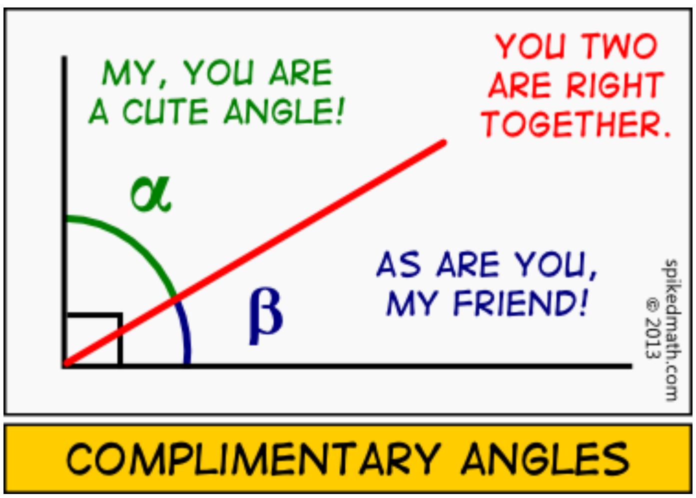
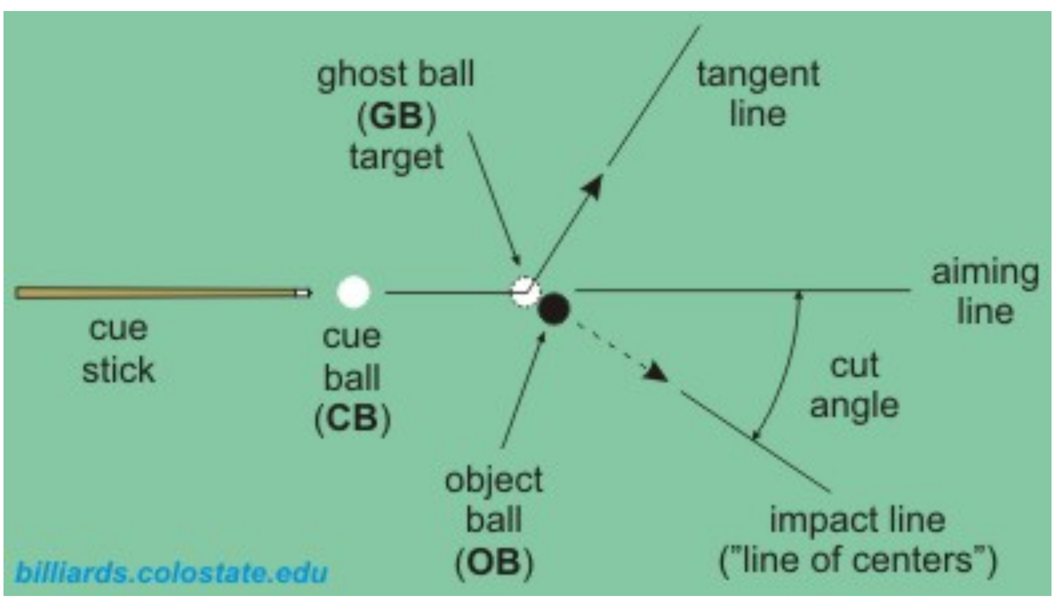
FULL NAME: ______________________ BLOCK/PERIOD: ___

V. 1.0

BASEPACKET

CHAPTER 1
THE BASICS OF GEOMETRY:
POINTS, LINES, PLANES, ANGLES, AND MORE.





LESSON 1.1

NAME ______________________________ DATE ____________

Practice with Examples

For use with pages 3–9

GOAL **Find and describe patterns and use inductive reasoning**

VOCABULARY

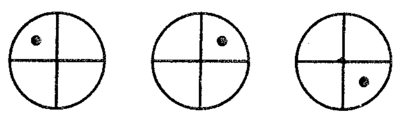
A **conjecture** is an unproven statement that is based on observations.

Inductive reasoning is a process that involves looking for patterns and making conjectures.

A **counterexample** is an example that shows a conjecture is false.

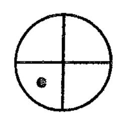
EXAMPLE 1 *Describing a Visual Pattern*

Sketch the next figure in the pattern.



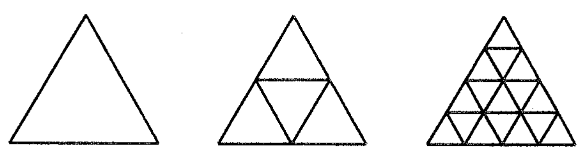
SOLUTION

Each figure looks like the one before it except that it has rotated 90°. The next figure will have the smaller circle in the lower-left quarter of the bigger circle.



Exercise for Example 1

1. Sketch the next figure in the pattern.



NAME ______________________ DATE __________

Practice with Examples

For use with pages 3–9

EXAMPLE 2 Describing a Number Pattern

Describe a pattern in the sequence of numbers. Predict the next number.

a. $5, 3, 1, -1, \ldots$ **b.** $1, -4, 9, -16, \ldots$ **c.** $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

SOLUTION

a. These are consecutive odd numbers, but listed backwards starting with 5. The next number is -3.

b. These numbers look like consecutive perfect squares, except that every other one is negative. The next number is 25.

c. Each number is $\frac{1}{2}$ times the previous number. The next number is $\frac{1}{16}$.

