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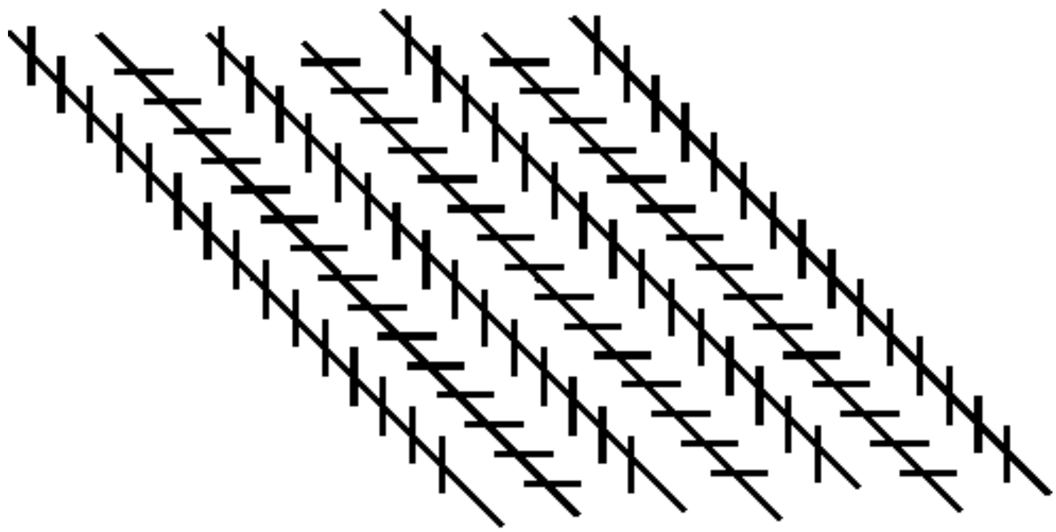
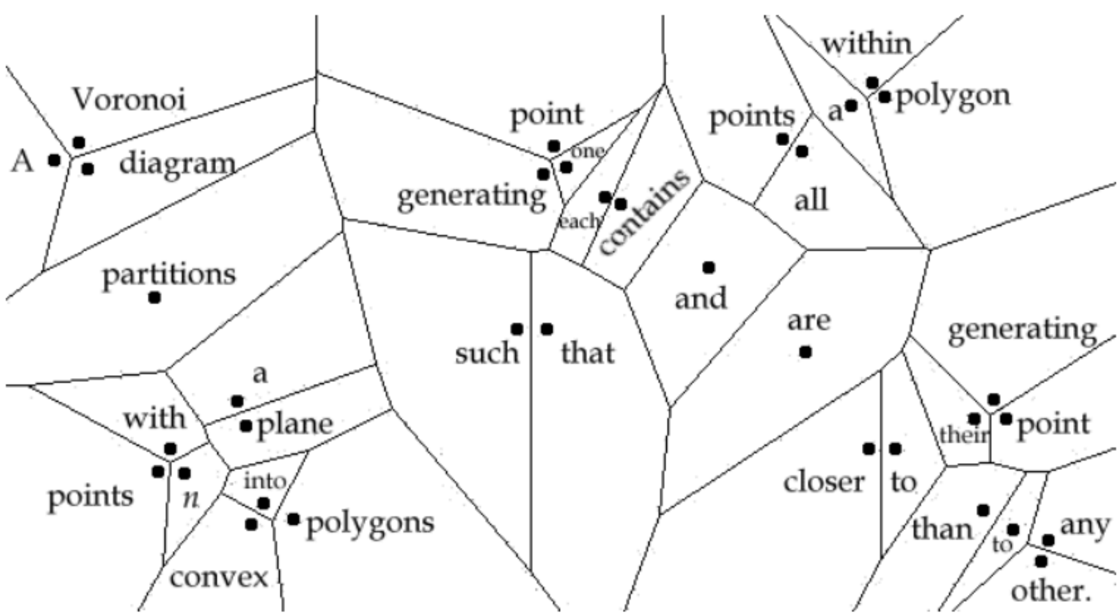
ParallelPacket

Chapter 3

Parallel Lines, Triangle and Polygon Angles



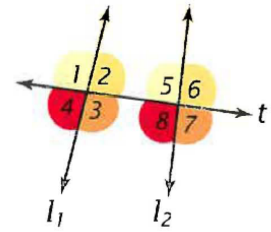


Sheet#0351: Postulates, Properties, and Theorems 2

Segment Addition Postulate (#2)	$AB + BC = AC$
Angle Addition Postulate (#4)	$m\angle AOB + m\angle BOC = m\angle AOC$
Addition Property of Equality (1)	If $a = b$ and $c = d$, then $a + c = b + d$
Subtraction Property of = (2)	If $a = b$ and $c = d$, then $a - c = b - d$
Multiplication Property of = (3)	If $a = b$ then $ca = cb$
Division Property of = (4)	If $a = b$ then $a/c = b/c$ for $c \neq 0$
Substitution Property of = (5)	If $a = b$ then a or b may be substituted for the other in any equation.
Reflexive Property of Equality (6)	$a = a$
Symmetric Property of = (7)	If $a = b$, then $b = a$
Transitive Property of = (8)	If $a = b$ and $b = c$ then $a = c$
Reflexive Property of Congruence (9)	$\overline{AB} \cong \overline{AB}$ (and likewise for angles)
Symmetric Property of $\cong$ (10)	If $\overline{AB} \cong \overline{CD}$ then $\overline{CD} \cong \overline{AB}$
Transitive Property of $\cong$ (11)	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$ then $\overline{AB} \cong \overline{EF}$
Midpoint Theorem (2–1)	If M is the midpoint of $\overline{AB}$ then $AM = ½\ AB$ and $MB = ½\ AB$
Angle Bisector Theorem (2–2)	If $\overrightarrow{BX}$ is the bisector of $\angle ABC$ then $m\angle ABX = ½ m\angle ABC$ and $m\angle XBC = ½ m\angle ABC$
Vertical Angles Theorem (2–3)	Vertical angles are congruent.
Perpendicular Lines Theorem (2–4)	If two lines are perpendicular, then they form congruent adjacent angles.
Perpendicular Lines Theorem Converse (2–5)	If two lines form congruent adjacent angles, then the lines are perpendicular.
Corresponding Angles Postulate (#10)	If two parallel lines are cut by a transversal, then corresponding angles are congruent.
Corresponding Angles Postulate Converse (#11)	If two lines are cut by a transversal and corresponding angles are congruent, then the lines are parallel.
AIA Theorem (3–2)	If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
AIA Theorem Converse (3–5)	If two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.
SSIA Theorem (3–3)	If two parallel lines are cut by a transversal, then same-side interior angles are supplementary.
SSIA Theorem Converse (3–6)	If two lines are cut by a transversal and same-side interior angles are supplementary, then the lines are parallel.
Triangle Sum Theorem (3–11)	The sum of the measures of the interior angles of a triangle is 180º.
Exterior Angle Theorem (3–12)	The measure of an exterior angle of a triangle equals the sum of the measures of the two remote (non-adjacent) interior angles.
Polygon Interior Angles Theorem (3–13)	The sum of the measures of the interior angles of a convex polygon with n sides is $(n-2)180^\circ$.
Polygon Exterior $\angle$s Theorem (3–14)	The sum of the measures of the exterior angles of any convex polygon, one angle at each vertex, is 360º.

