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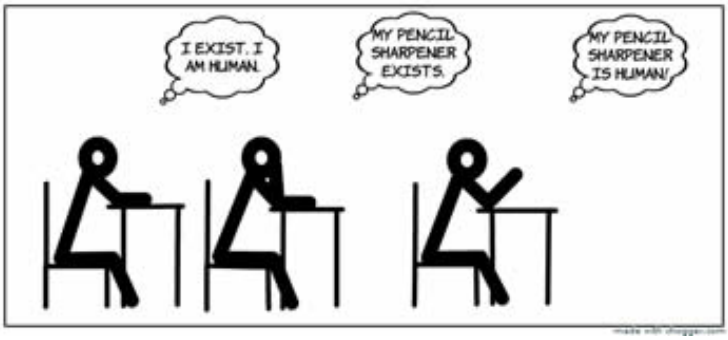
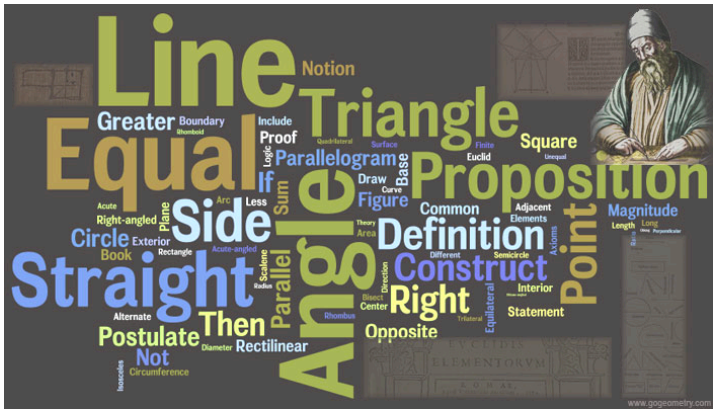
LogicPacket

Chapter 2
Logic and Deductive Reasoning



Euclid of Alexandria

Euclid was active during the reign of Ptolemy I (323–283 BCE). In Greek Εὐκλείδης means renowned.



The Premises for Logical Arguments

I) Given information.
II) Definitions and undefined terms.
III) Postulates. Properties of arithmetic, algebra, equality, and congruence.
IV) Theorems that have already been proved.

Euclid's Postulates

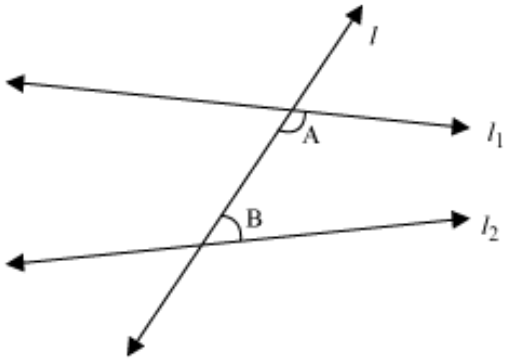
1. A straight line segment can be drawn joining any two points.

2. Any straight line segment can be extended indefinitely in a straight line.

3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.

4. All right angles are congruent.

5. If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.



*Postulate 5 is the parallel postulate. Non-Euclidean geometries can be created in which the parallel postulate does not hold.

Sheet#0251: Beginning Postulates, Properties, and Theorems

	Segment Addition Postulate (#2)	$AB + BC = AC$
	Angle Addition Postulate (#4)	$m\angle AOB + m\angle BOC = m\angle AOC$
1	Addition Property of Equality	If $a = b$ and $c = d$, then $a + c = b + d$
2	Subtraction Property of =	If $a = b$ and $c = d$, then $a - c = b - d$
3	Multiplication Property of =	If $a = b$ then $ca = cb$
4	Division Property of =	If $a = b$ then $a/c = b/c$ for $c \neq 0$
5	Substitution Property of =	If $a = b$ then a or b may be substituted for the other in any equation.
6	Reflexive Property of Equality	$a = a$
7	Symmetry Property of =	If $a = b$, then $b = a$
8	Transitive Property of =	If $a = b$ and $b = c$ then $a = c$
9	Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$ (and likewise for angles)
10	Symmetric Property of $\cong$	If $\overline{AB} \cong \overline{CD}$ then $\overline{CD} \cong \overline{AB}$
11	Transitive Property of $\cong$	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$ then $\overline{AB} \cong \overline{EF}$
	Midpoint Theorem (2–1)	If M is the midpoint of $\overline{AB}$ then $AM = \frac{1}{2} AB$ and $MB = \frac{1}{2} AB$
	Angle Bisector Theorem (2–2)	If $\overrightarrow{BX}$ is the bisector of $\angle ABC$ then $m\angle ABX = \frac{1}{2} m\angle ABC$ and $m\angle XBC = \frac{1}{2} m\angle ABC$
	Vertical Angle Theorem (2–3)	Vertical angles are congruent.
	Theorems on Perpendicular Lines (2–4 & 2–5, and 2–6)	• Two lines are perpendicular *if and only if* they form congruent adjacent angles. • If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.

1–5 Postulates and Theorems Relating Points, Lines, and Planes

Objective: Use postulates and theorems relating points, lines, and planes.

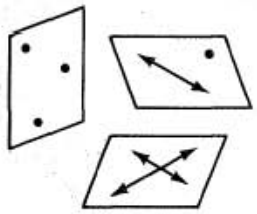
postulate	a basic assumption accepted without proof
theorem	a statement that can be proved using postulates, definitions, and previously proved theorems
exists	there is at least one
unique	there is no more than one
one and only one	exactly one
determine	to define or specify

Relationships between points

Two points must be collinear. (Postulate 6)
Three points may be collinear or noncollinear.
Three points must be coplanar. (Postulate 7)
Three noncollinear points determine a plane. (Postulate 7)
Four points may be coplanar or noncoplanar.
Four noncoplanar points determine space. (Postulate 5)
Space contains at least four noncoplanar points.

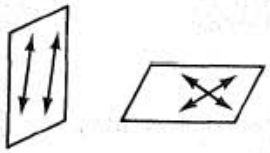
Three ways to determine a plane

Three noncollinear points determine a plane. (Postulate 7)
A line and a point not on the line determine a plane. (Theorem 1–2)
Two intersecting lines determine a plane. (Theorem 1–3)



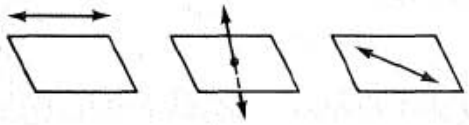
Relationships between two lines in the same plane

Either two lines are parallel, or they intersect in exactly one point.



