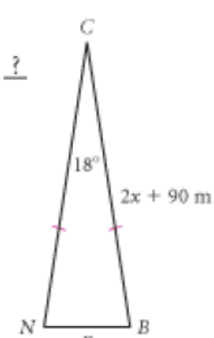
6. The perimeter of $\triangle MUD$ is 36.6 cm.
$m\angle D =$ _?_
$m\angle U =$ _?_
$MD =$ _?_



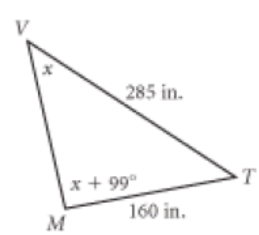
7. $m\angle T =$ _?_
perimeter of $\triangle TBS$ _?_



8. The perimeter of $\triangle NBC$ is 555 m.
$NB =$ _?_
$m\angle N =$ _?_

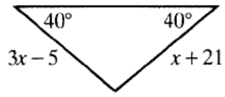


9. The perimeter of $\triangle MTV$ is 605 in.
$MV =$ _?_
$m\angle M =$ _?_

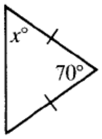


Example 3 Find the value of x.

a.



b.



Solution

a. Since two angles are congruent, the sides opposite these angles must be congruent.

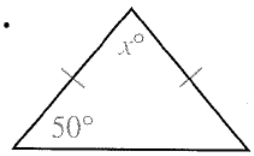
$$3x - 5 = x + 21$$
$$2x = 26$$
$$x = 13$$

b. By the Isosceles Triangle Theorem, the third angle must also be $x°$.

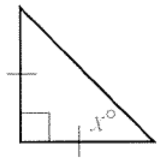
$$x + x + 70 = 180$$ (The sum of the measures of the ∠s of a △ is 180.)
$$2x = 110$$
$$x = 55$$

Find the value of x.

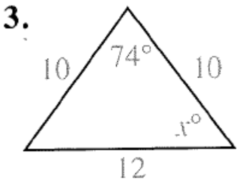
1.



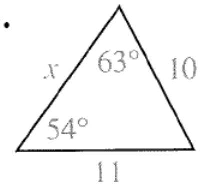
2.



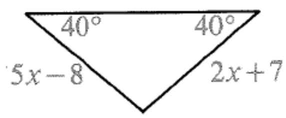
3.



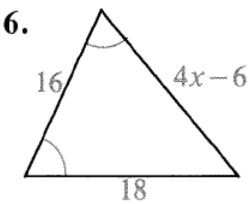
4.



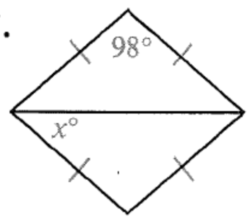
5.



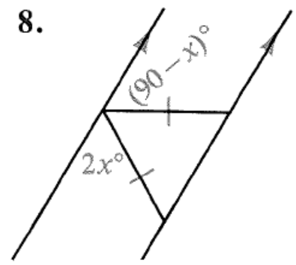
6.



7.

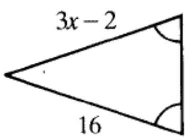


8.

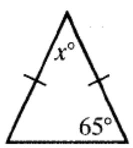


Find the value of x.

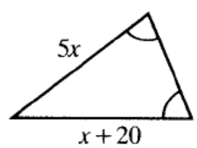
9.



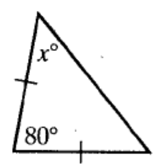
10.



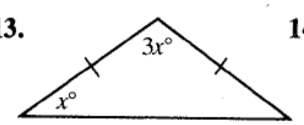
11.



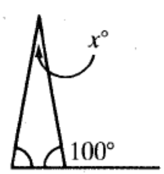
12.



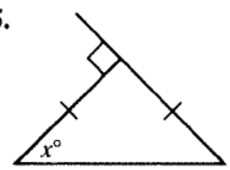
13.



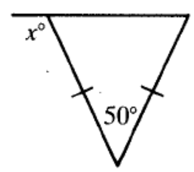
14.



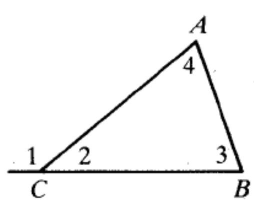
15.



16.

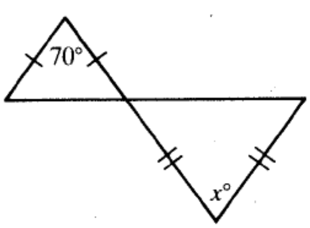


17. Given: $\overline{BC} \cong \overline{AC}$;
$m\angle 1 = 140$
Find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

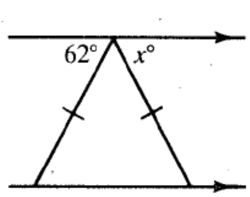


Find the value of x.

18.

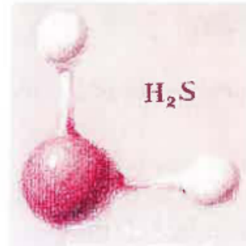


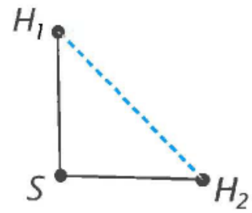
19.



Hydrogen Sulfide. One of the reasons that rotten eggs stink is hydrogen sulfide. The hydrogen sulfide molecule consists of a sulfur atom bonded to two hydrogen atoms. (*The Architecture of Molecules*, by Linus Pauling and Roger Hayward (W. H. Freeman and Company, 1964).) H stands for hydrogen. The two hydrogen atoms are labeled H_1 and H_2.

In $\triangle SH_1H_2$, S represents the center of the sulfur atom; H_1 and H_2 represents, the centers of the hydrogen atoms; $SH_1 = SH_2$.





36. What kind of triangle is $\triangle SH_1H_2$ with respect to its sides?

The measure of $\angle H_1$ is 43.9°. Find the measure of

37. $\angle H_2$.

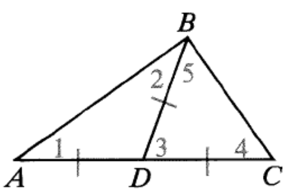
38. $\angle S$.

39. What kind of triangle is $\triangle SH_1H_2$ with respect to its angles?

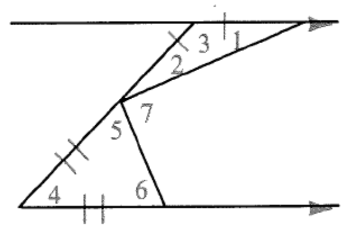
40. If you knew that $SH_1 = SH_2 = 1.34$ angstroms, could you use the Pythagorean Theorem to find the length of H_1H_2? Explain.

40b. Find the distance between H_1 and H_2.

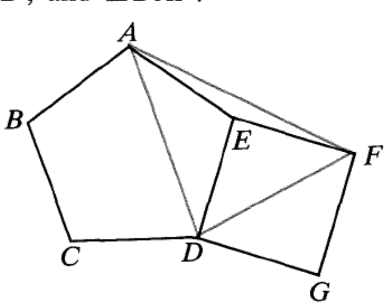
24. **a.** If $m\angle 1 = 35$, find $m\angle ABC$.
b. If $m\angle 1 = k$, find $m\angle ABC$.



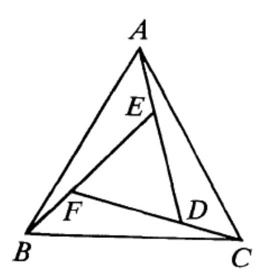
25. **a.** If $m\angle 1 = 23$, find $m\angle 7$.
b. If $m\angle 1 = k$, find $m\angle 7$.



33. *ABCDE* is a regular pentagon and *DEFG* is a square. Find the measures of $\angle EAF$, $\angle AFD$, and $\angle DAF$.



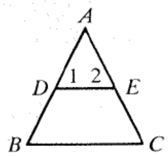
34. Given: $\triangle ABC$ is equilateral;
$\angle CAD \cong \angle ABE \cong \angle BCF$
Prove something interesting about $\triangle DEF$.



Example 2

Given: $\overline{AB} \cong \overline{AC}$;
$\angle B \cong \angle 2$;
$\angle C \cong \angle 1$

Prove: $\triangle ADE$ is isosceles.



Solution

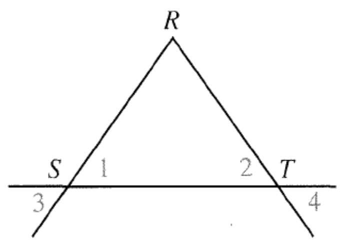
Plan for Proof:
$\triangle ADE$ is isosceles if $\overline{AD} \cong \overline{AE}$.
$\overline{AD} \cong \overline{AE}$ if $\angle 1 \cong \angle 2$.
Using the given, $\angle 1 \cong \angle 2$ if $\angle B \cong \angle C$. $\angle B \cong \angle C$ if $\overline{AB} \cong \overline{AC}$, which is also given.

3. Complete the proof of Example 2.

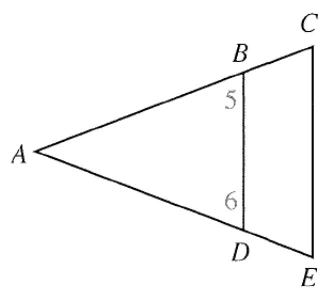
Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$	1. ______
2. $\angle B \cong \angle C$	2. ______
3. $\angle B \cong \angle 2$; $\angle C \cong \angle 1$	3. ______
4. ______	4. Substitution Prop.
5. $\overline{AD} \cong \overline{AE}$	5. ______
6. ______	6. Def. of isosceles $\triangle$

For each exercise place the statements in an appropriate order for a proof. (There may be more than one correct order.)