Exercises for Example 2

Describe a pattern in the sequence of numbers. Predict the next number.

2. $1, 2, 6, 24, \ldots$ **3.** $0, 3, 8, 15, 24, \ldots$

EXAMPLE 3 Making a Conjecture

Complete the conjecture.

Conjecture: The product of two consecutive even integers is divisible by ___?___.

SOLUTION

List some specific examples and look for a pattern.

Examples:

$2 \times 4 = 8 = 8 \times 1$ $6 \times 8 = 48 = 8 \times 6$ $10 \times 12 = 120 = 8 \times 15$

$4 \times 6 = 24 = 8 \times 3$ $8 \times 10 = 80 = 8 \times 10$ $12 \times 14 = 168 = 8 \times 21$

Conjecture: The product of two consecutive even integers is divisible by 8.

NAME ______________________ DATE ____________

Practice with Examples

For use with pages 3–9

Exercises for Example 3

Complete the conjecture based on the pattern you observe in the specific cases.

4. **Conjecture**: For any two numbers a and b, the product of $(a + b)$ and $(a - b)$ is always equal to ___?___.

   $(2 + 1) \times (2 - 1) = 3 = 2^2 - 1^2$ $(4 + 2) \times (4 - 2) = 12 = 4^2 - 2^2$

   $(3 + 2) \times (3 - 2) = 5 = 3^2 - 2^2$ $(6 + 3) \times (6 - 3) = 27 = 6^2 - 3^2$

EXAMPLE 4 *Finding a Counterexample*

Show the conjecture is false by finding a counterexample.

Conjecture: All odd numbers are prime.

SOLUTION

The conjecture is false. Here is a counterexample: The number 9 is odd and is a composite number, not a prime number.

Exercise for Example 4

Show the conjecture is false by finding a counterexample.

5. The square of the sum of two numbers is equal to the sum of the squares of the two numbers. That is, $(a + b)^2 = a^2 + b^2$.

NAME ______________________________ DATE ____________

Practice with Examples

For use with pages 10–16

GOAL **Understand and use the basic undefined terms and defined terms of geometry and sketch intersections of lines and planes**

VOCABULARY

A **point** has no dimension, a **line** extends in one dimension, and a **plane** extends in two dimensions.

Collinear points are points that lie on the same line.

Coplanar points are points that lie on the same plane.

On a line passing through points *A* and *B*, **segment** *AB* consists of all points between *A* and *B* and **endpoints** *A* and *B*.

On a line passing through points *A* and *B*, **ray** *AB* consists of the **initial point** *A* and all points on the same side of *A* as point *B*.

If point *C* is between *A* and *B*, then ray *CA* and ray *CB* are **opposite rays.**

Two or more geometric figures **intersect** if they have one or more points in common. The **intersection** of the figures is the set of points the figures have in common.

EXAMPLE 1 Drawing and Naming Lines, Segments, and Rays

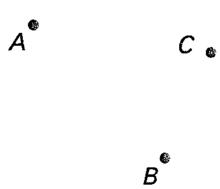
a. Draw three noncollinear points, *A*, *B*, and *C*. Then draw point *D* on line *AB* between points *A* and *B*. Draw segment *CD*. Draw ray *CA* and ray *CB*.

b. Are points *A*, *B*, and *D* collinear? Are points *B*, *C*, and *D* collinear?

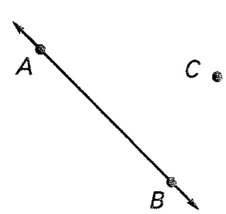
c. Are ray *CA* and ray *CB* opposite rays? Are ray *DA* and ray *DB* opposite rays?

SOLUTION

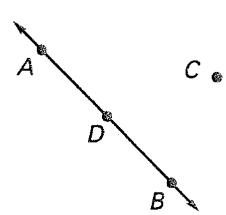
a.



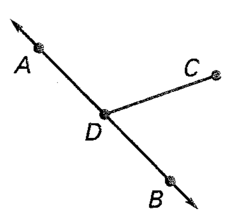
1. First, draw *A*, *B*, and *C*.



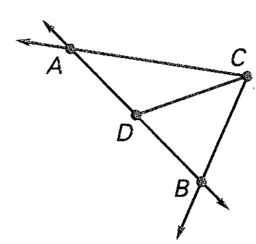
2. Draw line *AB*.



3. Draw *D*.



4. Draw segment *CD*.



5. Draw ray *CA* and ray *CB*.

NAME ______________________ DATE __________

Practice with Examples

For use with pages 10–16

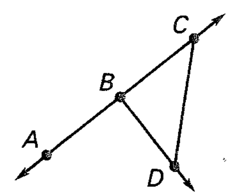
b. Yes, points *A*, *B*, and *D* are collinear because they lie on line *AB*. No, points *B*, *C*, and *D* are noncollinear because a straight line cannot be drawn through all three points.

c. No, ray *CA* and ray *CB* are not opposite rays. Point *C* is not between *A* and *B*. Yes, ray *DA* and ray *DB* are opposite rays. Point *D* is between *A* and *B*.

Exercise for Example 1

1. Draw collinear points *A*, *B*, and *C*, with point *B* between *A* and *C*. Draw point *D* not on line *AC*. Draw line *AD*. Draw point *E* on line *AD* between point *A* and point *D*. Draw segment *EC*. Draw ray *BD*. Draw ray *EB*.