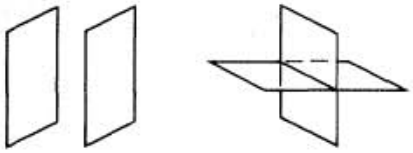
Relationships between a line and a plane

Either a line and a plane are parallel, or they intersect in exactly one point, or the plane contains the line.



Relationships between two planes

Either two planes are parallel, or they intersect in a line.



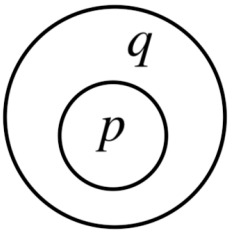
Classify each statement as true or false.

16. The intersection of a line and a plane may be the line itself.
17. Three noncollinear points determine exactly one line.
18. Two points can determine two lines.
19. Two lines can intersect in exactly one point.
20. Postulates are statements to be proved.
21. Two points determine a plane.
22. A line and a point not on it determine one plane.
23. A plane contains at least three noncollinear points.
24. Line l always has at least two points on it.
25. Theorems are statements to be proved.
26. Any three points are always coplanar.
27. It is possible that points P and Q are in plane M but $\overleftrightarrow{PQ}$ is not.
28. Two intersecting lines determine a plane.
29. Two planes can intersect in two lines.
30. If points A, B, C, and D are noncoplanar, then no one plane contains all four of them.
31. Two planes can intersect in exactly one point.
32. Three planes can intersect in exactly one point.
33. A line and a plane can intersect in exactly one point.

2–1 If-Then Statements; Converses

A **conditional statement**, or **conditional**, is often written in if-then form. We often use p for the **hypothesis** and q for the **conclusion**.

Conditional statement	If p, then q. ↑ ↑ hypothesis conclusion	If it is snowing, then it is cold.
Other forms	p implies q. p only if q. q if p.	It is snowing implies it is cold. It is snowing only if it is cold. It is cold if it is snowing.



Example 1 In each of the following conditionals, underline the hypothesis once and the conclusion twice.

a. If it rains, then the game will be canceled. **b.** $\angle A$ is acute if $m\angle A = 60$.

Notice that *if*, *then*, *implies*, and *only if* are not part of the hypothesis or the conclusion.

In each of the following conditionals, underline the hypothesis once and the conclusion twice.

1. $AB = BC$ if B is the midpoint of $\overline{AC}$.

2. We will go only if it is sunny.

3. $\angle AOC$ is a right angle implies $m\angle AOC = 90$.

4. If $x = -2$, then $x^2 = 4$.

The **converse** of a conditional statement is formed by interchanging the hypothesis and the conclusion.

Conditional Statement	If p, then q.	If today is Tuesday, then tomorrow is Wednesday.
Converse	If q, then p.	If tomorrow is Wednesday, then today is Tuesday.

Example 2 Give the converse of each of the following conditionals.

a. If points are coplanar, then they lie in the same plane. (True)

b. If $m\angle X = 110$, then $\angle X$ is obtuse. (True)

Solution

a. If points lie in the same plane, then they are coplanar. (True)

b. If $\angle X$ is obtuse, then $m\angle X = 110$. (False)

Notice that you cannot assume a converse is true just because the original statement is true.

One way of proving a statement false is to give a **counterexample.**

Example 3 Tell whether the statement is true or false. Then write the converse and tell whether it is true or false. If the statement or the converse is false, give a counterexample.

Two angles are adjacent if they have a common vertex.

Solution

Statement is false. (In if-then form, the statement would be: If two angles have a common vertex, then they are adjacent angles.)
Counterexample: The angles may not have a common side.
Converse: If two angles are adjacent angles, then they have a common vertex. True

Tell whether each statement is true or false. Then write the converse and tell whether it is true or false. If the statement or the converse is false, give a counterexample.

5. If two angles are right angles, then they are congruent.

6. $x > 7$ implies $x > 2$.

7. If a number is divisible by 2, then it is divisible by 4.

8. An animal is a penguin only if it is a bird.

21. Write the conditional statement represented by the Euler diagram below in the form "if a, then b."

A statement combining a conditional and its converse is called a **biconditional.** As is shown in Example 4, definitions are biconditionals.

Conditional statement	If p, then q.	If today is Tuesday, then tomorrow is Wednesday.
Converse	If q, then p.	If tomorrow is Wednesday, then today is Tuesday.
Biconditional	p if and only if q.	Today is Tuesday if and only if tomorrow is Wednesday.

Example 4 Write the biconditional as two conditionals that are converses.
Line k is a bisector of $\overline{XY}$ if and only if k intersects $\overline{XY}$ at its midpoint.

Solution
If line k is a bisector of $\overline{XY}$, then k intersects $\overline{XY}$ at its midpoint.
If line k intersects $\overline{XY}$ at its midpoint, then k is a bisector of $\overline{XY}$.

Write each biconditional as two conditionals that are converses of each other.

9. An angle is a right angle if and only if its measure is 90.

10. $DE = FG$ if and only if $\overline{DE} \cong \overline{FG}$.

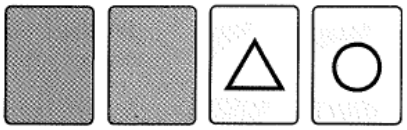
11. B is on $\overline{AC}$ if and only if B is on $\overrightarrow{AC}$ and $\overrightarrow{CA}$.

Rewrite each pair of conditionals as a biconditional.

7. If B is between A and C, then $AB + BC = AC$.
If $AB + BC = AC$, then B is between A and C.

8. If $m\angle AOC = 180$, then $\angle AOC$ is a straight angle.
If $\angle AOC$ is a straight angle, then $m\angle AOC = 180$.

Here is a puzzle in logical reasoning called the Wason test, named after the British psychologist Peter Wason who created it.†



Four cards lie on a table; each card is either red or blue on one side and has a triangle or circle on the other side.

The following conditional statement supposedly describes the way that the cards are made:

> If a card is blue on one side, it has a circle on the other side.

The puzzle is to figure out which cards have to be turned over to find out if this statement is true. It turns out that the "obvious" answer, chosen by most people, is wrong.

Which cards would you choose? Explain your reasoning.

2–2 Properties from Algebra

Properties of Equality: Addition Property of Equality, Subtraction Property of Equality, Multiplication Property of Equality, Division Property of Algebra, and Substitution Property of Equality.