9. Given: $\overline{RS} \cong \overline{RT}$
Prove: $\angle 3 \cong \angle 4$
(a) $\angle 3 \cong \angle 4$
(b) $\angle 3 \cong \angle 1$; $\angle 2 \cong \angle 4$
(c) $\overline{RS} \cong \overline{RT}$
(d) $\angle 1 \cong \angle 2$



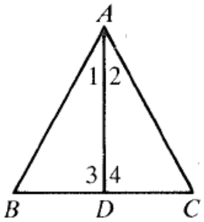
10. Given: $\overline{BD} \parallel \overline{CE}$; $\angle 5 \cong \angle 6$
Prove: $\overline{AC} \cong \overline{AE}$
(a) $\overline{BD} \parallel \overline{CE}$
(b) $\overline{AC} \cong \overline{AE}$
(c) $\angle 5 \cong \angle C$; $\angle 6 \cong \angle E$
(d) $\angle 5 \cong \angle 6$
(e) $\angle C \cong \angle E$



Prove Corollary 3 to Theorem 4–1.
(Number the statements and the reasons in an appropriate order.)

6. The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.

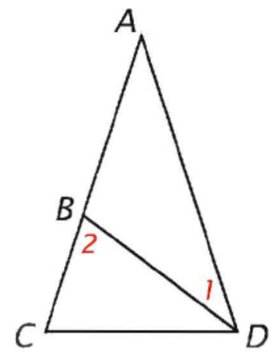
Given: $\overline{AB} \cong \overline{AC}$;
$\overrightarrow{AD}$ bisects $\angle BAC$.
Prove: $\overline{BD} \cong \overline{CD}$;
$\overline{AD} \perp \overline{BC}$



Statements	Reasons
(__) $\angle 1 \cong \angle 2$	(__) Given
(__) $\angle B \cong \angle C$	(__) Given
(__) $\overrightarrow{AD}$ bisects $\angle BAC$.	(__) Def. of $\angle$ bisector
(__) $\overline{BD} \cong \overline{CD}$; $\angle 3 \cong \angle 4$	(__) Isos. $\triangle$ Thm.
(__) $\overline{AB} \cong \overline{AC}$	(__) ASA Post.
(__) $\triangle ABD \cong \triangle ACD$	(__) If 2 lines form $\cong$ adj. $\angle$s, then the lines are $\perp$.
(__) $\overline{AD} \perp \overline{BC}$	
	(__) Corr. parts of $\cong$ $\triangle$s are $\cong$.

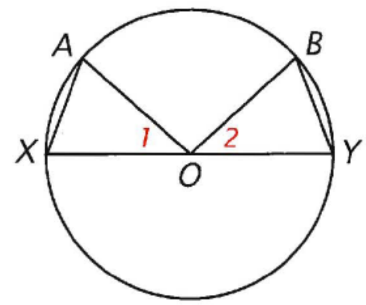
51. Write a complete proof.

Given: ∠A = ∠1 and ∠2 = ∠C.
Prove: AB = CD.

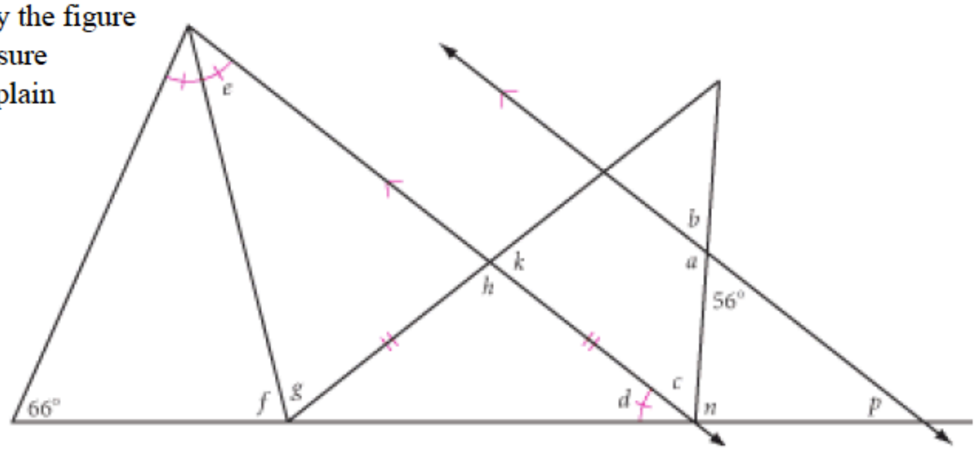


52. Write a complete proof.

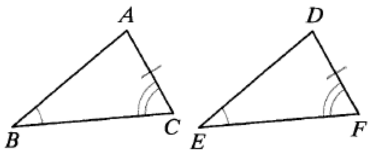
Given: O is the midpoint of XY;
∠1 = ∠2 and OA = OB.
Prove: ∠A = ∠B.



10. ***Developing Proof*** Copy the figure at right. Calculate the measure of each lettered angle. Explain how you determined the measures *d* and *h*.



4–5 Other Methods of Proving Triangles Congruent: AAS, HL



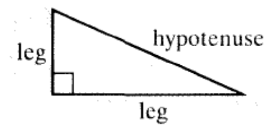
Ways to Prove Two Triangles Congruent

Postulates:	SSS SAS ASA	Similar AA(A)
Theorems:	AAS HL (LL,HA,LA)	No Conclusion SSA

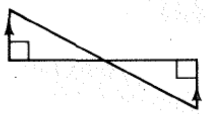
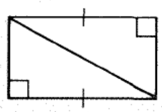
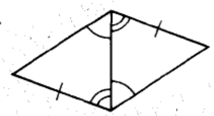
Theorem 4–3: ***AAS Theorem***
If two angles and a non-included side of one triangle are congruent to the corresponding parts of another triangle, then the triangles are congruent.

Proved using the triangle sum theorem ($\angle A$ and $\angle D$ must be $\cong$) and ASA Postulate.

Theorem 4–4: ***HL Theorem***
If the hypotenuse and a leg of one right triangle are congruent to the corresponding parts of another right triangle, then the triangles are congruent.



Example 1 State which congruence method(s) can be used to prove the triangles congruent. If no method applies, write *none*.

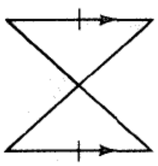
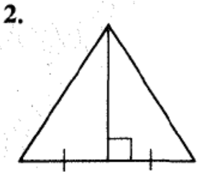
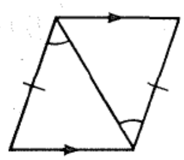
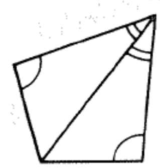
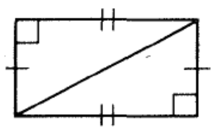
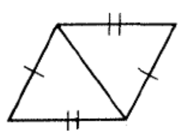
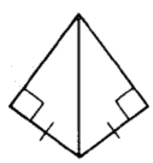
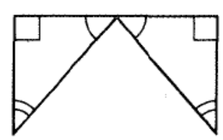
a.

b.

c.


Solution

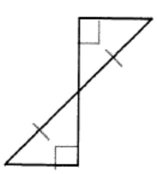
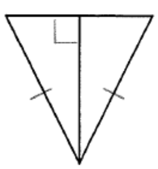
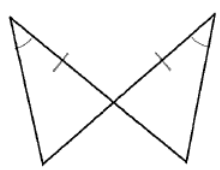
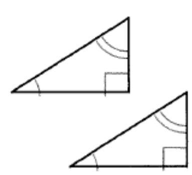
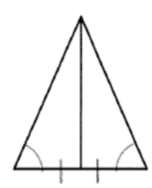
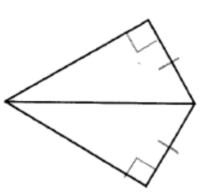
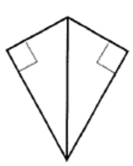
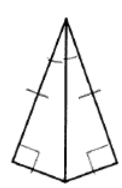
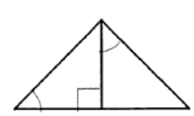
a. none (AAA is not a congruence method.)
b. HL
c. ASA, AAS, SAS

State which congruence method(s) can be used to prove the triangles congruent. If no method applies, write *none*.

Set A:

1.

2.

3.

4.

5.

6.

7.

8.


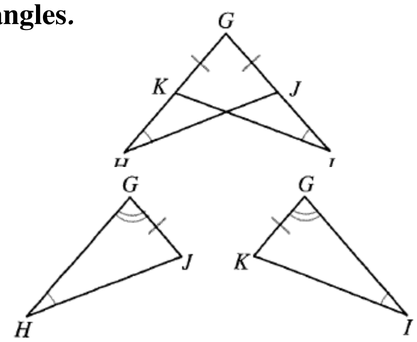
Set B: *Yes, it's really true: the question numbers are duplicates. Can you handle it?*

1.

2.

3.

4.

5.

6.

7.

8.

9.


Example **For overlapping triangles, redraw the triangles.**

Given: $\overline{GJ} \cong \overline{GK}$;
$\angle H \cong \angle I$

Prove: $\triangle GHJ \cong \triangle GIK$

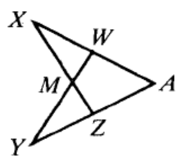


You may find it helps you visualize the congruence if you redraw the two triangles, as shown below. Now you can see that since $\angle G$ is common to both triangles, the triangles must be congruent by the AAS Theorem.

For each exercise, name a pair of overlapping congruent triangles. State which congruence method can be used to prove the triangles congruent. If no triangles can be proved congruent, write none.

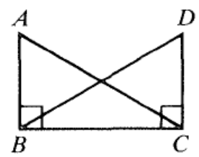
9. Given: $\overline{XZ} \cong \overline{YW}$;
$\angle X \cong \angle Y$

10. Given: $XA = YA$;
$\angle X \cong \angle Y$



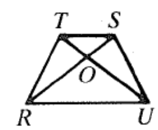
11. Given: $\overline{AB} \cong \overline{DC}$;
$\overline{AB} \perp \overline{BC}$; $\overline{DC} \perp \overline{BC}$

12. Given: $AC = BD$;
$\overline{AB} \perp \overline{BC}$; $\overline{BC} \perp \overline{DC}$



13. Given: $\angle RTS \cong \angle UST$;
$\overline{RS} \cong \overline{UT}$

14. Given: $\overline{RT} \cong \overline{US}$; $\overline{TU} \cong \overline{SR}$
(*Hint:* Two different pairs are possible.)

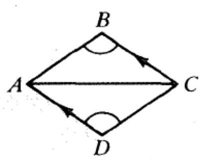


Redraw!

Example 1

Given: $\angle B \cong \angle D$;
$\overline{BC} \parallel \overline{AD}$

Prove: $\overline{DC} \cong \overline{AB}$



Solution

Plan for Proof:
$\overline{DC} \cong \overline{AB}$ if they are corresponding parts of congruent triangles.
You can prove this congruence by proving $\triangle ABC \cong \triangle CDA$.