Use the diagram to name the figures.



2. Three noncollinear points

3. Two opposite rays

4. One line segment

5. Three collinear points

6. Two rays which are *not* opposite rays

7. Two line segments that are on the same line

NAME ______________________ DATE __________

Practice with Examples

For use with pages 10–16

Chapter 1

EXAMPLE 2 Sketching Intersections

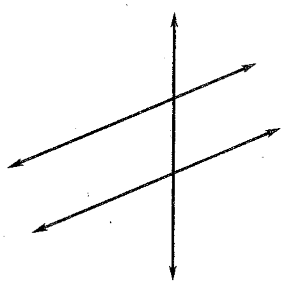
Sketch the figure described.

a. Three lines that lie in the same plane, but two of the lines do not intersect with each other and the third line intersects with each of the other lines in a point.

b. Two planes which do not intersect, and a line which intersects each plane in a point.

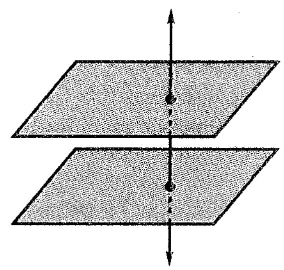
SOLUTION

a.



Draw two lines which do not intersect. Draw a third line, crossing each of the other lines.

b.



Draw two planes which do not intersect. Draw a line through both planes. Emphasize the points where the line intersects.

Exercises for Example 2

Sketch the figure described.

8. Three planes which intersect in a line

9. Two planes which intersect in a line, and a third plane which intersects each of the other two planes in a line, but not the same line

NAME ______________________ DATE ____________

Practice with Examples

For use with pages 17–24

GOAL **Use segment postulates and use the distance formula to measure distances**

VOCABULARY

A **postulate** or **axiom** is a rule that is accepted without proof.

Postulate 1 Ruler Postulate:
The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the **coordinate** of the point.

The **distance** between points A and B, written as AB, is the absolute value of the difference between the coordinates of A and B.

AB is also called the **length** of $\overline{AB}$.

When three points lie on a line, you can say that one of them is **between** the other two.

Postulate 2 Segment Addition Postulate:
If B is between A and C, then $AB + BC = AC$. If $AB + BC = AC$, then B is between A and C.

The **Distance Formula** is a formula for computing the difference between two points in a coordinate plane.

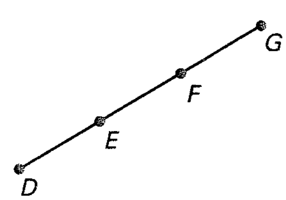
The Distance Formula:
If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Segments that have the same length are called **congruent segments.**

EXAMPLE 1 *Using the Segment Addition Postulate*

In the diagram of the collinear points, $DE = 2$, $EF = 3$, and $DE = FG$. Find each length.

FG
DF
DG
EG



SOLUTION

Since $DE = FG$ and $DE = 2$, $FG = 2$.

Since $DF = DE + EF$, $DF = 2 + 3 = 5$.

Since $DG = DF + FG$, $DG = 5 + 2 = 7$.

Since $EG = EF + FG$, $EG = 3 + 2 = 5$.

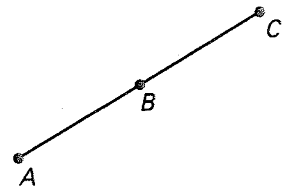
NAME ______________________ DATE ________

Practice with Examples

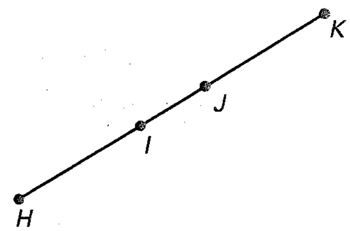
For use with pages 17–24

Exercises for Example 1

1. In the diagram of the collinear points, $BC = 5$ and $BC = AB$. Find the following lengths.
   a. AC
   b. AB
   c. Are any segments congruent?



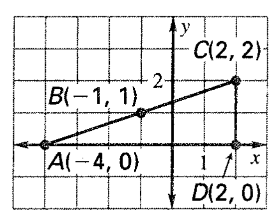
2. In the diagram of the collinear points, $HK = 9$, $HI = JK$, and $IJ = 1$. Find the following lengths.
   a. HI
   b. JK
   c. HJ
   d. IK



EXAMPLE 2 Using the Distance Formula

Find the following distances. State whether any of the segments are congruent.

a. AB
b. BC
c. CD
d. AC



SOLUTION

Use the Distance Formula.

a. $AB = \sqrt{[(-1) - (-4)]^2 + (1 - 0)^2} = \sqrt{3^2 + 1^2} = \sqrt{9 + 1} = \sqrt{10}$

b. $BC = \sqrt{[2 - (-1)]^2 + (2 - 1)^2} = \sqrt{3^2 + 1^2} = \sqrt{9 + 1} = \sqrt{10}$

c. $CD = \sqrt{(2 - 2)^2 + (0 - 2)^2} = \sqrt{0^2 + (-2)^2} = \sqrt{0 + 4} = \sqrt{4} = 2$

d. $AC = \sqrt{[2 - (-4)]^2 + (2 - 0)^2} = \sqrt{6^2 + 2^2} = \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$

Segments $\overline{AB}$ and $\overline{BC}$ are congruent because they have the same length.