Substitution Property	If $x = 3$ and $x + y = 7$, then $3 + y = 7$.	If $m\angle A = m\angle 1$ and $m\angle 1 + m\angle B = 90$, then $m\angle A + m\angle B = 90$.

Justify each statement with a property from algebra, or a definition or postulate from geometry.

1. If $AB = CD$ and $BC = BC$, then $AB + BC = CD + BC$.
2. If $2m\angle 1 = 72$, then $m\angle 1 = 36$.
3. If $m\angle A = \frac{1}{2}m\angle X$ and $\frac{1}{2}m\angle X = m\angle B$, then $m\angle A = m\angle B$.
4. If point B is in the interior of $\angle XOY$, then $m\angle XOB + m\angle BOY = m\angle XOY$.
5. If $2 + YZ = 8$, then $YZ = 6$.

Notice that the properties of equality listed above are used only on numbers and variables. Some properties used in geometry apply to both equality and congruence.

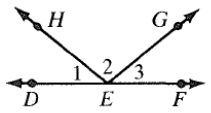
Properties of Equality and Congruence

	Equality (numbers, variables, lengths, angle measures)	**Congruence** (segments, angles, polygons)
Reflexive Property	$DE = DE$ $\quad m\angle 1 = m\angle 1$	$\overline{DE} \cong \overline{DE}$ $\quad \angle 1 \cong \angle 1$
Symmetric Property	If $DE = AB$, then $AB = DE$. If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.	If $\overline{DE} \cong \overline{AB}$, then $\overline{AB} \cong \overline{DE}$. If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.
Transitive Property	If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Example Complete the proof by supplying the missing statements and reasons.

Given: $m\angle 1 = m\angle 3$

Prove: $m\angle DEG = m\angle HEF$

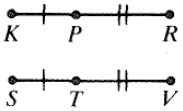


Statements	Reasons
1. $m\angle 1 = m\angle 3$	1. Given
2. $m\angle 2 = m\angle 2$	2. Reflexive Prop.
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3. Addition Prop. of =
4. $m\angle DEG = m\angle 1 + m\angle 2$; $m\angle HEF = m\angle 3 + m\angle 2$	4. Angle Addition Post.
5. $m\angle DEG = m\angle HEF$	5. Substitution Prop.

Complete the following proof by supplying the missing statements and reasons.

6. Given: $KP = ST$; $PR = TV$

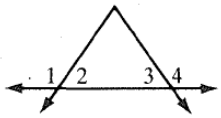
Prove: $KR = SV$



Statements	Reasons
1. _____	1. Given
2. $KP + PR = ST + TV$	2. _____
3. $KP + PR = KR$; $ST + TV = SV$	3. _____
4. _____	4. Substitution Prop.

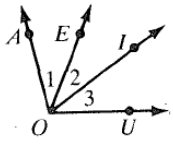
Complete the following proofs by supplying the missing statements and reasons.

7. Given: $m\angle 1 = m\angle 4$
Prove: $m\angle 2 = m\angle 3$



Statements	Reasons
1. $m\angle 1 + m\angle 2 = 180$; $m\angle 3 + m\angle 4 = 180$	1. Angle Addition Post.
2. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	2. _____
3. _____	3. Given
4. $m\angle 2 = m\angle 3$	4. _____

8. Given: $m\angle AOI = m\angle EOU$
Prove: $m\angle 1 = m\angle 3$



Statements	Reasons
1. _____	1. Given
2. $m\angle 2 = m\angle 2$	2. _____
3. $m\angle 1 + m\angle 2 = m\angle AOI$; $m\angle 2 + m\angle 3 = m\angle EOU$	3. _____
4. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	4. _____
5. _____	5. _____

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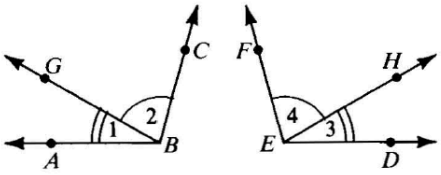
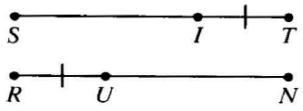
11. Given: $m\angle 1 = m\angle 3$; $m\angle 2 = m\angle 4$
Prove: $m\angle ABC = m\angle DEF$



Proof:

Statements	Reasons
1. $m\angle 1 = m\angle 3$; $m\angle 2 = m\angle 4$	1. ?
2. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	2. ?
3. $m\angle 1 + m\angle 2 = m\angle ABC$; $m\angle 3 + m\angle 4 = m\angle DEF$	3. ?
4. $m\angle ABC = m\angle DEF$	4. ?

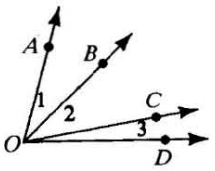
12. Given: $ST = RN$; $IT = RU$
Prove: $SI = UN$



Proof:

Statements	Reasons
1. $ST = RN$	1. ?
2. ? $= SI + IT$; ? $= RU + UN$	2. ?
3. $SI + IT = RU + UN$	3. ?
4. $IT = RU$	4. ?
5. ?	5. ?

7. Given: $\angle AOD$ as shown
Prove: $m\angle AOD = m\angle 1 + m\angle 2 + m\angle 3$



Proof:

Statements	Reasons
1. $m\angle AOD = m\angle AOC + m\angle 3$	1. ?
2. $m\angle AOC = m\angle 1 + m\angle 2$	2. ?
3. ?	3. ?

8. Given: $FL = AT$
Prove: $FA = LT$

Proof:

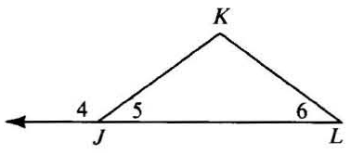
Statements	Reasons
1. ?	1. Given
2. $LA = LA$	2. ?
3. $FL + LA = AT + LA$	3. ?
4. $FL + LA = FA$; $LA + AT = LT$	4. ?
5. ?	5. Substitution Prop.

9. Given: $DW = ON$
Prove: $DO = WN$

Proof:

Statements	Reasons
1. $DW = ON$	1. ?
2. $DW = DO + OW$; $ON =$? + ?	2. ?
3. ?	3. Substitution Prop.
4. $OW = OW$	4. ?
5. ?	5. ?