1. Complete the proof of Example 1.

Statements	Reasons
1. $\angle B \cong \angle D$; $\overline{BC} \parallel \overline{AD}$	1. ______
2. $\angle CAD \cong \angle ACB$	2. ______
3. $\overline{AC} \cong \overline{AC}$	3. ______
4. $\triangle$ ______ $\cong \triangle$ ______	4. ______
5. $\overline{DC} \cong \overline{AB}$	5. ______

14. To prove that right triangles are congruent, some geometry books also use the methods stated below. For each method, draw two right triangles that appear to be congruent. Mark the given information on your triangles. Use your marks to determine which of our methods (SSS, SAS, ASA, AAS, or HL) could be used instead of each method listed.

a. Leg-Leg Method (LL) If two legs of one right triangle are congruent to the two legs of another right triangle, then the triangles are congruent.

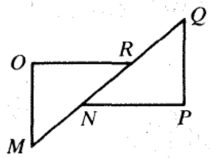
b. Hypotenuse-Acute Angle Method (HA) If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle of another right triangle, then the triangles are congruent.

c. Leg-Acute Angle Method (LA) If a leg and an acute angle of one right triangle are congruent to the corresponding parts in another right triangle, then the triangles are congruent.

Example 2

Given: $m\angle O = m\angle P = 90$;
$MN = QR$;
$OM = PQ$

Prove: $\triangle MOR \cong \triangle QPN$



Solution

Plan for Proof:
By the Segment Addition Postulate and the given $MN = QR$, you can get $MR = QN$. Since SSA is not a congruence method, show that the triangles are right triangles and use HL.

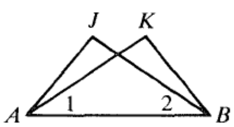
2. Complete the proof of Example 2.

Statements	Reasons
1. $m\angle O = m\angle P = 90$; $MN = QR$; $OM = PQ$	1. ________
2. $\triangle MOR$ and $\triangle QPN$ are rt. $\triangle$s.	2. ________
3. ________ = ________	3. Reflexive Prop.
4. $MN + NR = QR + NR$	4. ________
5. $MN + NR =$ ______; $QR + NR =$ ______	5. Segment Add. Post.
6. $MR = QN$	6. ________
7. $\triangle MOR \cong \triangle QPN$	7. ________

All of the statements and reasons for the following proofs have been provided. Number the statements and the reasons in an appropriate order. (There may be more than one correct order.)

3. Given: $\angle JAB \cong \angle KBA$; $\angle 1 \cong \angle 2$

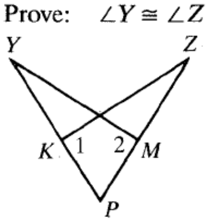
Prove: $\angle J \cong \angle K$



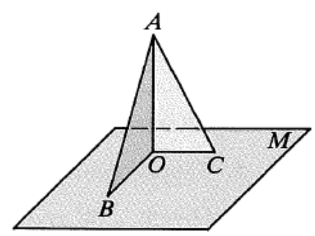
Statements	Reasons
(___) $\angle JAB \cong \angle KBA$; $\angle 1 \cong \angle 2$	(___) Reflexive Prop.
(___) $\triangle JAB \cong \triangle KBA$	(___) Corr. parts of $\cong$ $\triangle$s are $\cong$.
(___) $\overline{AB} \cong \overline{AB}$	(___) ASA Post.
(___) $\angle J \cong \angle K$	(___) Given

4. Given: $\angle 1$ and $\angle 2$ are right angles; $\overline{KZ} \cong \overline{MY}$; $\overline{PY} \cong \overline{PZ}$

Prove: $\angle Y \cong \angle Z$



Statements	Reasons
(___) $\angle Y \cong \angle Z$	(___) Def. of rt. $\triangle$
(___) $\angle 1$ and $\angle 2$ are rt. $\angle$s; $\overline{KZ} \cong \overline{MY}$; $\overline{PY} \cong \overline{PZ}$	(___) Corr. parts of $\cong$ $\triangle$s are $\cong$.
(___) $\triangle YMP \cong \triangle ZKP$	(___) HL Theorem
(___) $\triangle YMP$ and $\triangle ZKP$ are rt. $\triangle$s.	(___) Given



Use the information given in each exercise to name the method (SSS, SAS, ASA, AAS, or HL) you could use to prove $\triangle AOB \cong \triangle AOC$. You need not write the proofs.

5. Given: $\overline{AO} \perp$ plane M; $\overline{BO} \cong \overline{CO}$

6. Given: $\overline{AO} \perp$ plane M; $\angle B \cong \angle C$

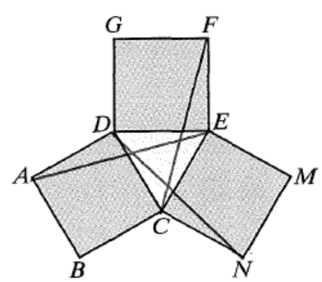
7. Given: $\overline{AO} \perp$ plane M; $\overline{AB} \cong \overline{AC}$

8. Given: $\overline{AB} \cong \overline{AC}$; $\overline{OB} \cong \overline{OC}$

a. Is it possible to prove that $\angle AOB \cong \angle AOC$?

b. Is it possible to prove that $\angle AOB$ and $\angle AOC$ are right angles?

Write proofs in two-column form. A square has congruent sides and right angles.



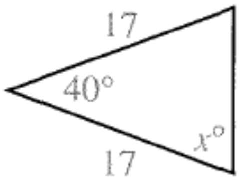
21. The diagram shows three squares and an equilateral triangle.
Prove: $\overline{AE} \cong \overline{FC} \cong \overline{ND}$

22. Use the results of Exercise 21 to prove that $\triangle FAN$ is equilateral.

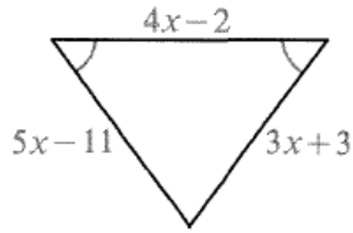
Self-Test 2

Find the value of x.

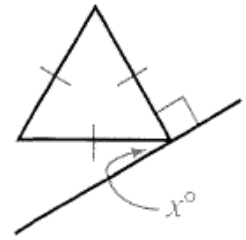
1.



2.



3.

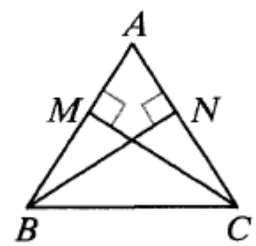


4. Given: $\overline{AB} \cong \overline{AC}$; $\overline{BN} \perp \overline{AC}$; $\overline{CM} \perp \overline{AB}$

Explain how you could prove that $\triangle ABN \cong \triangle ACM$.

5. Given: $\overline{MB} \cong \overline{NC}$; $\overline{BN} \perp \overline{AC}$; $\overline{CM} \perp \overline{AB}$

Prove: $\overline{CM} \cong \overline{BN}$



4–6 Using More than One Pair of Congruent Triangles

Prove two triangles congruent by first proving two other triangles congruent.

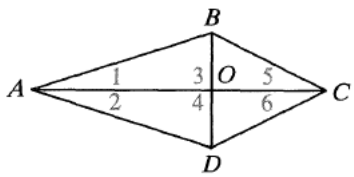
Don't rush through this page even though it has no questions.

Sometimes two triangles that you want to prove congruent have common parts with two *other* triangles that you can easily prove congruent. You may then be able to use corresponding parts of these other triangles to prove the original triangles congruent.

Example

Given: $\angle 1 \cong \angle 2$; $\angle 5 \cong \angle 6$

Prove: $\overline{AC} \perp \overline{BD}$



Plan for Proof: It may be helpful here to *reason backward* from what you want to prove. You can show $\overline{AC} \perp \overline{BD}$ if you can show that $\angle 3 \cong \angle 4$. You can prove $\angle 3 \cong \angle 4$ if you can prove that the angles are corresponding parts of congruent triangles. To prove $\triangle ABO \cong \triangle ADO$, you need $\overline{AB} \cong \overline{AD}$. You can prove this congruence by proving that $\triangle ABC \cong \triangle ADC$. You should prove this congruence first.

Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$; $\angle 5 \cong \angle 6$	1. Given
2. $\overline{AC} \cong \overline{AC}$	2. Reflexive Property
3. $\triangle ABC \cong \triangle ADC$	3. ASA Postulate
4. $\overline{AB} \cong \overline{AD}$	4. Corr. parts of $\cong$ $\triangle$s are $\cong$.
5. $\overline{AO} \cong \overline{AO}$	5. Reflexive Property
6. $\triangle ABO \cong \triangle ADO$	6. SAS Postulate (Steps 1, 4, and 5)
7. $\angle 3 \cong \angle 4$	7. Corr. parts of $\cong$ $\triangle$s are $\cong$.
8. $\overline{AC} \perp \overline{BD}$	8. If two lines form $\cong$ adj. $\angle$s, then the lines are $\perp$.

Use two congruency postulates

Use CPCTC twice

If you were to outline this two-column proof, you might pick out the following *key steps*.

Key steps of proof:

1. $\triangle ABC \cong \triangle ADC$	(ASA Postulate)
2. $\overline{AB} \cong \overline{AD}$	(Corr. parts of $\cong$ $\triangle$s are $\cong$.)
3. $\triangle ABO \cong \triangle ADO$	(SAS Postulate)
4. $\angle 3 \cong \angle 4$	(Corr. parts of $\cong$ $\triangle$s are $\cong$.)
5. $\overline{AC} \perp \overline{BD}$	(If two lines form $\cong$ adj. $\angle$s, then the lines are $\perp$.)

In mathematics a proof is often given in paragraph form rather than in two-column form. A *paragraph proof* usually focuses on the key ideas and omits details that the writer thinks will be clear to the reader. The following paragraph proof might be given for the example above.