NAME ______________________ DATE ________

Practice with Examples

For use with pages 17–24

Exercises for Example 2

Find the distance between the points whose coordinates are given.

3. $(6, 4), (-8, 11)$

4. $(-5, 8), (-10, 14)$

5. $(-4, -20), (-10, 15)$

6. $(40, 32), (36, 20)$

7. $(5, -8), (0, 0)$

8. $(a, b), (-a, -b)$

LESSON 1.4

NAME ______________________ DATE __________

Practice with Examples

For use with pages 26–32

GOAL **Use angle postulates and classify angles as acute, right, obtuse, or straight**

VOCABULARY

An **angle** consists of two different rays that have the same initial point.

The rays are the **sides** of the angle.

The initial point is the **vertex** of the angle.

Angles that have the same measure are called **congruent angles.**

A point is in the **interior** of an angle if it is between points that lie on each side of the angle.

A point is in the **exterior** of an angle if it is not on the angle or in its interior.

An **acute** angle has measure greater than 0° and less than 90°.

A **right** angle has measure equal to 90°.

An **obtuse** angle has measure greater than 90° and less than 180°.

A **straight** angle has measure equal to 180°.

Two angles are **adjacent angles** if they share a common vertex and side, but have no common interior points.

Postulate 3 Protractor Postulate:
Consider a point A on one side of $\overleftrightarrow{OB}$. The rays of the form $\overrightarrow{OA}$ can be matched one to one with the real numbers from 0 to 180.

The **measure** of $\angle AOB$ is equal to the absolute value of the difference between the real numbers for $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

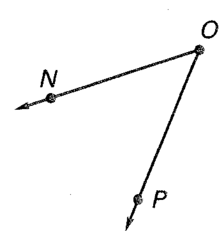
Postulate 4 Angle Addition Postulate:

If P is in the interior of $\angle RST$, then $m\angle RSP + m\angle PST = m\angle RST$.

EXAMPLE 1 *Naming Angles*

a. Write three names for the angle and name the vertex and sides of the angle.

b. Suppose R is in the interior of $\angle NOP$, with $m\angle NOR = 23°$ and $m\angle ROP = 27°$. Find $m\angle NOP$.



SOLUTION

a. $\angle NOP$, $\angle PON$, and $\angle O$ are all appropriate names for this angle.
The vertex of this angle is point O and the sides are $\overrightarrow{ON}$ and $\overrightarrow{OP}$.

b. By Angle Addition Postulate, $m\angle NOP = m\angle NOR + m\angle ROP = 23° + 27° = 50°$.

NAME ______________________ DATE __________

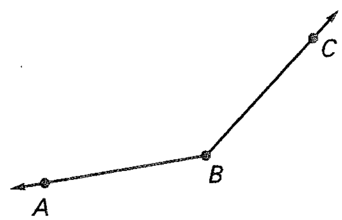
Practice with Examples

For use with pages 26–32

Exercises for Example 1

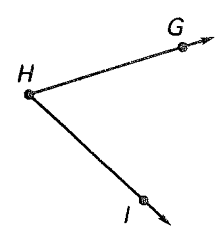
Write three names for the angles and name the vertex and sides of each.

1.



2.

3. Suppose that the angle at the right measures 60° and that there is a point K in the interior of the angle such that $m\angle GHK = 25°$. Find $m\angle KHI$.



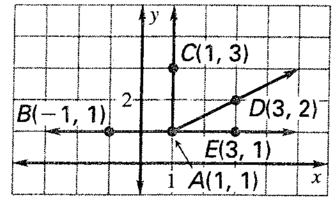
EXAMPLE 2 Classifying Angles in a Coordinate Plane

Plot the points $A(1, 1)$, $B(-1, 1)$, $C(1, 3)$, $D(3, 2)$, and $E(3, 1)$. Then classify the following angles as acute, right, obtuse, or straight.

a. $\angle CAB$ **b.** $\angle DAE$ **c.** $\angle BAD$ **d.** $\angle EAB$

SOLUTION

Begin by plotting the points, then observe whether each angle is less than 90°, equal to 90°, between 90° and 180°, or equal to 180°.



a. right angle **b.** acute angle

c. obtuse angle **d.** straight angle

NAME ______________________ DATE ____________

Practice with Examples

For use with pages 26–32

Exercises for Example 2

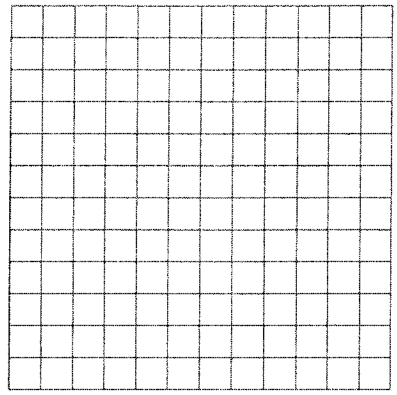
Plot the given points and classify the given angles as *acute*, *right*, *obtuse*, or *straight*.