10. Given: $m\angle 4 + m\angle 6 = 180$
Prove: $m\angle 5 = m\angle 6$

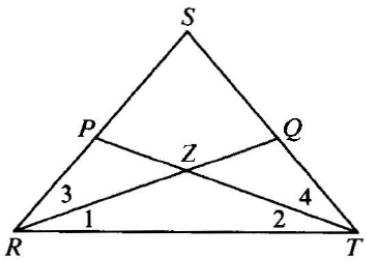


Proof:

Statements	Reasons
1. $m\angle 4 + m\angle 6 = 180$	1. ?
2. $m\angle 4 + m\angle 5 = 180$	2. ?
3. $m\angle 4 + m\angle 5 = m\angle 4 + m\angle 6$	3. ?
4. $m\angle 4 = m\angle 4$	4. ?
5. ?	5. ?

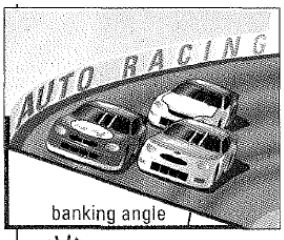
Write two-column proofs.

11. Given: $m\angle 1 = m\angle 2$;
$m\angle 3 = m\angle 4$
Prove: $m\angle SRT = m\angle STR$



Exs. 11–14

14. Given: $m\angle SRT = m\angle STR$;
$m\angle 3 = m\angle 4$
Prove: $m\angle 1 = m\angle 2$



REAL LIFE **AUTO RACING** Banked turns help the cars travel around the track at high speeds. The angles provide an inward force that helps keep the cars from flying off the track.

28. Some facts about the maximum banking angles of Daytona International Speedway at comers 1, 2, 3, and 4 are at the right. Find $m\angle 3$. Explain your steps.

$m\angle 1 + m\angle 3 + m\angle 4 = 93°$

$m\angle 2 + m\angle 4 = 62°$

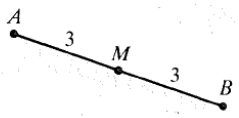
$m\angle 2 = m\angle 3$

$m\angle 1 = m\angle 2$

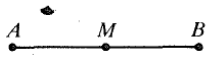
2–3 Proving Theorems

Midpoint Theorem and Angle Bisector Theorem.

If M is the midpoint of $\overline{AB}$, you may conclude that $AM = MB$ by the definition of midpoint. If you are also told that $AB = 6$, you may realize that $AM = MB = \frac{1}{2}(6) = 3$. This idea is generalized in the **Midpoint Theorem.**

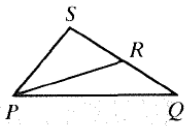


Midpoint Theorem **If M is the midpoint of $\overline{AB}$, then $AM = \frac{1}{2}AB$ and $MB = \frac{1}{2}AB$.**



Example 1

Given: R is the midpoint of $\overline{SQ}$. Name the definition, postulate, or theorem that justifies each statement about the diagram.



a. $\overline{SR} \cong \overline{RQ}$

b. $SR = \frac{1}{2}SQ$

c. $SR + RQ = SQ$

d. $\overline{PR}$ bisects $\overline{SQ}$.

Solution

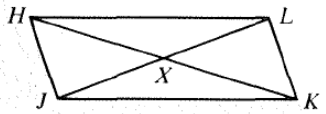
a. Def. of midpoint

b. Midpoint Thm.

c. Segment Addition Post.

d. Def. of segment bisector

Given: $\overline{HK}$ bisects $\overline{JL}$; X is the midpoint of $\overline{HK}$. State the definition, postulate, or theorem that justifies each statement about the diagram.



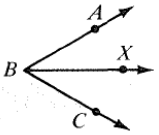
1. $HX = XK$

2. $HX = \frac{1}{2}HK$

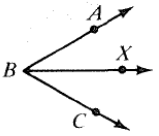
3. X is the midpoint of $\overline{JL}$.

4. $JL = JX + XL$

If $\overrightarrow{BX}$ bisects $\angle ABC$, you may conclude that $m\angle ABX = m\angle XBC$ by the definition of an angle bisector. If you are also told that $m\angle ABC = 60$, you may realize that $m\angle ABX = m\angle XBC = \frac{1}{2}(60) = 30$. This idea is generalized in the **Angle Bisector Theorem.**

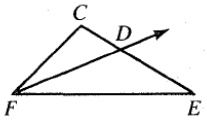


Angle Bisector Theorem **If $\overrightarrow{BX}$ is the bisector of $\angle ABC$, then $m\angle ABX = \frac{1}{2}m\angle ABC$ and $m\angle XBC = \frac{1}{2}m\angle ABC$.**



Example 2

Given: $\overrightarrow{FD}$ bisects $\angle CFE$. Name the definition, postulate, or theorem that justifies each statement.



a. $m\angle CFD = \frac{1}{2}m\angle CFE$

b. $m\angle CFD = m\angle DFE$

c. $CD + DE = CE$

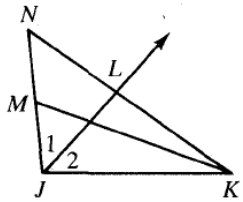
Solution

a. Angle Bisector Thm.

b. Def. of angle bisector

c. Segment Addition Post.

Given: $\overrightarrow{JL}$ bisects $\angle NJK$; M is the midpoint of $\overline{NJ}$. State the definition, postulate, or theorem that justifies each statement.



5. $m\angle 1 = \frac{1}{2}m\angle NJK$

6. $MJ = \frac{1}{2}NJ$

7. $m\angle 1 = m\angle 2$

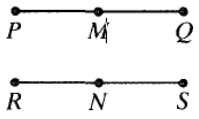
8. $m\angle NJK = m\angle 1 + m\angle 2$

9. $\overline{NM} \cong \overline{MJ}$

10. $NL + LK = NK$

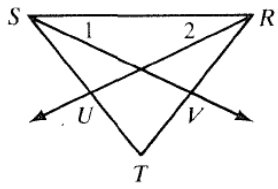
Complete the following proofs by supplying the missing statements and reasons.