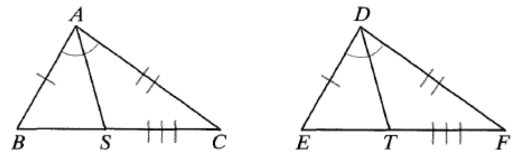
Paragraph proof:

$\triangle ABC \cong \triangle ADC$ by the ASA Postulate. Therefore, corresponding parts $\overline{AB}$ and $\overline{AD}$ are congruent. $\overline{AB}$ and $\overline{AD}$ are also corresponding parts of $\triangle ABO$ and $\triangle ADO$, which can now be proved congruent by the SAS Postulate. So corresponding parts $\angle 3$ and $\angle 4$ are congruent, and $\overline{AC} \perp \overline{BD}$.

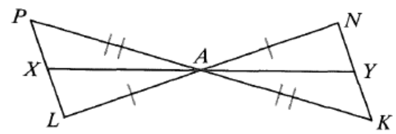
Paragraph proofs focus on key ideas

In Exercises 1–3 you are given a diagram that is marked with given information. Give the reason for each key step of the proof.

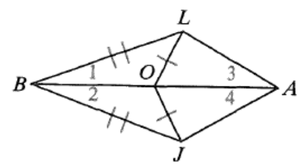
1. Prove: $\overline{AS} \cong \overline{DT}$
   Key steps of proof:
   a. $\triangle ABC \cong \triangle DEF$
   b. $\angle C \cong \angle F$
   c. $\triangle ACS \cong \triangle DFT$
   d. $\overline{AS} \cong \overline{DT}$



2. Prove: $\overline{AX} \cong \overline{AY}$
   Key steps of proof:
   a. $\triangle PAL \cong \triangle KAN$
   b. $\angle L \cong \angle N$
   c. $\triangle LAX \cong \triangle NAY$
   d. $\overline{AX} \cong \overline{AY}$

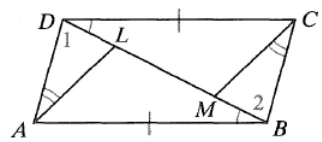


3. Prove: $\angle 3 \cong \angle 4$
   Key steps of proof:
   a. $\triangle LOB \cong \triangle JOB$
   b. $\angle 1 \cong \angle 2$
   c. $\triangle LBA \cong \triangle JBA$
   d. $\angle 3 \cong \angle 4$



☺

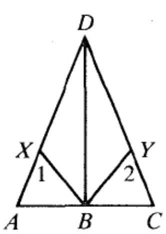
4. Prove: $\overline{AL} \cong \overline{CM}$
   Key steps of proof:
   a. $\triangle ABD \cong \triangle CDB$
   b. $\overline{AD} \cong \overline{CB}$; $\angle 1 \cong \angle 2$
   c. $\triangle ADL \cong \triangle CBM$
   d. $\overline{AL} \cong \overline{CM}$



☺

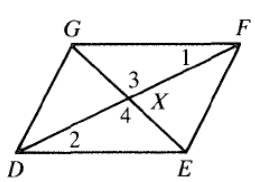
3. All of the statements and reasons for the following proof have been provided. Number them in an appropriate order. (There may be more than one correct order.)

Given: $\overline{DB} \perp \overline{AC}$; $\overline{AB} \cong \overline{CB}$; $\angle 1 \cong \angle 2$
Prove: $\overline{XB} \cong \overline{YB}$



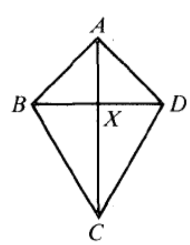
Statements	Reasons
(___) $\overline{DB} \cong \overline{DB}$	(___) SAS Post.
(___) $\angle ABD \cong \angle CBD$	(___) Given
(___) $\triangle ABD \cong \triangle CBD$	(___) If 2 lines are $\perp$, then they form $\cong$ adj. $\angle$s.
(___) $\overline{DB} \perp \overline{AC}$; $\overline{AB} \cong \overline{CB}$	(___) Reflexive Prop.
(___) $\angle A \cong \angle C$	(___) Given
(___) $\overline{XB} \cong \overline{YB}$	(___) Corr. parts of $\cong$ $\triangle$s are $\cong$.
(___) $\triangle AXB \cong \triangle CYB$	(___) Corr. parts of $\cong$ $\triangle$s are $\cong$.
(___) $\angle 1 \cong \angle 2$	(___) AAS Theorem

4. Given: X is the midpoint of $\overline{GE}$ and of $\overline{DF}$.
   Prove: $\triangle DEF \cong \triangle FGD$



Statements
(___) $\angle 3 \cong \angle 4$
(___) $\overline{GX} \cong \overline{XE}$; $\overline{DX} \cong \overline{XF}$
(___) $\triangle DXE \cong \triangle FXG$
(___) X is the midpoint of $\overline{GE}$ and of $\overline{DF}$.
(___) $\triangle DEF \cong \triangle FGD$
(___) $\angle 1 \cong \angle 2$; $\overline{GF} \cong \overline{DE}$
(___) $\overline{DF} \cong \overline{DF}$

5. Given: $\overline{AB} \cong \overline{AD}$; $\overline{BC} \cong \overline{DC}$
   Prove: $\overline{AC} \perp \overline{BD}$

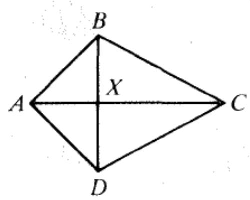


Statements
(___) $\triangle BCX \cong \triangle DCX$
(___) $\angle BCX \cong \angle DCX$
(___) $\angle BXC \cong \angle DXC$
(___) $\overline{AC} \perp \overline{BD}$
(___) $\overline{XC} \cong \overline{XC}$
(___) $\overline{AC} \cong \overline{AC}$
(___) $\overline{AB} \cong \overline{AD}$; $\overline{BC} \cong \overline{DC}$.
(___) $\triangle ABC \cong \triangle ADC$

Example

Given: X is the midpoint of $\overline{BD}$; $\overline{AC} \perp \overline{BD}$

Prove: $\angle ABC \cong \angle ADC$



Solution

Plan for Proof:

$\angle ABC \cong \angle ADC$ if $\triangle ABC \cong \triangle ADC$. But you do not yet have enough information to prove them congruent. You first need to prove $\triangle ABX \cong \triangle ADX$ (or $\triangle BCX \cong \triangle DCX$). Then use congruent corresponding parts to prove $\triangle ABC \cong \triangle ADC$.

1. Complete the proof of the example above.

Statements	Reasons
1. X is the midpoint of $\overline{BD}$; $\overline{AC} \perp \overline{BD}$	1. ______
2. ______ ≅ ______	2. Def. of midpoint
3. ∠______ ≅ ∠______	3. If 2 lines are ⊥, then they form ≅ adj. ∠s.
4. ______ ≅ ______	4. Reflexive Prop.
5. $\triangle ABX \cong \triangle ADX$	5. ______
6. $\overline{AB} \cong \overline{AD}$; $\angle BAC \cong \angle DAC$	6. ______
7. $\overline{AC} \cong \overline{AC}$	7. ______
8. △______ ≅ △______	8. ______
9. $\angle ABC \cong \angle ADC$	9. ______

2. Use the diagram for the example above and write a proof.

Given: X is the midpoint of $\overline{BD}$; $\overline{BC} \cong \overline{DC}$

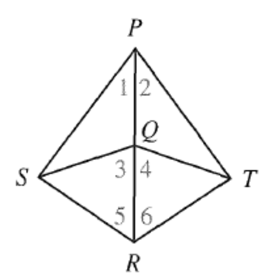
Prove: $\angle BAC \cong \angle DAC$

8. Given: $\overline{PR}$ bisects $\angle SPT$ and $\angle SRT$.

Prove: $\overline{PR}$ bisects $\angle SQT$.

a. List the key steps of a proof.

b. Write a proof in paragraph form.



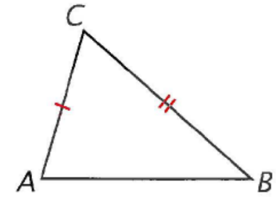
6–4 Inequalities for One Triangle

Theorem 6–2 ***Triangle Angle-Side Size-Ordering Theorem***
If two sides of a triangle are unequal, the angles opposite them are unequal in the same order.

Theorem 6–3 ***Triangle Angle-Side Size-Ordering Converse***
If two angles of a triangle are unequal, the sides opposite them are unequal in the same order.

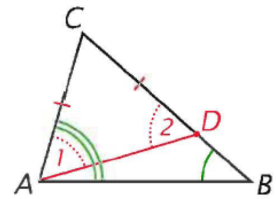
> **Corollary 1**: The perpendicular segment from a point to a line is the shortest segment from the point to the line.

Theorem 6–2



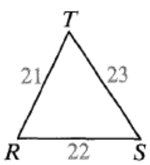
Given: △ABC with BC > AC.
Prove: ∠A > ∠B.

Plan for proof of Theorem 6–2:
Euclid observed the following. The shorter side, AC, is copied along the longer side CB as CD. AD is then drawn to form an isosceles triangle. Euclid's proof is based on observing that $m\angle CAB > m\angle 1$ and $m\angle 1 = m\angle 2$; so $m\angle CAB > m\angle 2$. But $m\angle 2 > m\angle B$; so $m\angle CAB > m\angle B$.

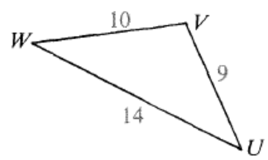


Name the largest angle and the smallest angle of the triangle.

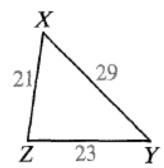
1.



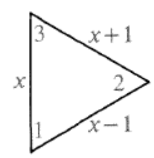
2.



3.

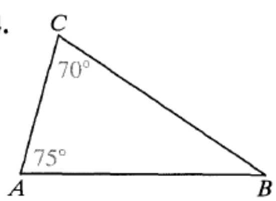


8.

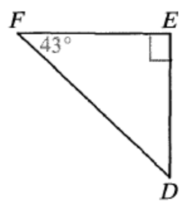


Name the longest side and the shortest side of the triangle.

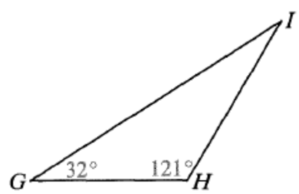
4.



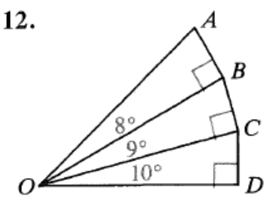
5.



6.