4. $A(-2, 4)$, $B(-5, 1)$, $C(0, 0)$, and $D(3, 0)$

a. $\angle ACB$

b. $\angle BCD$

c. $\angle ACD$



5. $E(4, 0)$, $F(3, 2)$, $G(1, 0)$, $H(-1, -2)$, $I(-1, 2)$

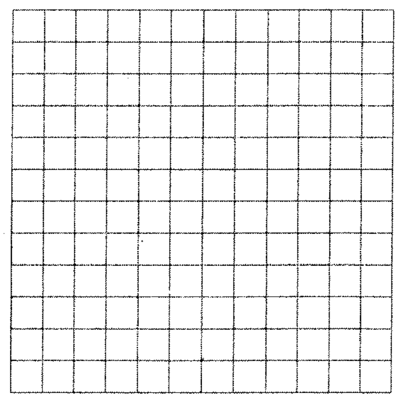
a. $\angle HGF$

b. $\angle EGF$

c. $\angle EGI$

d. $\angle FGI$

e. $\angle HGI$



Chapter 1

LESSON 1.5

NAME ______________________ DATE __________

Practice with Examples

For use with pages 34–42

GOAL Bisect a segment and bisect an angle

VOCABULARY

The **midpoint** of a segment is the point that divides, or **bisects,** the segment into two congruent segments.

A **segment bisector** is a segment, ray, line, or plane that intersects a segment at its midpoint.

A **construction** is a geometric drawing that uses a limited set of tools, usually a **compass** and a **straightedge.**

An **angle bisector** is a ray that divides an angle into two adjacent angles that are congruent.

The Midpoint Formula:

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint of $\overline{AB}$ has coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

EXAMPLE 1 *Finding the Coordinates of the Midpoint of a Segment*

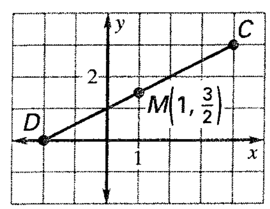
Find the coordinates of the midpoint of $\overline{CD}$ with endpoints $C(4, 3)$ and $D(-2, 0)$.

SOLUTION

Use the Midpoint Formula as follows.

$$M = \left(\frac{4 + (-2)}{2}, \frac{3 + 0}{2}\right)$$

$$= \left(1, \frac{3}{2}\right)$$



Exercises for Example 1

Find the coordinates of the midpoint of the segment whose endpoints are given.

1. $E(4, -4), F(1, 7)$ **2.** $G(2, 9), H(-3, 6)$ **3.** $I(-8, 3), J(3, 0)$

NAME ______________________ DATE __________

Practice with Examples

For use with pages 34–42

EXAMPLE 2 Finding the Coordinates of the Endpoint of a Segment

The midpoint of $\overline{KL}$ is $M(6, -2)$. One endpoint is $K(4, 3)$. Find the coordinates of the other endpoint.

SOLUTION

Let (x, y) be the coordinates of L. Use the Midpoint Formula to write equations involving x and y.

$$\frac{4 + x}{2} = 6 \qquad \frac{3 + y}{2} = -2$$

$$4 + x = 12 \qquad 3 + y = -4$$

$$x = 8 \qquad y = -7$$

So, the other endpoint of the segment is $L(8, -7)$.

Exercises for Example 2

Find the coordinates of the other endpoint of a segment with the given endpoint and midpoint *M*.

4. $N(-1, 5)$, $M(0, 1)$

5. $P(6, -4)$, $M(3, 10)$

6. $R(-7, -3)$, $M(0, 0)$

LESSON 1.5 CONTINUED

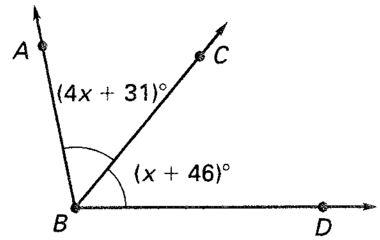
NAME ______________________ DATE __________

Practice with Examples

For use with pages 34–42

EXAMPLE 3 Finding the Measure of an Angle

In the diagram, $\overrightarrow{BC}$ bisects $\angle ABD$. Solve for x.



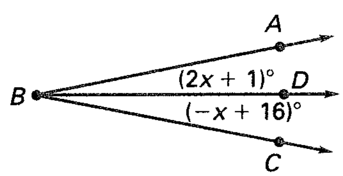
SOLUTION

$m\angle ABC = m\angle CBD$	Congruent angles have equal measures.
$(4x + 31)° = (x + 46)°$	Substitute given measures.
$4x = x + 15$	Subtract 31° from each side.
$3x = 15$	Subtract x from each side.
$x = 5$	Divide each side by 3.

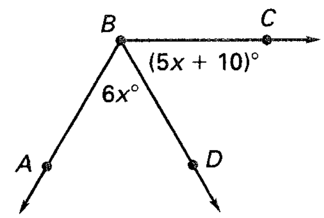
Exercises for Example 3

$\overrightarrow{BD}$ bisects $\angle ABC$. Find the value of x.

7.



8.



Chapter 1

NAME ______________________ DATE __________

Practice with Examples

For use with pages 44–50

GOAL **Identify vertical angles and linear pairs and identify complementary and supplementary angles**

VOCABULARY

Two angles are **vertical angles** if their sides form two pairs of opposite rays.

Two adjacent angles are a **linear pair** if their noncommon sides are opposite rays.

Two angles are **complementary angles** if the sum of their measures is 90°. Each angle is the **complement** of the other.