11. Given:
M is the midpoint of $\overline{PQ}$;
N is the midpoint of $\overline{RS}$;
$PQ = RS$
Prove: $PM = RN$



Statements	Reasons
1. M is the midpt. of $\overline{PQ}$; N is the midpt. of $\overline{RS}$.	1. _____
2. $PM = \frac{1}{2}PQ$; $RN = \frac{1}{2}RS$	2. _____
3. _____	3. Given
4. $\frac{1}{2}PQ = \frac{1}{2}RS$	4. _____
5. _____	5. Substitution Prop. (Steps _____ and _____)

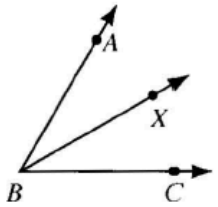
12. Given:
$\overrightarrow{SV}$ bisects $\angle RST$;
$\overrightarrow{RU}$ bisects $\angle SRT$;
$m\angle RST = m\angle SRT$
Prove: $m\angle 1 = m\angle 2$



Statements	Reasons
1. $\overrightarrow{SV}$ bisects $\angle RST$; $\overrightarrow{RU}$ bisects $\angle SRT$.	1. _____
2. $m\angle 1 = \frac{1}{2}m\angle RST$; $m\angle 2 = \frac{1}{2}m\angle SRT$	2. _____
3. _____	3. Given
4. $\frac{1}{2}m\angle RST = \frac{1}{2}m\angle SRT$	4. _____
5. _____	5. _____

☺

10. Complete the proof of Theorem 2-2.
Given: $\overrightarrow{BX}$ is the bisector of $\angle ABC$.
Prove: $m\angle ABX = \frac{1}{2}m\angle ABC$; $m\angle XBC = \frac{1}{2}m\angle ABC$



Proof:

Statements	Reasons
1. $\overrightarrow{BX}$ is the bisector of $\angle ABC$.	1. ?
2. $\angle ABX \cong$?, or $m\angle ABX =$?	2. ?
3. $m\angle ABX + m\angle XBC = m\angle ABC$	3. ?
4. $m\angle ABX + m\angle ABX = m\angle ABC$, or $2m\angle ABX = m\angle ABC$	4. ?
5. $m\angle ABX = \frac{1}{2}m\angle ABC$	5. ?
6. $m\angle XBC = \frac{1}{2}m\angle ABC$	6. Substitution Prop. (Steps ? and ?)

13. The coordinates of points L and X are 16 and 40, respectively. N is the midpoint of $\overline{LX}$, and Y is the midpoint of $\overline{LN}$. Sketch a diagram and find:
a. LN **b.** the coordinate of N **c.** LY **d.** the coordinate of Y

14. $\overrightarrow{SW}$ bisects $\angle RST$ and $m\angle RST = 72$. $\overrightarrow{SZ}$ bisects $\angle RSW$, and $\overrightarrow{SR}$ bisects $\angle NSW$. Sketch a diagram and find $m\angle RSZ$ and $m\angle NSZ$.

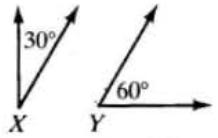
20. Fold down a corner of a rectangular sheet of paper. Then fold the next corner so that the edges touch as in the figure. Measure the angle formed by the fold lines. Repeat with another sheet of paper, folding the corner at a different angle. Explain why the angles formed are congruent.

2–4 Special Pairs of Angles

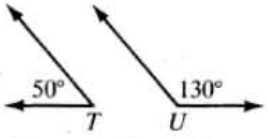
Objectives: Apply the definitions of complementary and supplementary angles.
State and use the theorem about vertical angles.

complementary angles Two angles whose measures have the sum 90 are complementary.

supplementary angles Two angles whose measures have the sum 180 are supplementary.

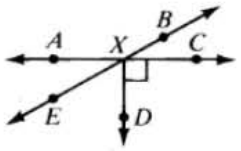


$m\angle X + m\angle Y = 90$
$\angle X$ and $\angle Y$ are complementary.
$\angle X$ is a complement of $\angle Y$.



$m\angle T + m\angle U = 180$
$\angle T$ and $\angle U$ are supplementary.
$\angle T$ is a supplement of $\angle U$.

In the diagram, $\angle CXD$ is a right angle. Name:



1. another right angle.
2. two congruent supplementary angles.
3. two noncongruent supplementary angles.
4. two complementary angles.

Example 1

$\angle C$ and $\angle D$ are complementary, $m\angle C = 3y - 5$, and $m\angle D = y + 15$. Find the value of y, $m\angle C$, and $m\angle D$.

Solution

$$\begin{aligned} m\angle C + m\angle D &= 90 \\ (3y - 5) + (y + 15) &= 90 \\ 4y + 10 &= 90 \\ 4y &= 80 \\ y &= 20 \end{aligned}$$

$$\begin{aligned} m\angle C &= 3y - 5 \\ &= 3(20) - 5 \\ &= 60 - 5 \\ &= 55 \end{aligned}$$

$$\begin{aligned} m\angle D &= y + 15 \\ &= 20 + 15 \\ &= 35 \end{aligned}$$

Example 2

A supplement of an angle is seven times a complement of the angle. Find the measures of the angle, its complement, and its supplement.

Solution

Let x be the measure of the angle. Then $90 - x$ is the measure of its complement, and $180 - x$ is the measure of its supplement.

$$\begin{aligned} 180 - x &= 7(90 - x) \\ 180 - x &= 630 - 7x \\ 6x &= 450 \\ x &= 75 \end{aligned}$$

measure of angle $= 75$
measure of complement $= 90 - x = 90 - 75 = 15$
measure of supplement $= 180 - x = 180 - 75 = 105$

$\angle A$ and $\angle B$ are supplementary. Find the value of x, $m\angle A$, and $m\angle B$.

5. $m\angle A = 3x, m\angle B = x + 20$

6. $m\angle A = x + 11, m\angle B = 2x - 5$

$\angle C$ and $\angle D$ are complementary. Find the value of y, $m\angle C$, and $m\angle D$.

7. $m\angle C = y + 11, m\angle D = 2y - 5$

8. $m\angle C = 3y + 5, m\angle D = 2y + 10$

Use the given information to write an equation and solve the problem.

9. Find the measure of an angle that is twice as large as its complement.

10. A supplement of an angle is four times as large as the angle. Find the measure of the angle.

11. The measure of a complement of an angle is three more than twice the measure of the angle. Find the measures of the angle and its complement.

Theorem 2-3

Vertical angles are congruent.

Given: $\angle 1$ and $\angle 2$ are vertical angles.

Prove: $\angle 1 \cong 2$

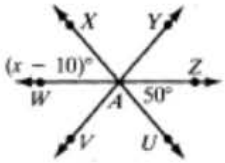
Proof:

Statements	Reasons
1. $m\angle 1 + m\angle 3 = 180$; $m\angle 2 + m\angle 3 = 180$	1. Angle Addition Postulate
2. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	2. Substitution Prop.
3. $m\angle 3 = m\angle 3$	3. Reflexive Prop.
4. $m\angle 1 = m\angle 2$, or $\angle 1 \cong \angle 2$	4. Subtraction Prop. of =

Example 3 In the diagram, $\overrightarrow{AZ}$ bisects $\angle YAU$.

a. Name three angles congruent to $\angle YAZ$.

b. Find the value of x.