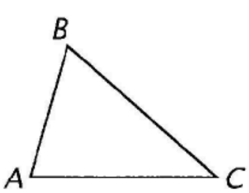


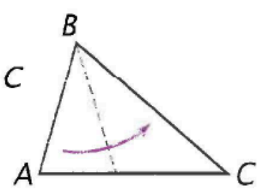
12.

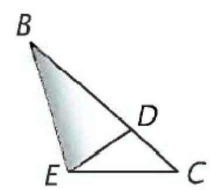


Folding Experiment. This experiment appeared in an old book titled *First Book of Geometry.**

29. Use your ruler to draw a large triangle similar in shape to the first figure below. Cut the triangle out and then fold the shorter side AB onto the longer side BC as shown in the second figure. The figure at the top of the next column shows the result. After answering the following questions, tape the triangle to your paper.







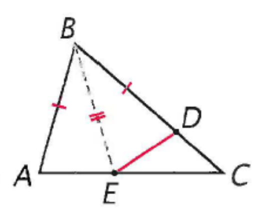
30. On what angle did ∠A fall?

31. In what way is ∠BDE related to △DEC?

32. What does this relation show about how ∠BDE compares with ∠C?

33. What theorem does this experiment illustrate?

Look at this figure.



34. What does the fold line BE do to ∠ABC?

35. How can your answer to exercise 34 be used to prove that ∠BDE = ∠A?

*Written by Grace Chisholm Young and W. H. Young and published in London in 1905.

As obvious as it may seem, the Triangle Inequality has found widespread use in many areas of mathematics including calculus.

From this and the triangle inequality,

$$\left|\frac{f(x)}{g(x)} - L\right| \le \left|\frac{f(x)}{g(x)} - Lu(x)\right| + |Lu(x) - L| \le \epsilon|u(x)| + |L|\,|u(x) - 1|.$$

Above is from a proof in a college course in Real Analysis.

Theorem 6–4 ***The Triangle Inequality***
The sum of any two sides of a triangle is greater than the third side.

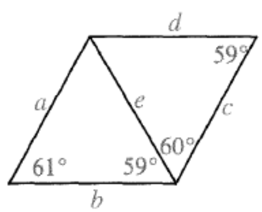
Is it possible for a triangle to have sides with the lengths indicated?

7. 10, 9, 8
8. 6, 6, 20
9. 7, 7, 14.1
10. 16, 11, 5
11. 0.6, 0.5, 1
12. 18, 18, 0.06

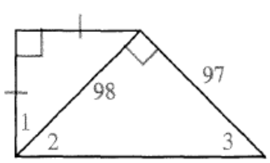
16. Two sides of a triangle have lengths 15 and 20. The length of the third side can be any number between _?_ and _?_.

☺

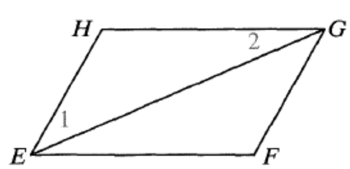
15. Use the lengths a, b, c, d, and e to complete:
_?_ > _?_ > _?_ > _?_ > _?_



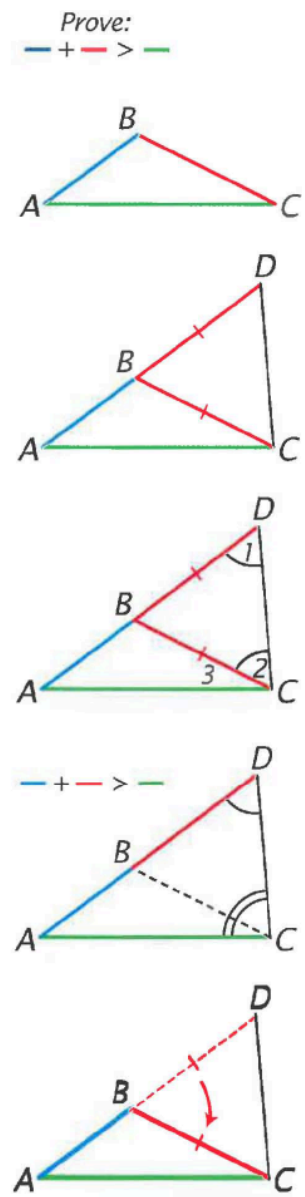
16. Use $m\angle 1$, $m\angle 2$, and $m\angle 3$ to complete:
_?_ > _?_ > _?_



19. Given: ▱$EFGH$; $EF > FG$
Prove: $m\angle 1 > m\angle 2$



Exercise: explain the steps of Euclid's proof of The Triangle Inequality.

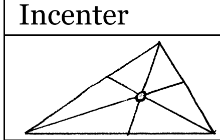
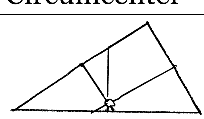
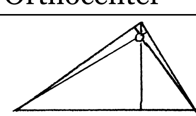
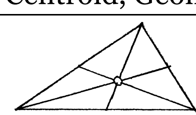
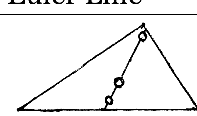


Sheet #471: Concurrent Lines and Centers of Triangles

v.2.4

Concurrent lines are three or more lines that intersect at a point.

Point of concurrency	A point where three or more lines meet	**Points of concurrency of a triangle**	**Triangle Centers**	**Theorems**	**More theorems / Constructions**
Bisector of an Angle	A ray or segment that divides an angle into two congruent adjacent angles	Angular Bisectors intersect at this point	Incenter (I)	The point equidistant from the three sides of a triangle (Theorem 10–1)	The center of the incircle (the circle inscribed in the triangle)
Perpendicular Bisector of a segment	A line or ray (or segment) that is perpendicular to a (another) segment at its midpoint	Perpendicular Bisectors intersect at this point	Circumcenter (C)	The point equidistant from the three vertices of a triangle (Theorem 10–2)	The center of the circumcircle (circle circumscribed about a triangle)
Altitude of a triangle	The perpendicular segment from a vertex to the line that contains the opposite side	Altitudes intersect at this point	Orthocenter (H)		
Median of a triangle	A segment from the vertex to the midpoint of the opposite side	Medians intersect at this point	Centroid, Geometric (G)	The point that is 2/3 of the distance from each vertex to the midpoint of the opposite side (Theorem 10–4)	Point of balance of a triangle with uniform mass (center of mass or center of gravity)
Euler Line	The line that passes through the circumcenter, orthocenter, and centroid				
Distance from a point to a line	The length of the perpendicular segment from the point to the line				

Incenter	Circumcenter	Orthocenter	Centroid, Geometric	Euler Line
				

Bisector-Distance Theorems	• A point is on the perpendicular bisector of a segment if and only if it is equidistant from the endpoints of the segment.	Theorems 4–5 and 4–6
	• A point is on the bisector of an angle if and only if it is equidistant from the sides of the angle.	Theorems 4–7 and 4–8

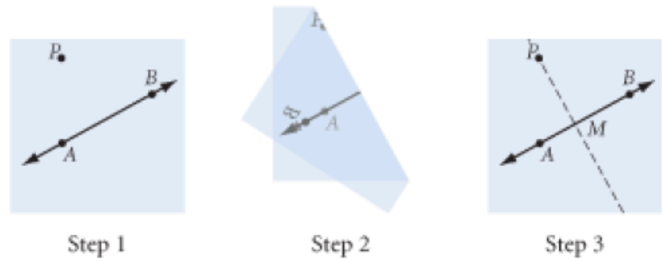
- For isosceles triangles only, the Euler line passes through the incenter, circumcenter, orthocenter, and centroid (I, C, H, G). All four centers coincide in an equilateral triangle.
- The distance from the Centroid to the Circumcenter is 1/2 the distance from the Centroid to the Orthocenter.
- There are more than one hundred named triangle centers.

1.3, 1.4, 4.7, 10.3, 13.7

Sheet 1031: Folding Perpendiculars and Bisectors

Investigation 1: Patty-Paper Perpendiculars

On a piece of patty paper, perform the steps below.



Step 1 Draw and label $\overleftrightarrow{AB}$ and a point P not on $\overleftrightarrow{AB}$.

Step 2 Fold the line onto itself, and slide the layers of paper so that point P appears to be on the crease. Is the crease perpendicular to the line? Check it with the corner of a piece of patty paper.

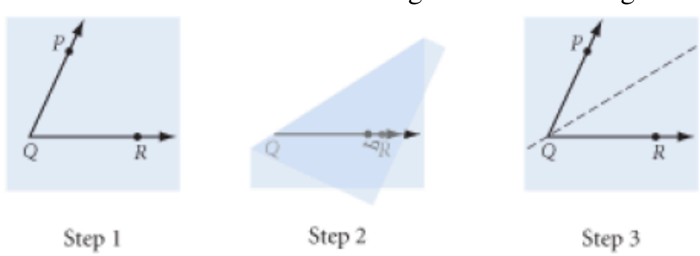
Step 3 Label the point of intersection M. Are $\angle AMP$ and $\angle BMP$ congruent? Supplementary? Why or why not?

Investigation 2: Perpendicular Bisectors of a Segment

To find the perpendicular bisector of $\overline{AB}$ instead, fold the paper so that points A and B **coincide** (land exactly on to of each other). The crease is the perpendicular bisector (a line). A point is on the perpendicular bisector of a segment if and only if it is equidistant from the endpoints of the segment.

Investigation 3: Angle Bisecting by Folding

Each person should draw his or her own acute angle for this investigation.



Step 1 On patty paper, draw a large-scale angle. Label it PQR.

Step 2 Fold your patty paper so that $\overrightarrow{QP}$ and $\overrightarrow{QR}$ coincide. Crease the fold.

Step 3 Unfold your patty paper. Draw a ray with endpoint Q along the crease. Does the ray bisect $\angle PQR$? How can you tell?

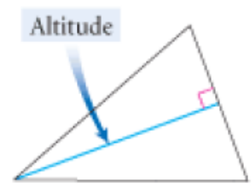
Step 4 Repeat Steps 1–3 with an obtuse angle. Do you use different methods for finding the bisectors of different kinds of angles?

Step 5 Place a point on your angle bisector. Label it A. Compare the distances from A to each of the two sides. Remember that "distance" means *shortest* distance! Try it with other points on the angle bisector. Compare your results with those of others. Copy and complete the conjecture.