3–1 Definitions

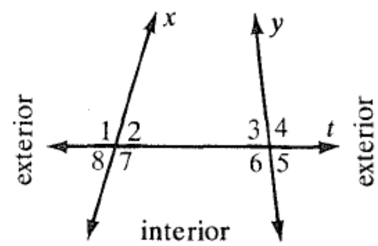
Two lines that do not intersect are either parallel or skew.
Parallel lines (|| lines) are coplanar lines that do not intersect.
Skew lines are non-coplanar lines. They do not intersect and are not parallel.
Parallel planes do not intersect.
A line and a plane are parallel if they do not intersect.

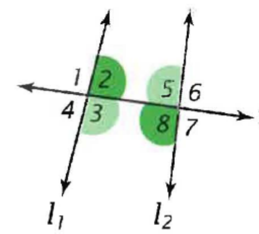


Corresponding angles

A transversal is a line that intersects two or more coplanar lines in different points. When two coplanar lines are cut by a transversal, some of the angles formed have special names. In the diagram, *t* is a transversal of *x* and *y*.

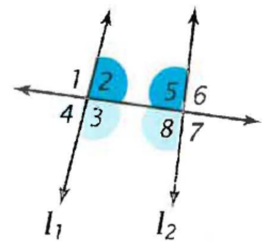
exterior angles	∠1, ∠4, ∠5, ∠8
interior angles	∠2, ∠3, ∠6, ∠7
alternate interior angles	∠2 and ∠6, ∠7 and ∠3
same-side interior angles	∠2 and ∠3, ∠7 and ∠6
corresponding angles	∠1 and ∠3, ∠2 and ∠4, ∠8 and ∠6, ∠7 and ∠5





Alternate interior angles

AIA Alternate interior angles used frequently and are abbreviated AIA.
SSIA Same-side interior angles can be abbreviated SSIA.



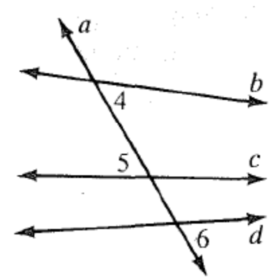
Interior angles on the same side of the transversal

Complete each statement with the word *always*, *sometimes*, or *never*.

1. Two lines in the same plane are _?_ parallel.
2. Two lines in the same plane are _?_ skew.
3. Two noncoplanar lines _?_ intersect.
4. Two planes _?_ intersect.
5. A line and a plane _?_ have exactly one point of intersection.
6. If two planes do not intersect, then they are _?_ parallel.

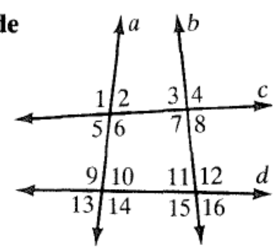
8. Use the figure to the right to name the two lines and the transversal that form each pair of angles.

a. ∠4 and ∠5
b. ∠4 and ∠6

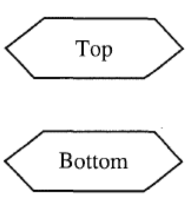


Classify each pair of angles as alternate interior angles, same-side interior angles, or corresponding angles.

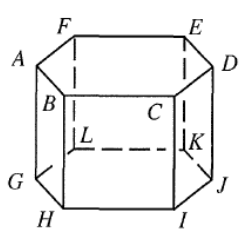
11. ∠2 and ∠4
12. ∠7 and ∠12
13. ∠10 and ∠11
14. ∠5 and ∠10
15. ∠14 and ∠15
16. ∠3 and ∠11



22. Draw a diagram of a six-sided box by following the steps below.



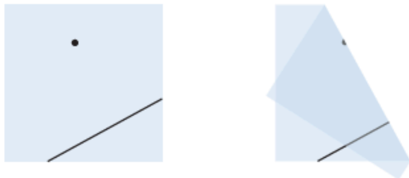
Step 1
Draw a six-sided top. Then draw an exact copy of the top directly below it.



Step 2
Draw vertical edges. Make invisible edges dashed.

Constructing Parallel Lines by Folding

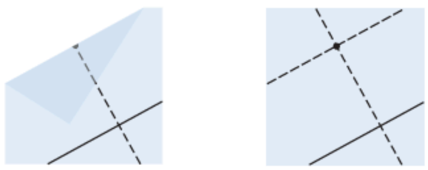
How would you check whether two lines are parallel? One way is to draw a transversal and compare corresponding angles. You can also use this idea to *construct* a pair of parallel lines.



Step 1 Step 2

Step 1 Draw a line and a point on patty paper as shown.

Step 2 Fold the paper to construct a perpendicular so that the crease runs through the point as shown. Describe the four newly formed angles.

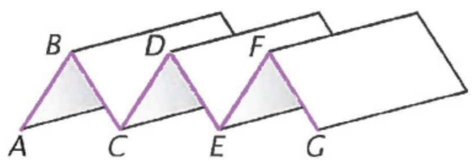


Step 3 Step 4

Step 3 Through the point, make another fold that is perpendicular to the first crease.

Step 4 Compare the pairs of corresponding angles created by the folds. Are they all congruent? Why? What conclusion can you make about the lines?

Folded Paper. The figure below shows a sheet of paper folded into an accordion shape. The pleated edge outlined in color lies in a vertical plane.



4. What are ∠B and ∠C called with respect to lines AB, BC, and CD?

In Exercises 18-20 use two parallel lines here or in your notebook and draw any transversal. Use a protractor to measure.

18. Measure one pair of corresponding angles. Repeat the experiment with another transversal. What appears to be true?

19. Measure one pair of alternate interior angles. Repeat the experiment with another transversal. What appears to be true?

20. Measure one pair of same-side interior angles. Repeat the experiment with another transversal. What appears to be true?

3–2 Properties of Parallel Lines

The parallel postulates and resulting theorems are central to Euclidian Geometry.

• If two parallel lines are cut by a traversal,
then **Corresponding angles are congruent** (Postulate 10),
Alternate Interior Angles are congruent (Theorem 3–2),
and Same-Side Interior Angles are supplementary (Theorem 3–3).