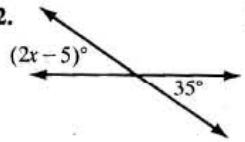
Solution

a. $\angle YAZ \cong \angle ZAU \cong \angle WAX \cong \angle WAV$

b. $x - 10 = 50$, so $x = 60$.

Find the value of x.

12.



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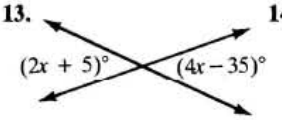

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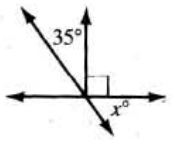
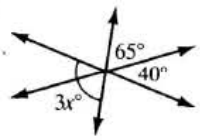


15.



In the diagram, $\overrightarrow{OC}$ bisects $\angle BOD$, $m\angle BOD = 90$, and $m\angle BOA = 40$. Find:

16. $m\angle BOC$

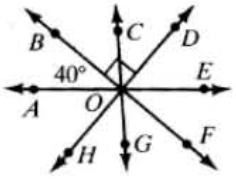
17. $m\angle FOG$

18. $m\angle AOH$

19. $m\angle HOE$

20. $m\angle DOE$

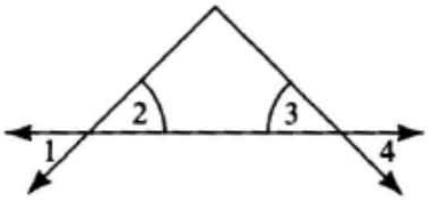
21. $m\angle AOE$



23. Copy everything shown. Complete the proof.

Given: $\angle 2 \cong \angle 3$

Prove: $\angle 1 \cong \angle 4$

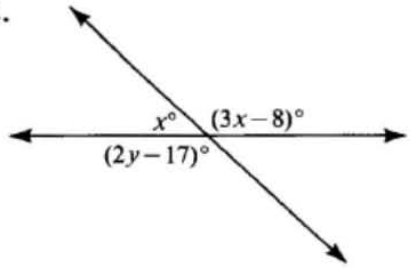


Proof:

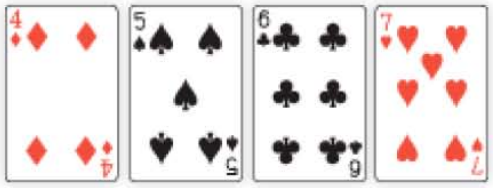
Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. ?
2. $\angle 2 \cong \angle 3$	2. ?
3. $\angle 3 \cong \angle 4$	3. ?
4. ?	4. Transitive Property (used twice)

Find the values of x and y

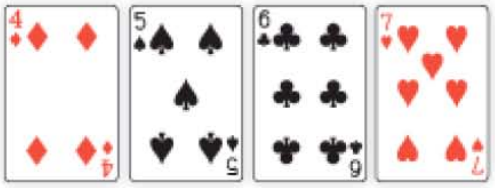
32.



18. Miriam the Magnificent placed four cards face up (the first four cards shown below). Blindfolded, she asked someone from her audience to come up to the stage and turn one card 180°.



Before turn

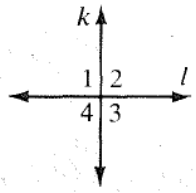


After turn

Miriam removed her blindfold and claimed she was able to determine which card was turned 180°. What is her trick? Can you figure out which card was turned? Explain.

2–5 Perpendicular Lines

Two lines that intersect to form right angles are **perpendicular lines.** If $k \perp l$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are right angles.



The definition of perpendicular lines, like all definitions, is a biconditional. It is given below as two conditionals.

If two lines are perpendicular, then they form right angles.
If two lines form right angles, then the lines are perpendicular.

Theorem 2-4

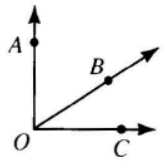
If two lines are perpendicular, then they form congruent adjacent angles.

Theorem 2-5

If two lines form congruent adjacent angles, then the lines are perpendicular.

Theorem 2-6

If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.



Given: $\overrightarrow{OA} \perp \overrightarrow{OC}$

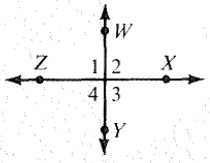
Prove: $\angle AOB$ and $\angle BOC$ are comp. $\triangleq$.

Example 1 Name the definition or state the theorem that justifies the statement about the diagram.

a. If $\overleftrightarrow{WY} \perp \overleftrightarrow{ZX}$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are right angles.

b. If $\angle 1 \cong \angle 4$, then $\overleftrightarrow{WY} \perp \overleftrightarrow{ZX}$.

c. If $m\angle 1 = 90$, then $\overleftrightarrow{WY} \perp \overleftrightarrow{ZX}$.



Solution

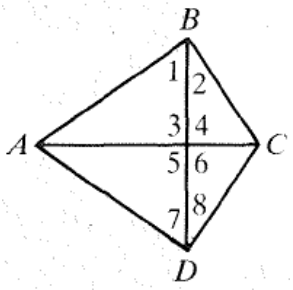
a. Def. of $\perp$ lines

b. If 2 lines form $\cong$ adj. $\triangleq$, then the lines are $\perp$.

c. Def. of $\perp$ lines

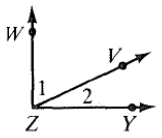
Name the definition or state the theorem that justifies each statement about the diagram.

1. If $\overline{AB} \perp \overline{BC}$, then $\angle ABC$ is a right angle.
2. If $\overline{BD} \perp \overline{AC}$, then $\angle 3 \cong \angle 4$.
3. If $\overline{DC} \perp \overline{DA}$, then $\angle 7$ and $\angle 8$ are complementary.
4. If $\angle 7$ and $\angle 8$ are complementary, then $m\angle 7 + m\angle 8 = 90$.
5. If $\angle 4 \cong \angle 6$, then $\overline{AC} \perp \overline{BD}$.
6. $\angle 4 \cong \angle 5$
7. If $\angle ADC$ is a right angle, then $m\ \angle ADC = 90$.



Example 2

If $\overrightarrow{ZW} \perp \overrightarrow{ZY}$, $m\angle 1 = 5x$, and $m\angle 2 = 2x - 1$, find the value of x.