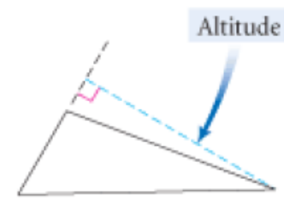
Angle Bisector Conjecture C-8

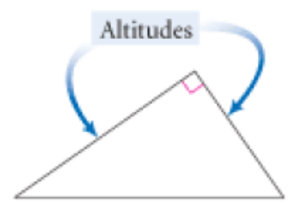
If a point is on the bisector of an angle, then it is __?__ from the sides of the angle.

You can use this construction to find an altitude of a triangle. An **altitude** of a triangle is a perpendicular segment from a vertex to the opposite side or to a line containing the opposite side.



An altitude can be inside the triangle.

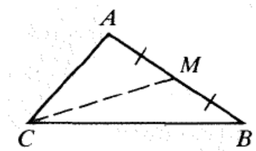




An altitude can be one of the sides of the triangle.

The **length** of the altitude is the height of the triangle. A triangle has three different altitudes, so it has three different heights.

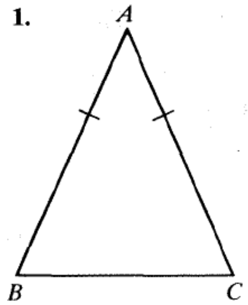
4–7 Medians, Altitudes, and Perpendicular Bisectors



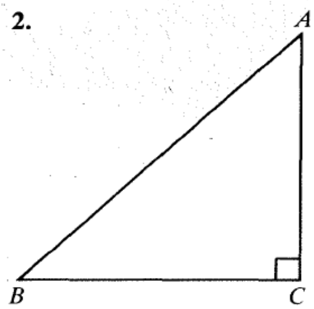
median of a triangle A segment from a vertex of a triangle to the midpoint of the opposite side is a median. $\overline{CM}$ is a median of $\triangle ABC$ since M is the midpoint of $\overline{AB}$.

For each triangle below, draw the median from *A*, the altitude from *A*, and the perpendicular bisector of $\overline{AB}$.

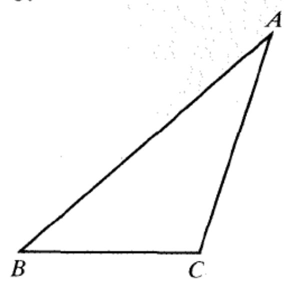
1.

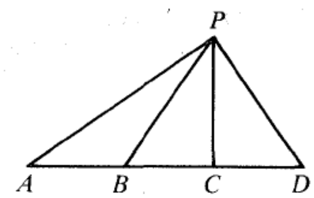


2.



3.

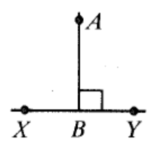




Complete.

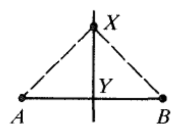
4. If $AB = BC$, then _?_ is a median of $\triangle APC$.
5. If $\overline{PC}$ is a perpendicular bisector of _?_, then $BC = DC$.
6. If $\angle APD$ is a right angle, then _?_ and _?_ are altitudes of $\triangle APD$.
7. If $\overline{PC}$ is a median of $\triangle PBD$, then _?_ = _?_.
8. If $BC = CD$ and $\overline{PC} \perp \overline{BD}$, then _?_ is a perpendicular bisector of _?_.
9. If $\overline{PC}$ and $\overline{AC}$ are both altitudes of $\triangle PCA$, then $\angle$_?_ is a right angle.

distance from a point to a line The length of the perpendicular segment from a point to a line is the distance from the point to the line. $\overline{AB} \perp \overleftrightarrow{XY}$, so AB is the distance from A to $\overleftrightarrow{XY}$.



A point lies on the perpendicular bisector of a segment if and only if the point is equidistant from the endpoints of the segment.

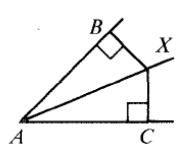
If $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$, then $XA = XB$.
If $XA = XB$, then X lies on the perpendicular bisector of $\overline{AB}$.



Biconditional of **Theorems 4–5 & 4–6**

A point lies on the bisector of an angle if and only if it is equidistant from the sides of the angle.

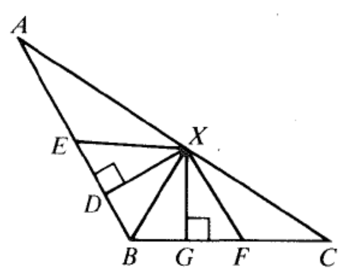
If $\overrightarrow{AX}$ bisects $\angle BAC$, $\overline{XB} \perp \overrightarrow{AB}$, and $\overline{XC} \perp \overrightarrow{AC}$, then $XB = XC$.
If $XB = XC$, $\overline{XB} \perp \overrightarrow{AB}$, and $\overline{XC} \perp \overrightarrow{AC}$, then $\overrightarrow{AX}$ bisects $\angle BAC$.



Biconditional of **Theorems 4–7 & 4–8**

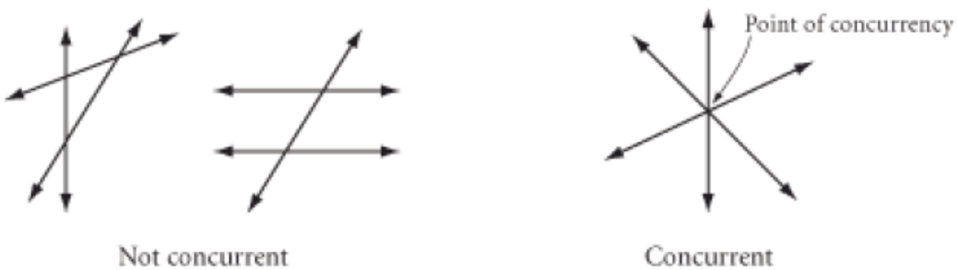
Complete.

10. If $\overrightarrow{BX}$ bisects $\angle ABC$, then $\angle$_?_ $\cong \angle$_?_ and $DX =$ _?_.
11. If $\overline{DX}$ is the perpendicular bisector of $\overline{EB}$, then $ED =$ _?_ and $XE =$ _?_.
12. If $XB = XF$, then _?_ is the perpendicular bisector of $\overline{BF}$, and $\angle XBF \cong$ _?_.
13. If $XD = XG$, then _?_ is the bisector of $\angle$_?_.



Sheet 1032: Folding Points of Concurrency

When three or more lines have a point in common, they are **concurrent.** Segments, rays, and even planes are concurrent if they intersect in a single point.



The point of intersection is the **point of concurrency.**

Investigation 4: Folding Concurrence

In this investigation you will discover that some special lines in a triangle have points of concurrency. As a group, you should investigate each set of lines on an acute triangle, an obtuse triangle, and a right triangle to be sure that your conjectures apply to all triangles.

Step 1 Draw a large triangle on patty paper. Make sure you have at least one acute triangle, one obtuse triangle, and one right triangle in your group.

Step 2 Construct the three angle bisectors for each triangle. Are they concurrent?

Compare your results with the results of others. State your observations as a conjecture.

> **Angle Bisector Concurrency Conjecture** C-9
>
> The three angle bisectors of a triangle _?_.

Step 3 Draw a large triangle on a new piece of patty paper. Make sure you have at least one acute triangle, one obtuse triangle, and one right triangle in your group.

Step 4 Construct the perpendicular bisector for each side of the triangle and complete the conjecture.

> **Perpendicular Bisector Concurrency Conjecture** C-10
>
> The three perpendicular bisectors of a triangle _?_.

Step 5 Draw a large triangle on a new piece of patty paper. Make sure you have at least one acute triangle, one obtuse triangle, and one right triangle in your group.

Step 6 Construct the lines containing the altitudes of your triangle and complete the conjecture.

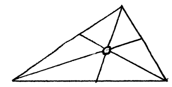
> **Altitude Concurrency Conjecture** C-11
>
> The three altitudes (or the lines containing the altitudes) of a triangle _?_.

Step 7 For what kind of triangle will the points of concurrency be the same point?

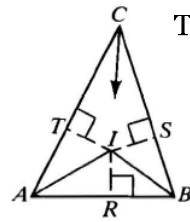
The point of concurrency for the three angle bisectors is the **incenter.** The point of concurrency for the perpendicular bisectors is the **circumcenter.** The point of concurrency for the three altitudes is called the **orthocenter.** Use these definitions to label each patty paper from the previous investigation with the correct name for each point of concurrency. Medians also form a point of concurrency.

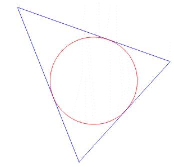
10–3 Concurrent Lines

Theorem 10–1 ***Incenter Theorem***
The bisectors of the angles of a triangle intersect in a point that is equidistant from the three sides of the triangle.

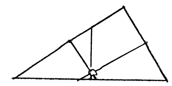


Thm 10–1

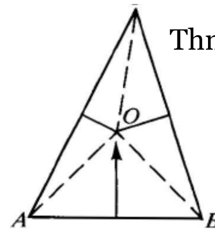


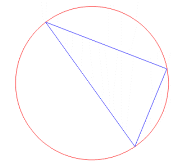


Theorem 10–2 ***Circumcenter Theorem***
The perpendicular bisectors of the sides of a triangle intersect in a point that is equidistant from the three vertices of the triangle.

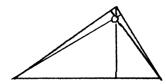


Thm 10–2

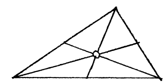




Theorem 10–3 ***Altitude Concurrency Theorem (Orthocenter)***
The lines that contain the altitudes of a triangle intersect in a point.

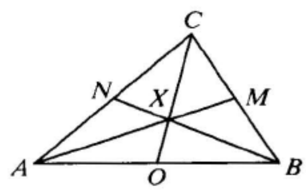


Theorem 10–4 ***Geometric Centroid Theorem***
The medians of a triangle intersect in a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.



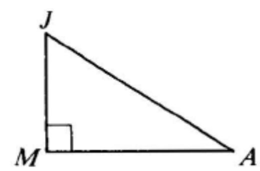
According to Theorem 10-4, if $\overline{AM}$, $\overline{BN}$, and $\overline{CO}$ are medians of $\triangle ABC$, then:

$$AX = \tfrac{2}{3}AM$$
$$XN = \tfrac{1}{3}BN$$
$$CX:XO:CO = 2:1:3$$

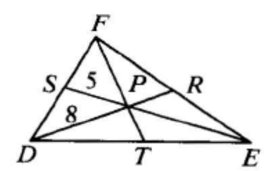


Set A.