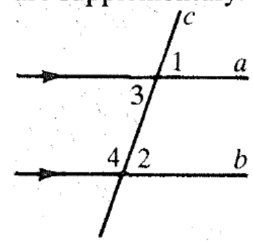
• The converses also hold (Postulate 11, Theorem 3–5, and Theorem 3–6).

• If a traversal is perpendicular to one of two parallel lines, then it is perpendicular to the other one also (Theorem 3–4).

If $a \parallel b$, then $\angle 1 \cong \angle 2$.

If $a \parallel b$, then $\angle 3 \cong \angle 2$.

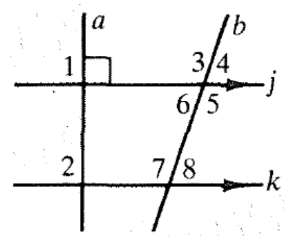
If $a \parallel b$, then $\angle 3$ and $\angle 4$ are supplementary.



Arrowheads (→) are used to indicate parallel lines. Double arrowheads are also used.

State the postulate or theorem that justifies each statement.

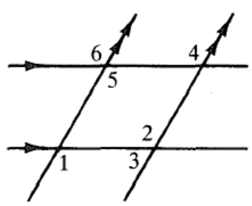
1. $\angle 3 \cong \angle 7$
2. $a \perp k$
3. $m\angle 6 + m\angle 7 = 180$
4. $\angle 6 \cong \angle 8$
5. $\angle 4 \cong \angle 6$
6. $\angle 1 \cong \angle 2$
7. $m\angle 5 = m\angle 7$
8. $\angle 5$ is supplementary to $\angle 8$.



Example

If $m\angle 1 = 120$, find the measure of:

a. $\angle 2$ **b.** $\angle 3$ **c.** $\angle 4$



Solution

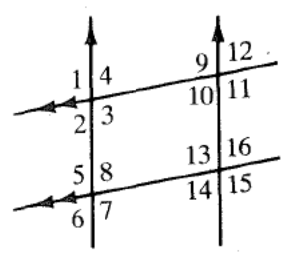
a. $m\angle 2 = m\angle 1 = 120$ (If 2 ∥ lines are cut by a transversal, then alt. int. ∠s are ≅.)

b. $\angle 1$ and $\angle 3$ are same-side interior angles, so $m\angle 1 + m\angle 3 = 180$. (If 2 ∥ lines are cut by a transversal, then same-side int. ∠s are supp.) $120 + m\angle 3 = 180$, so $m\angle 3 = 60$.

c. $\angle 4 \cong \angle 2$ (If 2 ∥ lines are cut by a transversal, then corr. ∠s are ≅.), and $\angle 2 \cong \angle 1$, so $\angle 4 \cong \angle 1$ and $m\angle 4 = 120$.

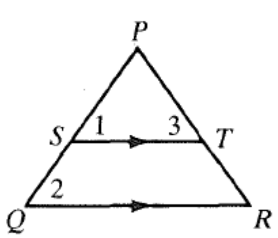
Complete.

12. If $m\angle 2 = 80$, then $m\angle 6 =$ _?_ and $m\angle 7 =$ _?_.
13. If $m\angle 9 = 105$, then $m\angle 10 =$ _?_ and $m\angle 16 =$ _?_.
14. If $m\angle 8 = 85$, then $m\angle 16 =$ _?_ and $m\angle 10 =$ _?_.
15. If $m\angle 15 = 95$, then $m\angle 8 =$ _?_ and $m\angle 1 =$ _?_.
16. If $m\angle 3 = m\angle 4 + 30$, find $m\angle 5$.
17. If $m\angle 16 = m\angle 13 - 20$, find $m\angle 11$.



Complete the proof.

18. Given: $\overline{ST} \parallel \overline{QR}$; $\angle 1 \cong \angle 3$
Prove: $\angle 2 \cong \angle 3$



Statements	Reasons
1. ____________	1. Given
2. $\angle 1 \cong \angle 2$	2. ____________
3. ____________	3. ____________

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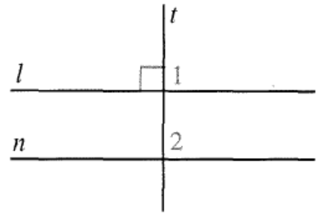
13. Copy and complete the proof of Theorem 3-4.

Given: Transversal t cuts l and n;
$t \perp l$; $l \parallel n$

Prove: $t \perp n$



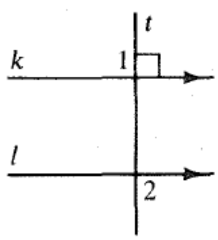
Proof:

Statements	Reasons
1. $t \perp l$	1. ?
2. $m\angle 1 = 90$	2. ?
3. ?	3. Given
4. $\angle 2 \cong \angle 1$ or $m\angle 2 = m\angle 1$	4. ?
5. ?	5. Substitution Property
6. $t \perp n$	6. ?

Complete the proof.

19. Given: $k \parallel l$; $k \perp t$

Prove: $\angle 1 \cong \angle 2$

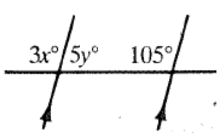


Statements	Reasons
1. ______	1. Given
2. $m\angle 1 = 90$	2. ______
3. $l \perp t$	3. ______
4. $m\angle 2 = 90$	4. ______
5. $m\angle 1 = m\angle 2$, or $\angle 1 \cong \angle 2$	5. ______

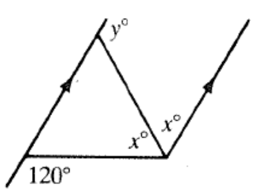
Example 1

Find the values of x and y.

a.



b.



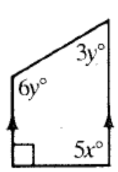
Solution

a. $3x = 105$ (If 2 ∥ lines are cut by a transversal,
$x = 35$ then corr. ∠s are ≅.)
$5y + 105 = 180$ (If 2 ∥ lines are cut by a transversal,
$5y = 75$ then same-side int. ∠s are supp.)
$y = 15$

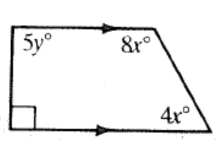
b. $120 = x + x$ (If 2 ∥ lines are cut by a transversal,
$120 = 2x$ then alt. int. ∠s are ≅.)
$60 = x$
$x + y = 180$ (If 2 ∥ lines are cut by a transversal,
$60 + y = 180$ then same-side int. ∠s are supp.)
$y = 120$

Find the values of x and y.

1.



2.



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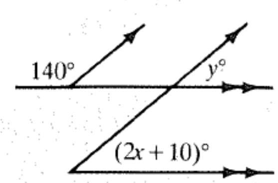
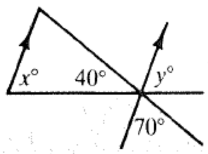


Find the values of x and y.