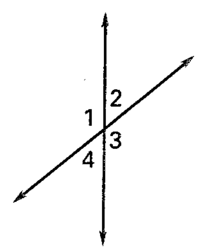
Two angles are **supplementary angles** if the sum of their measures is 180°. Each angle is the **supplement** of the other.

EXAMPLE 1 *Identifying Vertical Angles and Linear Pairs*

a. Are $\angle 1$ and $\angle 3$ vertical angles?

b. Are $\angle 2$ and $\angle 4$ a linear pair?

c. Are $\angle 1$ and $\angle 4$ a linear pair?



SOLUTION

a. Yes. The sides of the angles form two pairs of opposite rays.

b. No. The angles are not adjacent.

c. Yes. The angles are adjacent and their noncommon sides are opposite rays.

NAME ______________________________ DATE __________

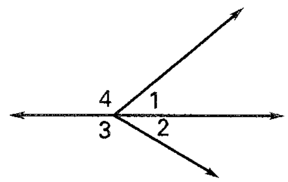
Practice with Examples

For use with pages 44–50

Exercises for Example 1

Use the figure to answer the questions.

1.



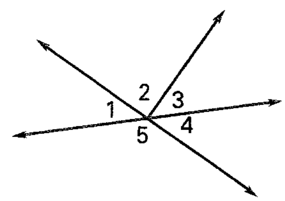
a. Are ∠1 and ∠2 a linear pair?

b. Are ∠1 and ∠3 vertical angles?

c. Are ∠1 and ∠4 a linear pair?

d. Are ∠2 and ∠4 vertical angles?

2.



a. Are ∠1 and ∠5 a linear pair?

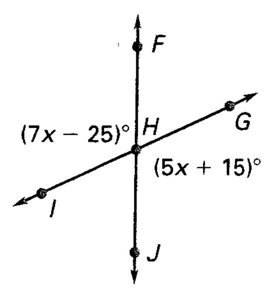
b. Are ∠1 and ∠2 a linear pair?

c. Are ∠1 and ∠4 vertical angles?

d. Are ∠3 and ∠5 vertical angles?

EXAMPLE 2 Finding Angle Measures

Solve for x in the diagram at the right. Then find the angle measures.



SOLUTION

Use the fact that vertical angles are congruent.

$$(7x - 25)^\circ = (5x + 15)^\circ$$

$$x = 20$$

Use substitution to find the angle measures.

$m\angle FHI = (7x - 25)^\circ = (7 \cdot 20 - 25)^\circ = 115^\circ$

$m\angle GHJ = (5x + 15)^\circ = (5 \cdot 20 + 15)^\circ = 115^\circ$

Next, realize that $\angle FHI$ and $\angle FHG$ are a linear pair. So, the measures of these two angles must sum to 180°. So, $m\angle FHG = 180^\circ - 115^\circ$, so $m\angle FHG = 65^\circ$.

Finally, notice that $\angle FHG$ and $\angle IHJ$ are vertical angles. So, $m\angle IHJ = 65^\circ$.

NAME ______________________ DATE __________

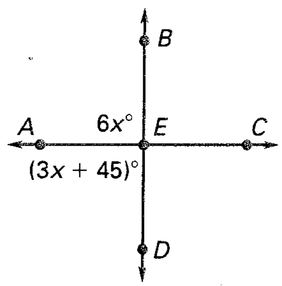
Practice with Examples

For use with pages 44–50

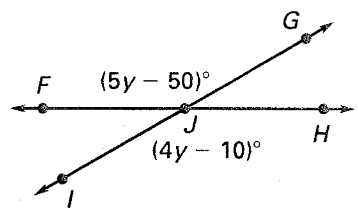
Exercises for Example 2

Solve for x and y, then find the angle measures.

3.



4.



EXAMPLE 3 Finding Measures of Complements and Supplements

a. Given that $\angle E$ is a complement of $\angle F$ and $m\angle E = 68°$, find $m\angle F$.

b. Given that $\angle G$ is a supplement of $\angle H$ and $m\angle G = 152°$, find $m\angle H$.

SOLUTION

a. $m\angle F = 90° - m\angle E = 90° - 68° = 22°$

b. $m\angle H = 180° - m\angle G = 180° - 152° = 28°$

Exercises for Example 3

Find the measure of the angle.

5. Given that $\angle A$ is a complement of $\angle B$ and $m\angle B = 81°$, find $m\angle A$.

6. Given that $\angle C$ is a supplement of $\angle D$ and $m\angle C = 27°$, find $m\angle D$.

LESSON 1.7

NAME ______________________ DATE ________

Practice with Examples

For use with pages 51–58

GOAL **Find the perimeter and area of common plane figures and use a general problem-solving plan**

VOCABULARY

Formulas for the perimeter P, area A, and circumference C of some common plane figures are given below.

Square

Side length s

$P = 4s$

$A = s^2$

Rectangle

length l and width w

$P = 2l + 2w$

$A = lw$

Triangle

Side lengths a, b, and c,

base b, and height h

$P = a + b + c$

$A = \frac{1}{2}bh$

Circle

radius r

$C = 2\pi r$

$A = \pi r^2$

A Problem-Solving Plan:

1. Ask yourself what you need to solve the problem. Write a **verbal model** or **draw a sketch** that will help you find what you need to know.
2. **Label known and unknown facts** on or near your sketch.
3. Use labels and facts to **choose related definitions, theorems, formulas,** or other results you may need.
4. **Reason logically** to link the facts, using a proof or other written argument.
5. Write a **conclusion** that answers the original problem. **Check** that your reasoning is correct.