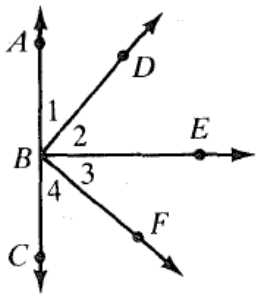


Solution

$\overrightarrow{ZW} \perp \overrightarrow{ZY}$, so $\angle 1$ and $\angle 2$ are complementary.

$$m\angle 1 + m\angle 2 = 90$$
$$(5x) + (2x - 1) = 90$$
$$7x - 1 = 90$$
$$7x = 91$$
$$x = 13$$

For the following questions, use the diagram to the right.



Given: $\overrightarrow{BE} \perp \overleftrightarrow{AC}$; $\overrightarrow{BD} \perp \overrightarrow{BF}$. Find the value of x.

8. $m\angle 2 = 2x + 10$, $m\angle 3 = 40$

9. $m\angle 3 = 2x + 5$, $m\angle 4 = 3x$

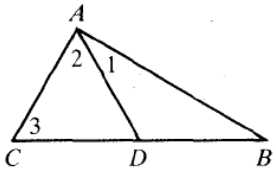
10. $m\angle 1 = 2x$, $m\angle 2 = 2x + 10$, $m\angle 3 = 3x - 20$, $m\angle 4 = 3x - 10$

12. $m\angle 1 = 3x + 1$, $m\angle 2 = 4x + 5$, $m\angle 3 = 2x + 13$

Complete the proof by supplying the missing statements and reasons.

13. Given: $\overline{BA} \perp \overline{AC}$; $\angle 1$ is complementary to $\angle 3$.

Prove: $m\angle 2 = m\angle 3$

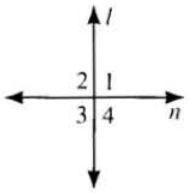


Statements	Reasons
1. $\overline{BA} \perp \overline{AC}$	1. _____
2. $\angle 1$ and $\angle 2$ are complementary.	2. _____
3. $\angle 1$ and $\angle 3$ are complementary.	3. _____
4. $m\angle 1 + m\angle 2 = 90$; $m\angle 1 + m\angle 3 = 90$	4. _____
5. $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$	5. _____
6. _____	6. Reflexive Prop.
7. $m\angle 2 =$ $m\angle 3$	7. _____

1. Complete the proof of Theorem 2-4: If two lines are perpendicular, then they form congruent adjacent angles.

Given: $l \perp n$

Prove: $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are congruent angles.



Proof:

Statements	Reasons
1. $l \perp n$	1. ?
2. $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ are $90°$ $\measuredangle$s.	2. Definition of ?
3. $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ are $\cong$ $\measuredangle$s.	3. Definition of ?

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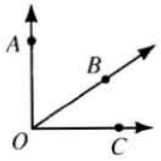
13. Copy and complete the proof of Theorem 2-6: If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.

Given: $\overrightarrow{OA} \perp \overrightarrow{OC}$

Prove: $\angle AOB$ and $\angle BOC$ are comp. $\measuredangle$s.



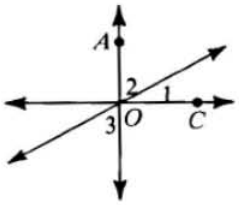
Proof:

Statements	Reasons
1. $\overrightarrow{OA} \perp \overrightarrow{OC}$	1. ?
2. $m\angle AOC = 90$	2. Def. of $\perp$ lines
3. $m\angle AOB + m\angle BOC = m\angle AOC$	3. ?
4. ?	4. Substitution Prop.
5. ?	5. Def. of comp. $\measuredangle$s

28. Copy everything shown and write a two-column proof.

Given: $\overleftrightarrow{AO} \perp \overleftrightarrow{CO}$

Prove: $\angle 1$ and $\angle 3$ are comp. $\measuredangle$s.



Here is an interesting definition:

> A *wolf pack* is two wolves or is a wolf pack together with a wolf.

36. According to this definition, how many wolves are in a pack?

37. Why is it appropriate that this definition is used as an example in a book titled *Keys to Infinity?**

**Keys to Infinity,* by Clifford A. Pickover (Wiley, 1995).

2–6 Planning a Proof

Theorems on angles supplementary or complementary to congruent angles.

As you have seen in the last few sections. a proof of a theorem consists of five parts:
1. *Statement* of the theorem
2. A *diagram* that illustrates the given information
3. A list, in terms of the figure, of what is *given*
4. A list, in terms of the figure, of what you are to *prove*
5. A series of *statements and reasons* that lead from the given information to the statement that is to be proved.

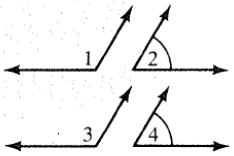
Before you can write a proof, you need to have a plan for the proof. Sometimes you will see immediately how to do the proof. Sometimes a previous proof will give you a plan, as in Exercise 1.

If two angles are supplements of congruent angles (or of the same angle), then the two angles are congruent.

Example 1

Given: $\angle 1$ and $\angle 2$ are supplementary; $\angle 3$ and $\angle 4$ are supplementary; $\angle 2 \cong \angle 4$

Prove: $\angle 1 \cong \angle 3$



Solution

Plan for Proof:

From the given, you can conclude $m\angle 1 + m\angle 2 = 180$ and $m\angle 3 + m\angle 4 = 180$. By substitution, $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$. $\angle 2 \cong \angle 4$, so $m\angle 2 = m\angle 4$, and by subtraction $m\angle 1 = m\angle 3$, or $\angle 1 \cong \angle 3$.

If two angles are complements of congruent angles (or of the same angle), then the two angles are congruent.

1. Write a proof of the theorem above.

(*Hint:* It will be very similar to the proof of the previous theorem.)

Given: $\angle 5$ and $\angle 6$ are complementary; $\angle 7$ and $\angle 8$ are complementary; $\angle 5 \cong \angle 7$

Prove: $\angle 6 \cong \angle 8$

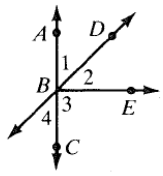
Sometimes you may need to try several times to find a plan that works. If you don't see immediately how to do a proof, here are two methods to try to find a plan.

Method 1	**Method 2**
Gather as much information as you can. Sometimes what you can see will show you a plan.	Work backward. Go to the conclusion, the part you would like to prove.
Reread the given. What does it tell you?	Think: This conclusion would be true if _?_.
Look at the diagram. What other information can you conclude?	And _?_ would be true if _?_. And so on, until you have a plan for a proof.

Example 2

Given: $\overrightarrow{BD}$ bisects $\angle ABE$.

Prove: $\angle 2 \cong \angle 4$



Solution

Plan for Proof (Method 1):

From the given, you can conclude that $\angle 1 \cong \angle 2$.