4.



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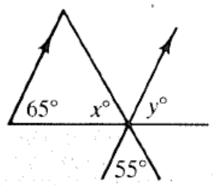

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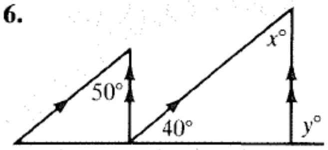
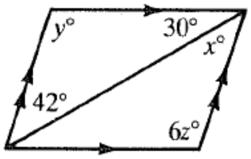


Find the values of x, y, and z.

7.



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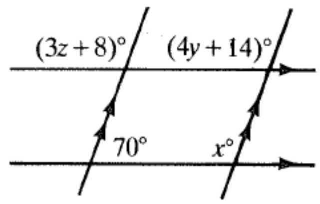

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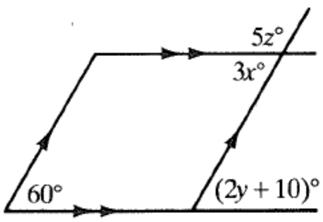
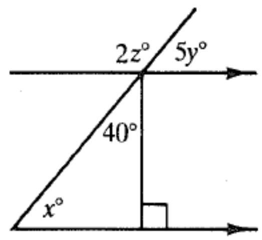


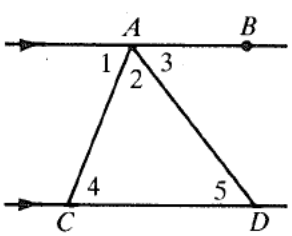
10.



11. Complete the following proof by supplying the missing statements and reasons.

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Prove: $m\angle 4 + m\angle 2 + m\angle 5 = 180$



Statements	Reasons
1. ______	1. ______
2. $m\angle 1 = m\angle$ _?_; $m\angle 3 = m\angle$ _?_	2. ______
3. $m\angle 1 + m\angle 2 + m\angle 3 =$ _?_	3. ______
4. ______	4. ______

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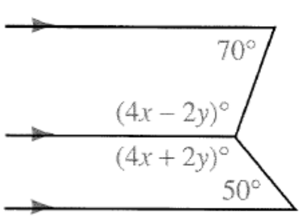
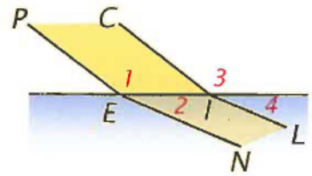
19. Find the values of x and y.



Bent Pencil. The pencil in this cup appears to be bent because light rays are bent when they go from air into water.

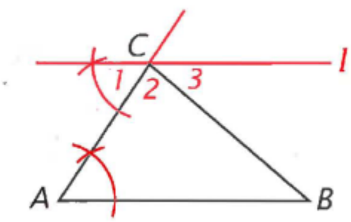
In the figure below, PE ∥ CI and ∠PEN = ∠CIL.



A note from your local physicist: this is not a light-ray diagram; it's a sketch of the pencil.

8. Why is ∠1 = ∠3?
9. Why is ∠PEN = ∠1 + ∠2 and ∠CIL = ∠3 + ∠4?
10. Why is ∠1 + ∠2 = ∠3 + ∠4?
11. Why is ∠2 = ∠4?
12. Why is EN ∥ IL?

Parallel Construction. In the figure below, line *l* has been constructed through point C by copying ∠A of △ABC as ∠1.



28. Draw a figure similar to this one and do the construction.

29. Why is *l* ∥ AB?
30. Why is line *l* the *only* line that can be constructed parallel to AB through point C?

31. Why is $\angle 3 \cong \angle B$?
32. What is $\angle 1 + \angle 2 + \angle 3$?
33. What is $\angle A + \angle 2 + \angle B$?

Paper Chain. The figure below shows a paper chain. The chain was produced by cutting out just one penguin.*

***Wild Animal Paper Chains,* by Stewart Walton and Sally Walton (Tupelo Books, 1993).

18. How many times do you think the paper was folded before it was cut?

20. How many times would a paper have to be folded to produce a chain of 16 penguins in this way?

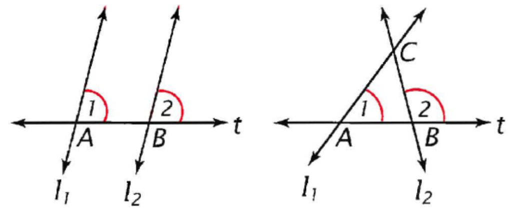
3–3 Proving Lines Parallel

Some of the following postulates and theorems are the converses of earlier ones.

The earlier ones said: If two parallel lines are cut by a transversal, then . . .

These say: If . . . , then the two lines are parallel.

Congruent corresponding angles mean that lines are parallel.



If two lines are cut by a transversal and corresponding angles are congruent, then the lines are parallel.

If two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.

If two lines are cut by a transversal and same-side interior angles are supplementary, then the lines are parallel.

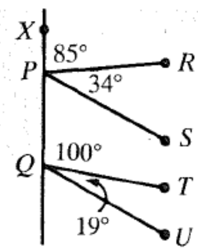
In a plane two lines perpendicular to the same line are parallel.

Two lines parallel to a third line are parallel to each other.

They are Postulate 11, and Theorems 3–5, 3–6, 3–7, and 3–10, respectively.

Example 1

State which segments (if any) are parallel.



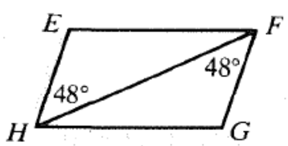
Solution

$\overline{PS} \parallel \overline{QU}$, since $\angle XPS$ and $\angle XQU$ are corresponding angles and $m\angle XPS = m\angle XQU = 119$.

$\overline{PR}$ is not parallel to $\overline{QT}$ because $m\angle XPR \neq m\angle XQT$.

State which segments (if any) must be parallel. State the postulate or theorem that justifies your answer.

1.



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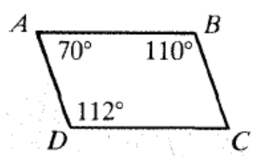

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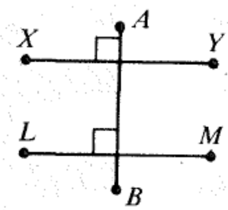

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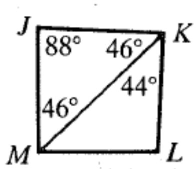


In each exercise, some information is given. Name the lines (if any) that must be parallel. If there are no such lines, write *none*.