Chapter 1

NAME ______________________ DATE ________

Practice with Examples

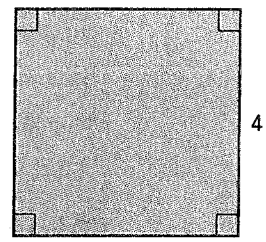
For use with pages 51–58

EXAMPLE 1 Finding the Perimeter and Area of a Square

Find the perimeter and area of a square with a side of 4 inches.

Solution

Begin by drawing a diagram and labeling one of the sides. Then, use the formulas for perimeter and area of a square.



$P = 4s$ $\quad\quad$ $A = s^2$

$= 4(4)$ $\quad\quad$ $= 4^2$

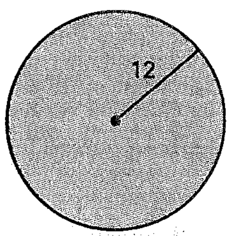
$= 16$ $\quad\quad$ $= 16$

So, the perimeter is 16 inches and the area is 16 square inches.

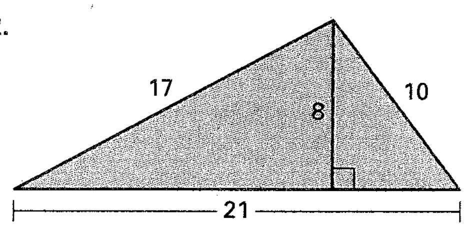
Exercises for Example 1

Find the perimeter (or circumference) and area of the figure. (Where necessary, use $\pi \approx 3.14$)

1.



2.



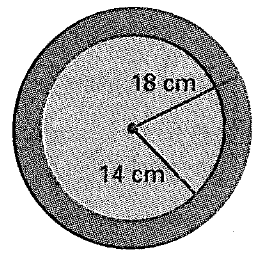
NAME ______________________ DATE __________

Practice with Examples

For use with pages 51–58

EXAMPLE 2 Using the Area of a Circle

You are making a cardboard model of a car. You make the tires with a radius of 18 centimeters. If the rim alone has a radius of 14 centimeters, what is the area of the rubber part of the tire?



Solution

Draw a Sketch From the diagram, you can see that the area of the rubber can be represented by the area of the larger circle minus the area of the smaller circle.

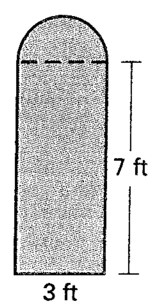
Verbal Model [Area of rubber] = [Area of large circle] − [Area of small circle]

Labels	Area of rubber = A	(square centimeters)
	Radius of whole tire = 18	(centimeters)
	Radius of rim = 14	(centimeters)
Reasoning	$A = \pi \cdot 18^2 - \pi \cdot 14^2$	Write model for rubber area.
	$\approx 3.14 \cdot 324 - 3.14 \cdot 196$	$\pi \approx 3.14$ and evaluate powers.
	$= 1017.36 - 615.44$	Multiply.
	$= 401.92$	Subtract.

The area of the rubber is about 401.92 square centimeters.

Exercises for Example 2

3. A window has the shape of a rectangle with a half-circle (see figure). The rectangle has a width of 3 feet and a height of 7 feet. Find the perimeter and area of the window. Use $\pi \approx 3.14$ where necessary.



Guided Practice

Vocabulary Check ✓

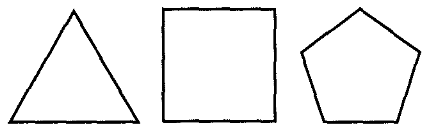
1. Explain what a *conjecture* is.

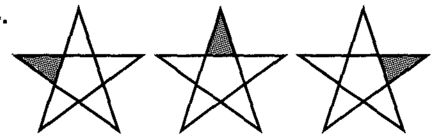
Concept Check ✓

2. How can you prove that a conjecture is false?

Skill Check ✓

Sketch the next figure in the pattern.

3. 

4. 

Describe a pattern in the sequence of numbers. Predict the next number.

5. 2, 6, 18, 54, . . .
6. 0, 1, 4, 9, . . .
7. 256, 64, 16, 4, . . .
8. 3, 0, −3, 0, 3, 0, . . .
9. 7.0, 7.5, 8.0, 8.5, . . .
10. 13, 7, 1, −5, . . .
11. Complete the conjecture based on the pattern you observe.

$3 + 4 + 5 = 4 \cdot 3$	$6 + 7 + 8 = 7 \cdot 3$	$9 + 10 + 11 = 10 \cdot 3$
$4 + 5 + 6 = 5 \cdot 3$	$7 + 8 + 9 = 8 \cdot 3$	$10 + 11 + 12 = 11 \cdot 3$
$5 + 6 + 7 = 6 \cdot 3$	$8 + 9 + 10 = 9 \cdot 3$	$11 + 12 + 13 = 12 \cdot 3$

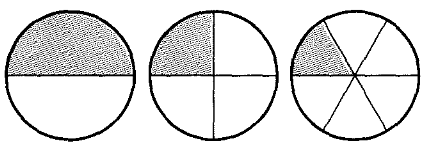
Conjecture: The sum of any three consecutive integers is __?__.