From the diagram, you can see that $\angle 1$ and $\angle 4$ are vertical angles, so $\angle 1 \cong \angle 4$.

From $\angle 1 \cong \angle 2$ and $\angle 1 \cong \angle 4$, you can conclude that $\angle 2 \cong \angle 4$.

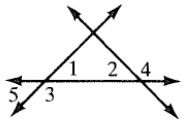
2. Complete the proof of Example 2.

Statements	Reasons
1. _____	1. Given
2. $\angle$ _____ $\cong \angle$ _____	2. Def. of _____
3. $\angle 1 \cong \angle 4$	3. _____
4. _____	4. _____

Example 3

Given: $m\angle 1 = m\angle 2$

Prove: $\angle 4$ is supplementary to $\angle 5$.



Solution

Plan for Proof (Method 2):

$\angle 4$ is supplementary to $\angle 5$ if $m\angle 4 + m\angle 5 = 180$. This is true if $m\angle 4 = m\angle 2$, since $m\angle 4 + m\angle 2 = 180$.

From the diagram $m\angle 1 = m\angle 5$, and $m\angle 1 = m\angle 2$, so $m\angle 2 = m\angle 5$.

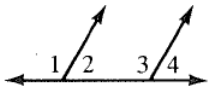
3. Complete the proof of Example 3.

Statements	Reasons
1. $m\angle 1 = m\angle 2$	1. _____
2. $m\angle 1 = m\angle 5$	2. _____
3. _____	3. _____
4. $m\angle 2 + m\angle 4 = 180$	4. _____
5. $m\angle 5 + m\angle 4 = 180$	5. _____
6. _____	6. _____

Write a two-column proof.

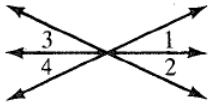
4. Given: $\angle 2$ is supplementary to $\angle 3$.

Prove: $\angle 1 \cong \angle 3$



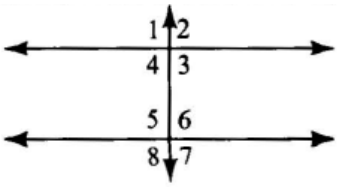
5. Given: $\angle 1 \cong \angle 3$

Prove: $\angle 3 \cong \angle 4$

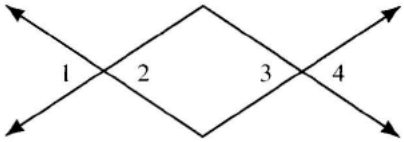


16. a. Are there any angles in the diagram that must be congruent to $\angle 4$? Explain.

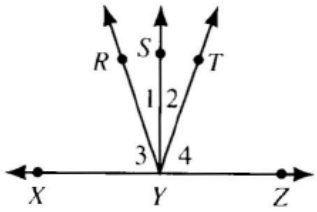
b. If $\angle 4$ and $\angle 5$ are supplementary, name all angles shown that must be congruent to $\angle 4$.



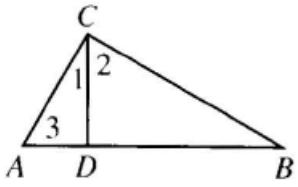
19. Given: $\angle 2 \cong \angle 3$
Prove: $\angle 1 \cong \angle 4$



22. Given: $m\angle 1 = m\angle 2$;
$m\angle 3 = m\angle 4$
Prove: $\overrightarrow{YS} \perp \overleftrightarrow{XZ}$



21. Given: $\overline{AC} \perp \overline{BC}$;
$\angle 3$ is comp. to $\angle 1$.
Prove: $\angle 3 \cong \angle 2$



(a) Write the proof.

(b) What do we know about $\angle ADC$, $\angle BDC$, and $\angle CBD$? Are they congruent to other angles?

(c) How is AC/AB related to AD/AC?

Chapter Review

Use the conditional: If $m\angle 1 = 120$, then $\angle 1$ is obtuse.

2–1

1. Write the hypothesis and the conclusion of the conditional.
2. Write the converse of the conditional.
3. Provide a counterexample to disprove the converse.
4. Write a definition of a straight angle as a biconditional.

Justify each statement with a property from algebra or a property of congruence.

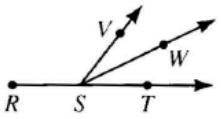
2–2

5. If $m\angle A + m\angle B + m\angle C = 180$ and $m\angle C = 50$, then $m\angle A + m\angle B + 50 = 180$.
6. If $m\angle A + m\angle B + 50 = 180$, then $m\angle A + m\angle B = 130$.
7. If $6x = 18$, then $x = 3$.
8. If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Name the definition, postulate, or theorem that justifies the statement.

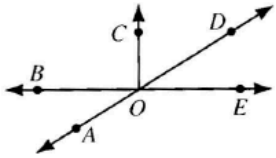
2–3

9. If $\overline{RS} \cong \overline{ST}$, then S is the midpoint of $\overline{RT}$.
10. If $\overrightarrow{SW}$ bisects $\angle VST$, then $\angle VSW \cong \angle WST$.
11. If $\overrightarrow{SW}$ bisects $\angle VST$, then $m\angle WST = \frac{1}{2}m\angle VST$.



2–4

12. If $\angle BOC$ is a right angle and $m\angle COD = 58$, then $m\angle DOE =$?, $m\angle BOA =$?, and $m\angle AOC =$?.
13. Name a supplement of $\angle AOE$.

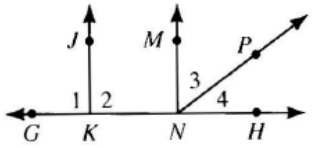


14. A supplement of a given angle is four times as large as a complement of the angle. Find the measure of the given angle.

Name the definition or state the theorem that justifies the statement about the diagram.

2–5

15. If $\overrightarrow{KJ} \perp \overleftrightarrow{GH}$, then $\angle 1$ is a right angle.
16. If $\angle 2$ is a 90° angle, then $\overrightarrow{KJ} \perp \overleftrightarrow{GH}$.
17. If $\overrightarrow{NM} \perp \overleftrightarrow{GH}$, then $\angle MNK \cong \angle MNH$.
18. If $\overrightarrow{NM} \perp \overleftrightarrow{GH}$, then $\angle 3$ and $\angle 4$ are complementary.



2–6

19. Write a plan for a proof.
Given: $\angle 3$ is a supplement of $\angle 1$;
$\angle 4$ is a supplement of $\angle 2$.
Prove: $\angle 3 \cong \angle 4$
20. Write a proof in two-column form for Exercise 19.