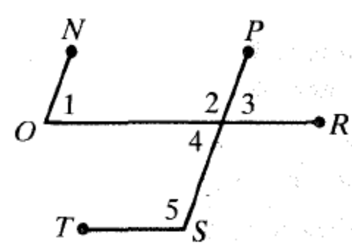
13. $\angle 1 \cong \angle 3$

14. $\angle 1 \cong \angle 4$

15. $\angle 2 \cong \angle 5$

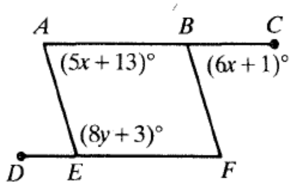
16. $\angle 3 \cong \angle 5$

17. $\angle 4$ is supplementary to $\angle 5$.



Example 2

a. Find the value of x that makes $\overline{AE} \parallel \overline{BF}$.



b. If $\overline{AE} \parallel \overline{BF}$, find the value of y that makes $\overline{AC} \parallel \overline{DF}$.

Solution

a. If $\angle CAE \cong \angle CBF$, then $\overline{AE} \parallel \overline{BF}$.

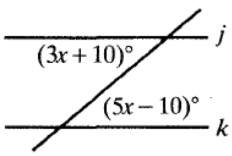
$5x + 13 = 6x + 1$
$5x + 12 = 6x$
$12 = x$

b. If $\angle CAE$ is supplementary to $\angle AEF$, then $\overline{AC} \parallel \overline{DF}$.

$(5x + 13) + (8y + 3) = 180$
$5(12) + 13 + 8y + 3 = 180$
$8y + 76 = 180$
$8y = 104$
$y = 13$

Find the value of x that makes $j \parallel k$.

18.



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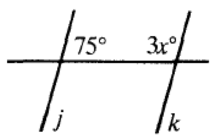

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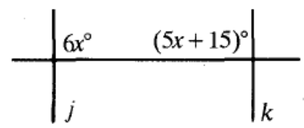
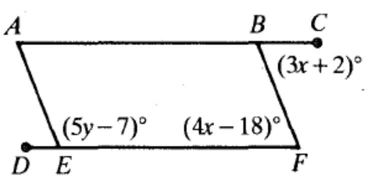


Find the values of x and y that make $\overline{AC} \parallel \overline{DF}$ and $\overline{AE} \parallel \overline{BF}$.

21.



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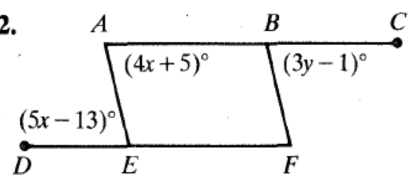
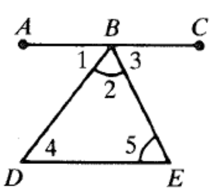


Ways to Prove Two Lines Parallel

1. Show that a pair of corresponding angles are congruent.
2. Show that a pair of alternate interior angles are congruent.
3. Show that a pair of same-side interior angles are supplementary.
4. In a plane show that both lines are perpendicular to a third line.
5. Show that both lines are parallel to a third line.

5. Number the statements and reasons for the following proof in an appropriate order. (There may be more than one correct order.)

Given: $\angle 2 \cong \angle 5$; $\overrightarrow{BE}$ bisects $\angle CBD$.
Prove: $\overline{AC} \parallel \overline{DE}$



Statements	Reasons
(__) $\angle 3 \cong \angle 5$	(__) Given
(__) $\overrightarrow{BE}$ bisects $\angle CBD$.	(__) Given
(__) $\angle 3 \cong \angle 2$	(__) Def. of angle bisector
(__) $\overline{AC} \parallel \overline{DE}$	(__) Substitution Prop.
(__) $\angle 2 \cong \angle 5$	(__) If 2 lines are cut by a trans. and alt. int. $\angle$s are $\cong$, then the lines are $\parallel$.

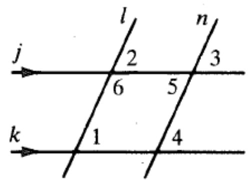
☺

19. True or false?

a. Two lines perpendicular to a third line must be parallel.
b. In a plane two lines perpendicular to a third line must be parallel.
c. In a plane two lines parallel to a third line must be parallel.
d. Any two lines parallel to a third line must be parallel.

Example 2

Given: $j \parallel k$;
$\angle 1 \cong \angle 3$
Prove: $l \parallel n$



Solution

Which pairs of angles, if shown congruent or supplementary, would give $l \parallel n$?
Four proofs are possible.
If $\angle 2 \cong \angle 3$, then $l \parallel n$.
If $\angle 2 \cong \angle 5$, then $l \parallel n$.
If $\angle 1 \cong \angle 4$, then $l \parallel n$.
If $\angle 6$ is supplementary to $\angle 5$, then $l \parallel n$.

Complete the following proofs of Example 2.

3.

Statements	Reasons
1. $j \parallel k$	1. ______
2. $\angle$___ $\cong$ $\angle$___	2. If 2 ∥ lines are cut by a transversal, then corr. ⦞ are ≅.
3. $\angle 1 \cong \angle 3$	3. ______
4. ______	4. Transitive Prop.
5. ______	5. ______

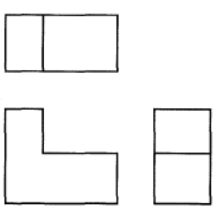
4.

Statements	Reasons
1. ______	1. ______
2. $\angle 3 \cong \angle 5$	2. ______
3. $\angle 1 \cong \angle 5$, or $m\angle 1 = m\angle 5$	3. ______
4. $\angle 1$ is supplementary to $\angle 6$.	4. ______
5. ______	5. Def. of supp. ⦞
6. $m\angle 5 + m\angle 6 = 180$	6. ______
7. $\angle 5$ is supplementary to $\angle 6$.	7. ______
8. ______	8. ______

☺

Match the orthographic projections with their isometric drawings. If there is no isometric drawing, then make one.

1.



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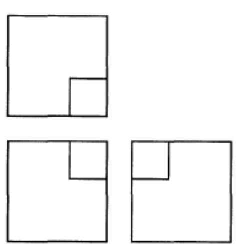

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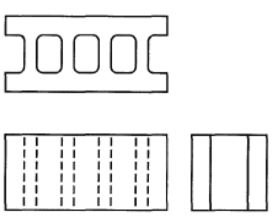

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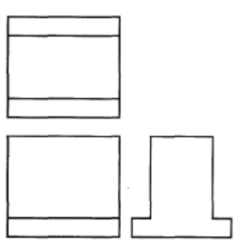

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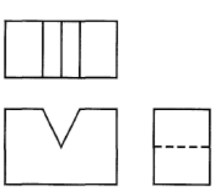

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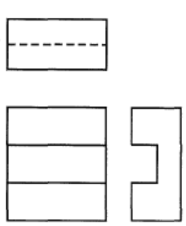
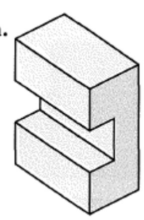
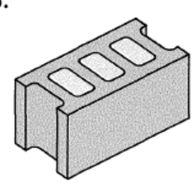


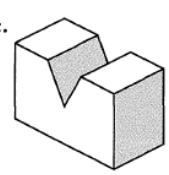
a.