4. $\triangle JAM$ is a right triangle.
- **a.** Is $\overline{JM}$ an altitude of $\triangle JAM$?
- **b.** Name another altitude shown.
- **c.** In what point do the three altitudes of $\triangle JAM$ meet?
- **d.** Where do the perpendicular bisectors of the sides of $\triangle JAM$ meet?
- **e.** Does your answer to (d) agree with Theorem 10-2? (duh)

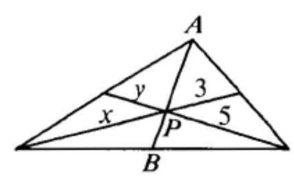


5. The medians of $\triangle DEF$ are shown. Find the lengths indicated.
a. EP = _?_ **b.** PR = _?_
c. If $FT = 9$, then PT = _?_ and FP = _?_.

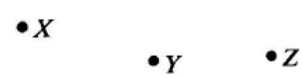


Set B. Exercises 2–5 refer the diagram in which the medians of a triangle are shown.

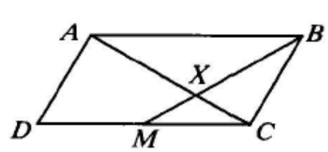
2. Find the values of x and y.
3. If $AB = 6$, then BP = _?_ and AP = _?_.
4. If $AB = 7$, then BP = _?_ and AP = _?_.
5. If $PB = 1.9$, then AP = _?_ and AB = _?_.



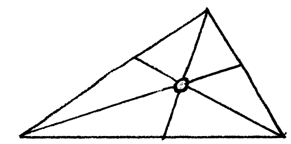
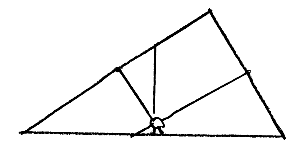
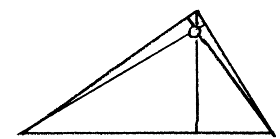
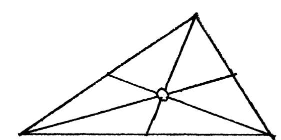
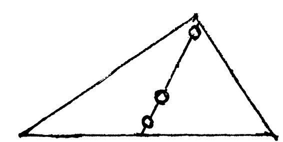
9. Three towns, located as shown, plan to build one recreation center to serve all three towns. They decide that the fair thing to do is to build the center equidistant from the three towns. Where would that be? Comment about the wisdom of the plan. Propose a fair location for the center.



15. $ABCD$ is a parallelogram with M the midpoint of $\overline{CD}$. If $\overline{BM}$ intersects $\overline{AC}$ at X, prove that $CX = \frac{1}{3}AC$.
(*Hint*: Draw $\overline{BD}$.)



Sheet #473: Triangle Center study cards

Points of concurrency of a triangle	**Triangle Centers**	**Theorems**	**More theorems / Constructions**	**Figures**
Angular Bisectors intersect at this point	Incenter (I)	The point equidistant from the 3 sides of a triangle	The center of the incircle	
Perpendicular Bisectors intersect at this point	Circumcenter (C)	The point equidistant from the 3 vertices of a triangle	The center of the circumcircle	
Altitudes intersect at this point	Orthocenter (H)			
Medians intersect at this point	Centroid (G)	The point that is 2/3 of the distance from each vertex to the midpoint of the opposite side	Point of balance of a triangle with uniform mass	
	Euler Line	The line that passes through the circumcenter, orthocenter, and centroid		

Sheet 474: Mix and Match Geometric Constructions

For Exercises 21–31, match each geometric construction with one of the figures below.

21. Construction of a perpendicular bisector

22. Construction of an angle bisector

23. Construction of a perpendicular from a point to a line

24. Construction of a perpendicular through a point on a line

25. Construction of a line parallel to a given line through a given point not on the line

26. Construction of an equilateral triangle

27. Construction of an altitude in a triangle

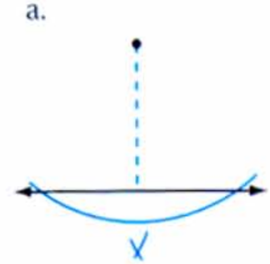
28. Construction of a centroid

29. Construction of an incenter

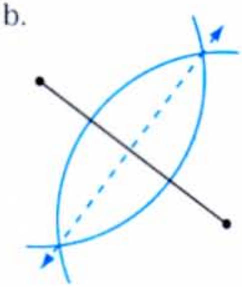
30. Construction of an orthocenter

31. Construction of a circumcenter

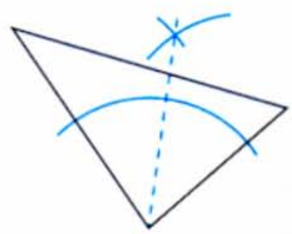
a.



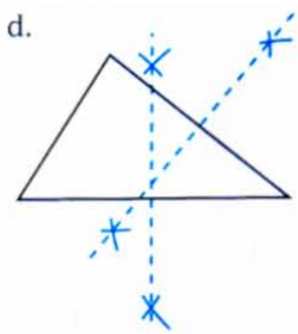
b.



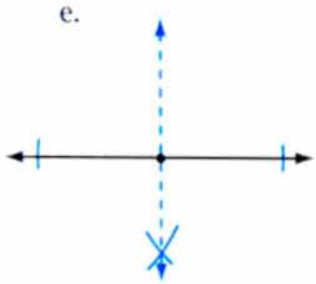
c.



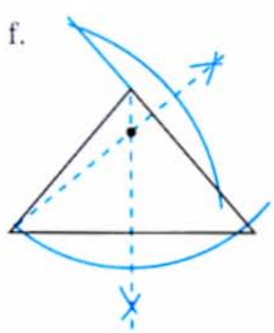
d.



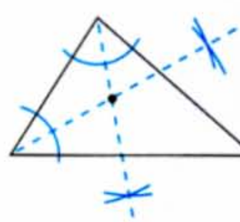
e.



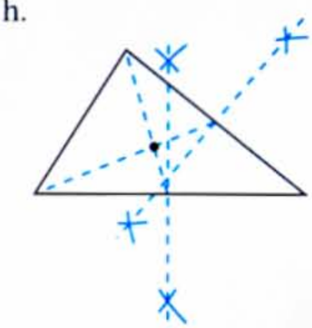
f.



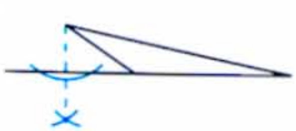
g.



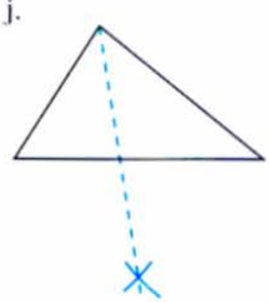
h.



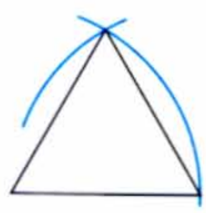
i.



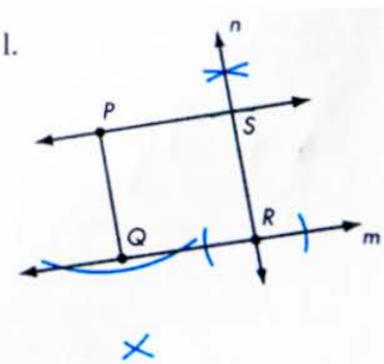
j.



k.



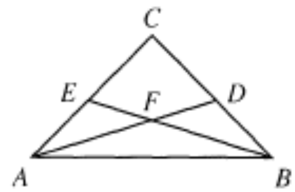
l.



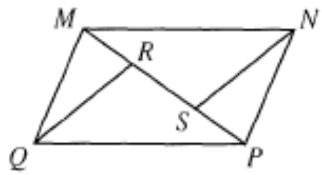
Self-Test 3

Set A (4–6, 4–7)

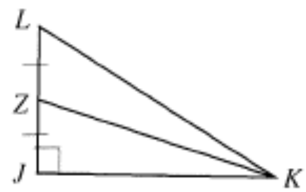
1. Suppose you wish to prove $\triangle AFE \cong \triangle BFD$. If you have already proved $\triangle ABE \cong \triangle BAD$, what corresponding parts from this second pair of congruent triangles would you use to prove the first pair of triangles congruent?



2. Given: $\triangle MPQ \cong \triangle PMN$; $\overline{MS} \cong \overline{PR}$
   Prove: $\triangle MSN \cong \triangle PRQ$



3. In $\triangle JKL$ name each of the following.
   a. an altitude **b.** a median
4. Note that $ZL = ZJ$. Can you deduce that $\overrightarrow{KZ}$ bisects $\angle LKJ$?



5. $\overrightarrow{UV}$ bisects $\angle WUX$. Write the theorem that justifies the statement that V is equidistant from $\overrightarrow{UW}$ and $\overrightarrow{UX}$.
6. In $\triangle ABC$, $AB = 7$ and $BC = 7$. Write the theorem that allows you to conclude that B is on the perpendicular bisector of $\overline{AC}$.

Set B (6–4)

1. In $\triangle XYZ$, $m\angle X = 50$, $m\angle Y = 60$, and $m\angle Z = 70$. Name the longest side of the triangle.
2. In $\triangle DOM$, $\angle O$ is a right angle and $m\angle D > m\angle M$. Which side of $\triangle DOM$ is the shortest side?

Set C (10–1, 10–2, and 10–3)

1. Draw any $\overline{CD}$. Construct the perpendicular bisector of $\overline{CD}$.
2. Construct a 60° angle, $\angle RST$, and its bisector, $\overrightarrow{SQ}$.
3. Draw a large acute $\triangle ABC$. Then construct altitude $\overline{AD}$ from vertex A.
4. Draw line t and choose any point P that is not on line t. Construct $\overleftrightarrow{PQ} \parallel t$.
5. Draw any $\overline{AB}$. Construct rectangle $JKLM$ so that $JK = 2AB$ and $KL = AB$.
6. Name four types of concurrent lines, rays, or segments that are associated with triangles.
7. The perpendicular bisectors of the sides of a right triangle intersect in a point located at _?_.
8. The medians of equilateral $\triangle ABC$ intersect at point X. If $\overline{AD}$ is a median and $AB = 12$, then $AX =$ _?_ and $XD =$ _?_.