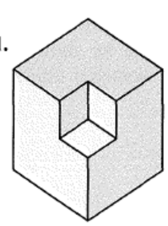
b.



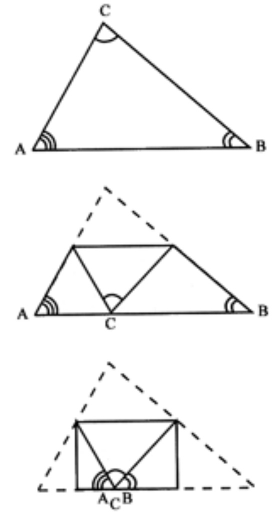
c.



d.



3–4 Angles of a Triangle



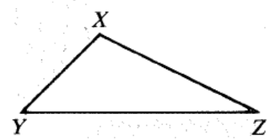
Exercise: Folding the triangle angle sum.

- Cut a triangle from a piece of paper and mark the three vertices as shown in the figure at the right.
- Fold the top vertex of the triangle, in this case vertex *C*, so that it touches the opposite side of the triangle, side *AB*, so that the ***crease in the paper is parallel*** to side *AB*.
- Fold the other two vertices, *A* and *B*, so that they fall on the same point where vertex *C* touches side *AB*.

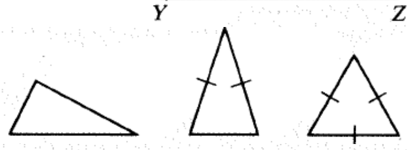
What is the sum of the measures of angles *A*, *B*, and *C*?

---

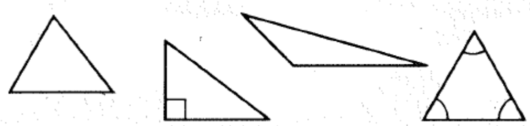
triangle A triangle is the figure formed by three segments joining three noncollinear points.
$\triangle XYZ$ has three **vertices:** X, Y, Z
three **sides:** $\overline{XY}, \overline{XZ}, \overline{YZ}$
three **angles:** $\angle X, \angle Y, \angle Z$



scalene triangle no sides congruent
isosceles triangle at least two sides congruent
equilateral triangle all sides congruent



acute triangle three acute angles
right triangle one right (90°) angle
obtuse triangle one obtuse angle
equiangular triangle all angles congruent



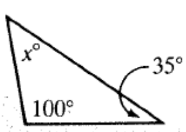
Draw a triangle that satisfies the conditions stated. If no triangle can satisfy the conditions, write *not possible.*

1. a right isosceles triangle **2.** an acute scalene triangle **3.** an equilateral equiangular triangle

Theorem 3–11. The sum of the measures of the angles of a triangle is 180°.

Find the value of *x*.

4.



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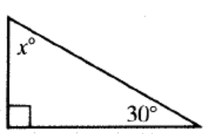

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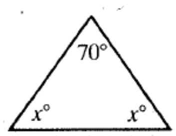
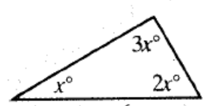
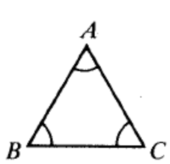


7.

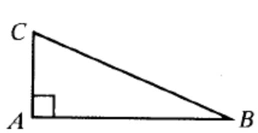


Complete.

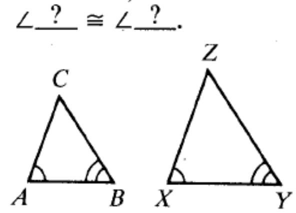
8. If $\angle A \cong \angle B \cong \angle C$, then $m\angle A = m\angle B = m\angle C = \underline{\ ?\ }$.



9. If $m\angle A = 90$, then $m\angle B + m\angle C = \underline{\ ?\ }$.

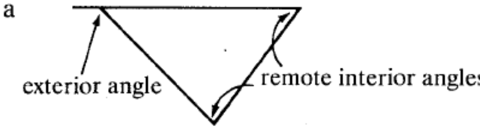


10. If $\angle A \cong \angle X$ and $\angle B \cong \angle Y$, then $\angle \underline{\ ?\ } \cong \angle \underline{\ ?\ }$.



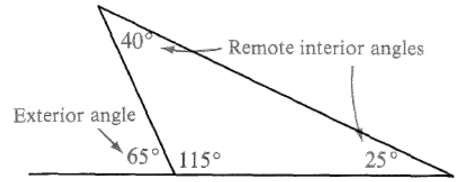
An **exterior angle** is the angle formed when one side of a triangle is extended.

The **remote interior angles** are the two angles of the triangle not adjacent to the exterior angle.



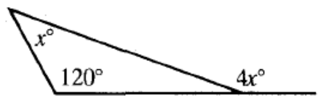
This theorem can serve as a shortcut to finding the missing angles in a triangle.

Theorem 3–12. The measure of an exterior angle of a triangle equals the sum of the measures of the two remote interior angles.



$65 = 40 + 25$

Example 2
Find the value of x.

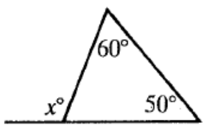


Solution

$4x = x + 120$
$3x = 120$
$x = 40$

Find the value of *x*.

11.



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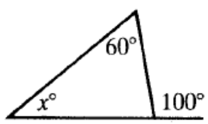

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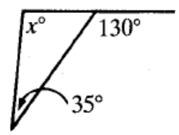

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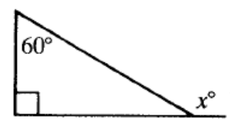
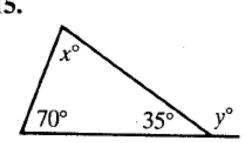


Find the values of *x* and *y*.

15.



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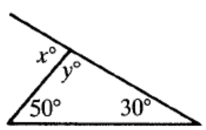

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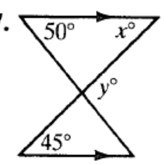

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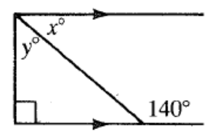

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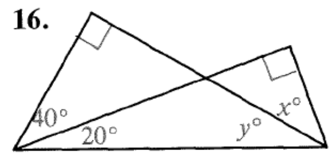
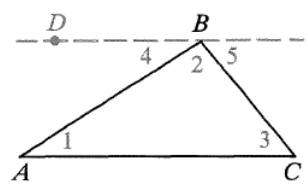
16.



Finish the proof. What proof is it?

Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$



Statements	Reasons
1. Through B draw $\overleftrightarrow{BD}$ parallel to $\overleftrightarrow{AC}$.	1. Through a point outside a line, there is exactly one line ∥ to the given line.

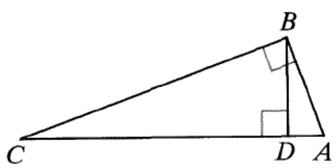
Complete each statement with the word *always*, *sometimes*, or *never*.

1. If a triangle is isosceles, then it is _?_ equilateral.
2. If a triangle is equilateral, then it is _?_ isosceles.
3. If a triangle is scalene, then it is _?_ isosceles.
4. If a triangle is obtuse, then it is _?_ isosceles.

18. The lengths of the sides of a triangle are $3t$, $5t - 12$, and $t + 20$.
a. Find the value(s) of t that make the triangle isosceles.
b. Does any value of t make the triangle equilateral? Explain.

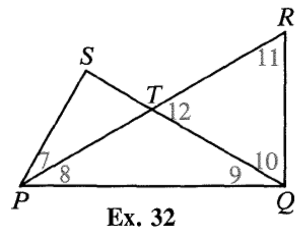
19. The largest two angles of a triangle are two and three times as large as the smallest angle. Find all three measures.

23. Given: $\overline{AB} \perp \overline{BC}$; $\overline{BD} \perp \overline{AC}$
a. If $m\angle C = 22$, find $m\angle ABD$.
b. If $m\angle C = 23$, find $m\angle ABD$.
c. Explain why $m\angle ABD$ always equals $m\angle C$.



32. Given: $\overrightarrow{PR}$ bisects $\angle SPQ$;
$\overline{PS} \perp \overline{SQ}$; $\overline{RQ} \perp \overline{PQ}$

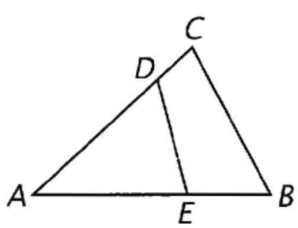
Which numbered angles must be congruent?



Ex. 32

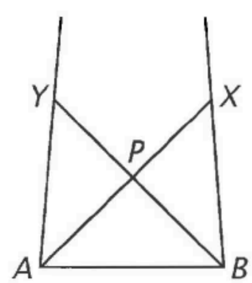
Write the proof.

45.



Given: In △ABC and △ADE,
∠ADE = ∠B.
Prove: ∠AED = ∠C.

Catch Question. This problem appeared in a collection of "catch" questions.*



Suppose lines AX and BY bisect two of the angles of a triangle whose third vertex is out of view. If AX ⊥ BY and the base AB of the triangle is 10 inches long, how tall is the triangle?

47. Mark the given information on the figure.
48. Reasoning from the figure, explain why this is a "catch" question.

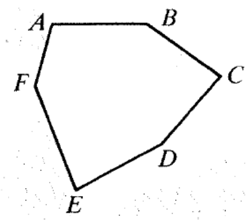
3–5 Angles of a Polygon

Polygons have vertices, sides, angles, and exterior angles.

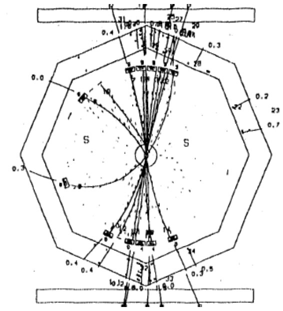
Polygons are named by listing consecutive vertices in order. *ABCDEF* is one name for this polygon.

A segment connecting two nonconsecutive vertices of a polygon is called a **diagonal.** $\overline{AD}$ and $\overline{FB}$ are diagonals.

Convex polygons are polygons such that no part of a diagonal is exterior to the polygon.



regular polygon A polygon that is both equiangular (all angles congruent) and equilateral (all sides congruent) is regular.

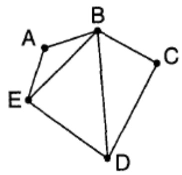


The Mark II detector at Stanford shows the decay of a *Z* particle into bottom quarks.
—Fred Kral's thesis

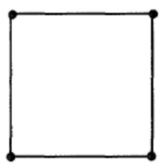
Divide and Count

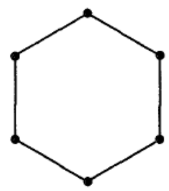
Draw diagonals to divide each polygon into triangles. Calculate the angle sum of each polygon.

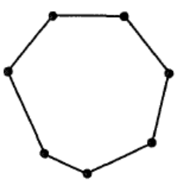
Example:

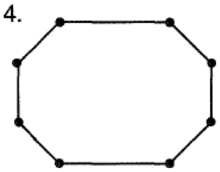


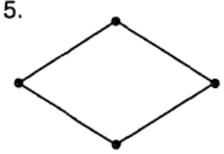
3 triangles x 180°/triangle = 540°

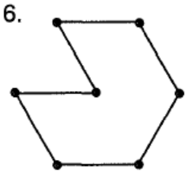
1.  ________

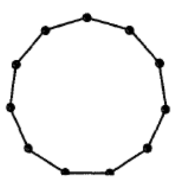
2.  ________

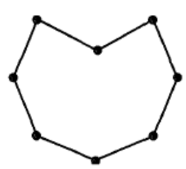
3.  ________

4.  ________

5.  ________

6.  ________

7.  ________

8.  ________

9. Write a math statement showing the relationship between the number of sides (N) and the number of triangles (T): ______________________________

10. Write a formula for the angle sum (S) in terms of N: ____________________

Names of polygons

3. Triangle

4.

5.

6.

7.

8.

9.

10.

11.

12.

20.

100.

n.

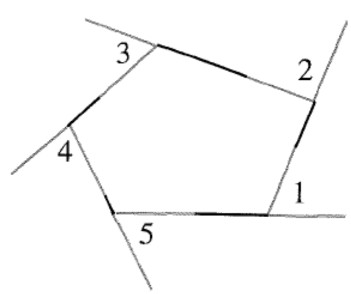
∞.

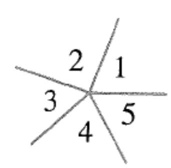
This formula is **Theorem 3–13**. The sum of the measures of the angles of a convex polygon with n sides is ______________________.

☺

7. Draw a pentagon with one exterior angle at each vertex. Cut out the exterior angles and arrange them so that they all have a common vertex, as shown at the far right. What is the sum of the measures of the exterior angles? ____________

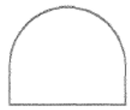
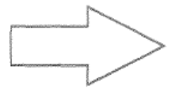
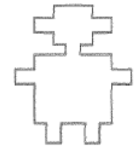
Repeat the experiment with a hexagon.