Self-Test Answers

Self-Test 1

1. $\angle P \cong \angle T$; CPCT **2.** $\overline{KO}$, $\overline{MA}$; $\overline{OP}$, $\overline{AT}$; $\overline{KP}$, $\overline{MT}$ **3.** $\triangle JKX \cong \triangle JKY$; SAS **4.** No $\cong$ can be deduced. **5.** $\triangle TRP \cong \triangle TRS$; ASA **6.** 1. $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$ (Given) 2. $\overline{DB} \cong \overline{DB}$ (Refl. Prop.) 3. $\triangle ADB \cong \triangle CBD$ (ASA Post.) **7.** 1. $\overline{CD} \cong \overline{AB}$; $\overline{CB} \cong \overline{AD}$ (Given) 2. $\overline{DB} \cong \overline{DB}$ (Refl. Prop.) 3. $\triangle ADB \cong \triangle CBD$ (SSS Post.) 4. $\angle 1 \cong \angle 2$ (CPCT) **8.** 1. $\overline{AD} \parallel \overline{BC}$ (Given) 2. $\angle 4 \cong \angle 3$ (If 2 $\parallel$ lines are cut by a trans., then alt. int. $\angle$s are $\cong$.) 3. $\overline{AD} \cong \overline{CB}$ (Given) 4. $\overline{DB} \cong \overline{DB}$ (Refl. Prop.) 5. $\triangle ADB \cong \triangle CBD$ (SAS Post.) 6. $\angle 1 \cong \angle 2$ (CPCT) 7. $\overline{DC} \parallel \overline{AB}$ (If 2 lines are cut by a trans. and alt. int. $\angle$s are $\cong$, then the lines are $\parallel$.)

Self-Test 2

1. 70 **2.** 7 **3.** 30 **4.** $\overline{AB} \cong \overline{AC}$, $\angle A \cong \angle A$, $\angle ANB \cong \angle AMC$, so $\triangle ABN \cong \triangle ACM$ by AAS. **5.** 1. $\overline{BN} \perp \overline{AC}$; $\overline{CM} \perp \overline{AB}$ (Given) 2. $\angle BMC$ and $\angle CNB$ are rt. $\angle$s. (Def. of $\perp$ lines) 3. $\triangle BMC$ and $\triangle CNB$ are rt. $\triangle$s. (Def. of rt. $\triangle$) 4. $\overline{MB} \cong \overline{NC}$ (Given) 5. $\overline{BC} \cong \overline{BC}$ (Refl. Prop.) 6. $\triangle BMC \cong \triangle CNB$ (HL) 7. $\overline{CM} \cong \overline{BN}$ (CPCT)

Self-Test 3

Set A

1. $\overline{EA} \cong \overline{DB}$ and $\angle AEB \cong \angle BDA$ **2.** 1. $\triangle MPQ \cong \triangle PMN$ (Given) 2. $\overline{MN} \cong \overline{QP}$; $\angle MPQ \cong \angle PMN$ (CPCT) 3. $\overline{MS} \cong \overline{PR}$ (Given) 4. $\triangle MSN \cong \triangle PRQ$ (SAS) **3. a.** $\overline{LJ}$ or $\overline{KJ}$ **b.** $\overline{KZ}$ **4.** No **5.** If a pt. lies on the bis. of an $\angle$, then the pt. is equidistant from the sides of the $\angle$. **6.** If a pt. is equidistant from the endpts. of a seg., then the pt. lies on the $\perp$ bis. of the seg.

Set B

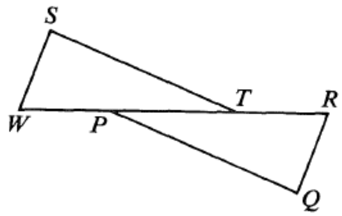
1. $\overline{XY}$ **2.** $\overline{OD}$

Set C

1. Const. 4 **2.** Draw $\overline{ST}$. With ctrs. S and T, and radius ST, draw arcs int. at R. Draw $\overrightarrow{SR}$; $m\angle RST = 60$. Use Const. 3. **3.** Const. 6 **4.** Const. 7 **5.** Methods may vary. Const. $\overline{JK}$ such that $JK = 2AB$ (Const. 1). Const. lines $\perp$ to $\overline{JK}$ at J and K. Const. $\overline{JM} \cong \overline{KL} \cong \overline{AB}$. Draw $\overline{ML}$. **6.** lines that contain the altitudes, medians, $\angle$ bis., $\perp$ bis. of the sides **7.** midpt. of the hyp. **8.** $4\sqrt{3}$, $2\sqrt{3}$

Chapter Review

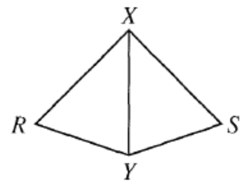
The two triangles shown are congruent. Complete.



1. $\triangle STW \cong$ _?_ **2.** $\triangle PQR \cong$ _?_ 4–1

3. $\angle R \cong$ _?_ **4.** _?_ $= RP$

Can you deduce from the given information that $\triangle RXY \cong \triangle SXY$? If so, what postulate can you use?



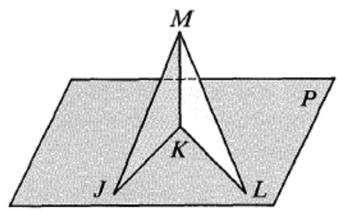
5. Given: $\overline{RX} \cong \overline{SX}$; $\overline{RY} \cong \overline{SY}$ 4–2

6. Given: $\overline{RY} \cong \overline{SY}$; $\angle R \cong \angle S$

7. Given: $\overline{XY}$ bisects $\angle RXS$ and $\angle RYS$.

8. Given: $\angle RXY \cong \angle SXY$; $\overline{RX} \cong \overline{SX}$

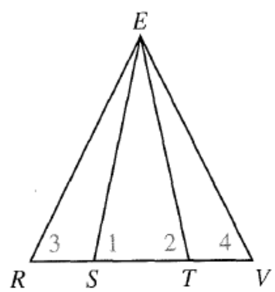
Write proofs in two-column form.



9. Given: $\overline{JM} \cong \overline{LM}$; $\overline{JK} \cong \overline{LK}$ 4–3
Prove: $\angle MJK \cong \angle MLK$

10. Given: $\angle JMK \cong \angle LMK$; $\overline{MK} \perp$ plane P
Prove: $\overline{JK} \cong \overline{LK}$

Complete.



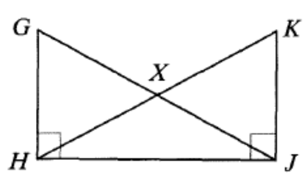
11. If $\angle 3 \cong \angle 4$, then which segments must be congruent? 4–4

12. If $\triangle REV$ is an equiangular triangle, then $\triangle REV$ is also a(n) _?_ triangle.

13. If $\overline{ES} \cong \overline{ET}$, $m\angle 1 = 75$, and $m\angle 2 = 3x$, then $x =$ _?_.

14. If $\angle 1 \cong \angle 2$, $ES = 3y + 5$, and $ET = 25 - y$, then $y =$ _?_.

Write proofs in two-column form.



15. Given: $\overline{GH} \perp \overline{HJ}$; $\overline{KJ} \perp \overline{HJ}$; 4–5
$\angle G \cong \angle K$
Prove: $\triangle GHJ \cong \triangle KJH$

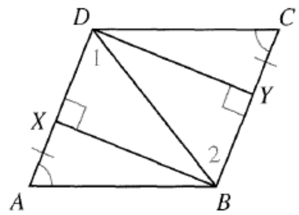
16. Given: $\overline{GH} \perp \overline{HJ}$; $\overline{KJ} \perp \overline{HJ}$;
$\overline{GJ} \cong \overline{KH}$
Prove: $\overline{GH} \cong \overline{KJ}$

17. Give the reason for each key step of the proof. 4–6

Given: $\overline{AX} \cong \overline{CY}$; $\angle A \cong \angle C$;
$\overline{BX} \perp \overline{AD}$; $\overline{DY} \perp \overline{BC}$

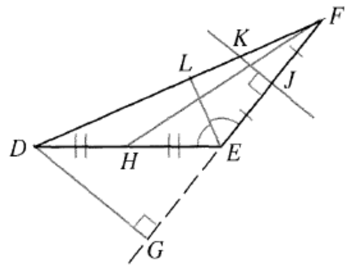
Prove: $\overline{AD} \parallel \overline{BC}$

1. $\triangle ABX \cong \triangle CDY$
2. $\overline{BX} \cong \overline{DY}$
3. $\triangle BDX \cong \triangle DBY$
4. $\angle 1 \cong \angle 2$
5. $\overline{AD} \parallel \overline{BC}$



18. Refer to $\triangle DEF$ and name each of the following: 4–7

a. an altitude
b. a median
c. the perpendicular bisector of a side of the triangle



19. Point G lies on the perpendicular bisector of $\overline{EF}$. Write the theorem that justifies the statement that $GE = GF$.

20. $\triangle ABC$ and $\triangle ABD$ are congruent right triangles with common hypotenuse $\overline{AB}$. Write the theorem that allows you to conclude that point B lies on the bisector of $\angle DAC$.

11. In $\triangle TOP$, if $OT > OP$, then $m\angle P >$ _?_. 6–4

12. In $\triangle RED$, if $m\angle D < m\angle E$, then $RD >$ _?_.

13. Points X and Y are in plane M. If $\overline{PX} \perp$ plane M, then PX _?_ PY.

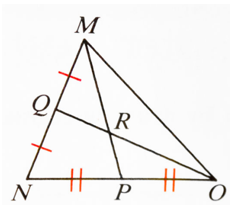
14. Two sides of a triangle have lengths 6 and 8. The length of the third side must be greater than _?_ and less than _?_.

8. The _?_ of a triangle intersect in a point that is equidistant from the vertices of the triangle. 10–3

9. The _?_ of a triangle intersect in a point that is equidistant from the sides of the triangle.

10. If $MR = 12$, then $MP =$ _?_.

11. $QR:RO =$ _?_ (numerical answer)



Exs. 10, 11