The result is **Theorem 3–14**. The sum of the measures of the exterior angles of ______ convex polygon, one angle at each vertex, is _______.

Is the figure a convex polygon, a nonconvex polygon, or neither?

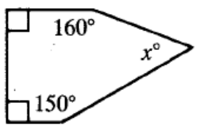
1. 
2. 
3. 
4. 
5. 
6. 

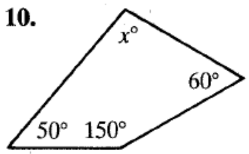
7. Imagine stretching a rubber band around each of the figures in Exercises 1–6. What is the relationship between the rubber band and the figure when the figure is a convex polygon?

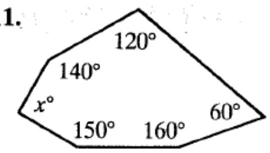
For each polygon, find (a) the interior angle sum and (b) the exterior angle sum.

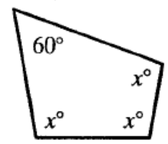
1. Quadrilateral **2.** Pentagon **3.** Hexagon
4. Octagon **5.** Decagon **6.** n-gon

Find the value of x.

9. 

10. 

11. 

12. 

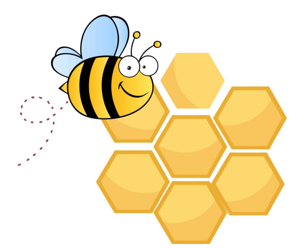
Two key facts for regular polygons: The sum of the measures of the exterior angles is 360°, so for *each exterior angle*, $m\angle X = 360° / (\text{number of sides})$. *Each interior angle is supplementary*, so $180° - m\angle X$. No complicated formulas to know!

8. Complete the table for regular polygons.

Number of sides	9	15	30	?	?	?	?
Measure of each ext. ∠	?	?	?	6	8	?	?
Measure of each int. ∠	?	?	?	?	?	165	178

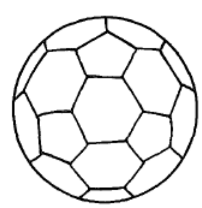
10. Four of the angles of a pentagon have measures 40, 80, 115, and 165. Find the measure of the fifth angle.

11. The face of a honeycomb consists of interlocking regular hexagons. What is the measure of each angle of these hexagons?



16. The sum of the measures of the interior angles of a polygon is five times the sum of the measures of its exterior angles, one angle at each vertex. How many sides does the polygon have?

20. Why is it not possible to tile a floor like a soccer ball?


Possible


Impossible

25. The sum of the measures of the interior angles of a polygon is known to be between 2100 and 2200. How many sides does the polygon have?

Self Test on Triangles and Polygons

Complete.

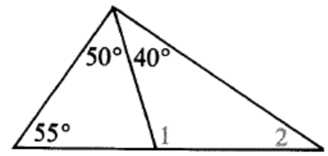
1. If the measure of each angle of a triangle is less than 90, the triangle is called __?__.

2. If a triangle has no congruent sides, it is called __?__.

3. Each angle of an equiangular triangle has measure __?__.

4. In the diagram, m∠1 = __?__ and m∠2 = __?__.

5. If the measures of the acute angles of a right triangle are $2x + 4$ and $3x - 9$, then x = __?__.



Ex. 4

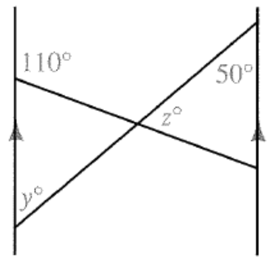
6. Find the values of y and z.

7. The lengths of the sides of a triangle are $2x + 5$, $3x + 10$, and $x + 12$. Find all values of x that make the triangle isosceles.

8. An octagon has __?__ sides.

9. A regular polygon is both __?__ and __?__.

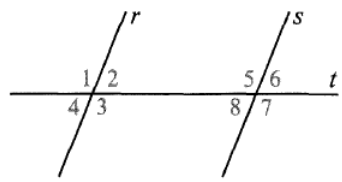
10. In a regular decagon, the sum of the measures of the exterior angles is __?__ and the measure of each interior angle is __?__.



Ex. 6

11. If the measure of each angle of a polygon is 174, then the measure of each exterior angle is __?__ and the polygon has __?__ sides.

Chapter Review

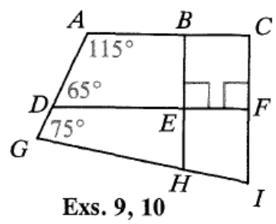


Exs. 1–7

1. $\angle 5$ and $\angle$ _?_ are same-side interior angles. 3–1
2. $\angle 5$ and $\angle 1$ are _?_ angles.
3. $\angle 5$ and $\angle 3$ are _?_ angles.
4. Line j, not shown, does not intersect line r. Must lines r and j be parallel?

In the diagram above, $r \parallel s$.

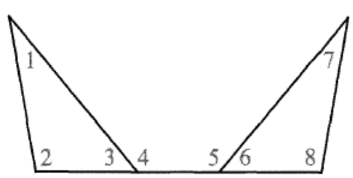
5. If $m\angle 1 = 105$, then $m\angle 5 =$ _?_ and $m\angle 7 =$ _?_. 3–2
6. Solve for x: $m\angle 2 = 70$ and $m\angle 8 = 6x - 2$
7. Solve for y: $m\angle 3 = 8y - 40$ and $m\angle 8 = 2y + 20$
8. Lines a, b, and c are coplanar, $a \parallel b$, and $a \perp c$. What can you conclude? Explain.



Exs. 9, 10

9. Which line is parallel to $\overleftrightarrow{AB}$? Why? 3–3
10. Name a pair of parallel lines other than the pair in Exercise 9. Why must they be parallel?
11. Name five ways to prove two lines parallel.

12. If x and $2x - 15$ represent the measures of the acute angles of a right triangle, find the value of x. 3–4



Exs. 13–15

13. $m\angle 6 + m\angle 7 + m\angle 8 =$ _?_
14. If $m\angle 1 = 30$ and $m\angle 4 = 130$, then $m\angle 2 =$ _?_.
15. If $\angle 4 \cong \angle 5$ and $\angle 1 \cong \angle 7$, name two other pairs of congruent angles and give a reason for each answer.

16. **a.** Sketch a hexagon that is equiangular but not equilateral. 3–5
    b. What is its interior angle sum?
    c. What is its exterior angle sum?
17. A regular polygon has 18 sides. Find the measure of each interior angle.
18. A regular polygon has 24 sides. Find the measure of each exterior angle.
19. Each interior angle of a regular polygon has measure 150. How many sides does the polygon have?