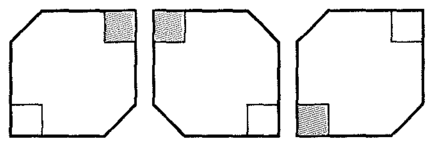
Practice and Applications

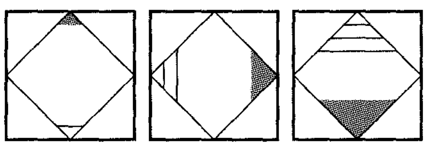
STUDENT HELP

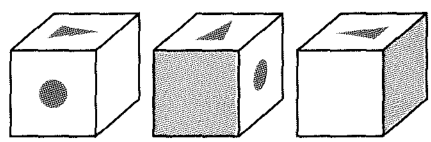
Extra Practice to help you master skills is on p. 803.

SKETCHING VISUAL PATTERNS **Sketch the next figure in the pattern.**

12. 

13. 

14. 

15. 

STUDENT HELP

HOMEWORK HELP

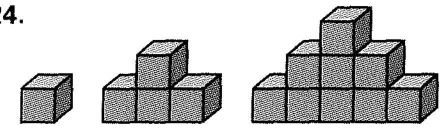
Example 1: Exs. 12–15, 24, 25
Example 2: Exs. 16–23, 26–28
Example 3: Exs. 29–33
Example 4: Exs. 34–39
Example 5: Exs. 40, 41
Example 6: Exs. 42, 43

DESCRIBING NUMBER PATTERNS **Describe a pattern in the sequence of numbers. Predict the next number.**

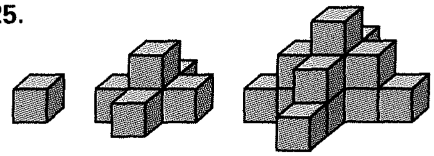
16. 1, 4, 7, 10, . . .
17. 10, 5, 2.5, 1.25, . . .
18. 1, 11, 121, 1331, . . .
19. 5, 0, −5, −10, . . .
20. 7, 9, 13, 19, 27, . . .
21. 1, 3, 6, 10, 15, . . .
22. 256, 16, 4, 2, . . .
23. 1.1, 1.01, 1.001, 1.0001, . . .

VISUALIZING PATTERNS The first three objects in a pattern are shown. How many blocks are in the next object?

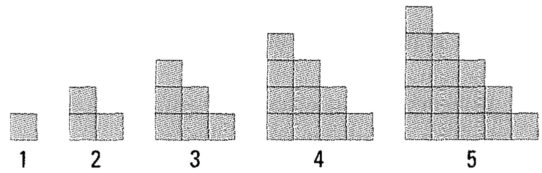
24.



25.



MAKING PREDICTIONS In Exercises 26–28, use the pattern from Example 1 shown below. Each square is 1 unit × 1 unit.



26. Find the distance around each figure. Organize your results in a table.

27. Use your table to describe a pattern in the distances.

28. Predict the distance around the twentieth figure in this pattern.

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with Exs. 29–31.

MAKING CONJECTURES Complete the conjecture based on the pattern you observe in the specific cases.

29. Conjecture: The sum of any two odd numbers is __?__.

$1 + 1 = 2$ $\quad$ $7 + 11 = 18$

$1 + 3 = 4$ $\quad$ $13 + 19 = 32$

$3 + 5 = 8$ $\quad$ $201 + 305 = 506$

30. Conjecture: The product of any two odd numbers is __?__.

$1 \times 1 = 1$ $\quad$ $7 \times 11 = 77$

$1 \times 3 = 3$ $\quad$ $13 \times 19 = 247$

$3 \times 5 = 15$ $\quad$ $201 \times 305 = 61{,}305$

31. Conjecture: The product of a number $(n - 1)$ and the number $(n + 1)$ is always equal to __?__.

$3 \cdot 5 = 4^2 - 1$ $\quad$ $6 \cdot 8 = 7^2 - 1$

$4 \cdot 6 = 5^2 - 1$ $\quad$ $7 \cdot 9 = 8^2 - 1$

$5 \cdot 7 = 6^2 - 1$ $\quad$ $8 \cdot 10 = 9^2 - 1$

CALCULATOR Use a calculator to explore the pattern. Write a conjecture based on what you observe.

32. $101 \times 34 =$ __?__

$101 \times 25 =$ __?__

$101 \times 97 =$ __?__

$101 \times 49 =$ __?__

33. $11 \times 11 =$ __?__

$111 \times 111 =$ __?__

$1111 \times 1111 =$ __?__

$11{,}111 \times 11{,}111 =$ __?__

FINDING COUNTEREXAMPLES Show the conjecture is false by finding a counterexample.

34. All prime numbers are odd.

35. The sum of two numbers is always greater than the larger number.

36. If the product of two numbers is even, then the two numbers must be even.

37. If the product of two numbers is positive, then the two numbers must both be positive.

38. The square root of a number x is always less than x.

39. If m is a nonzero integer, then $\frac{m+1}{m}$ is always greater than 1.

GOLDBACH'S CONJECTURE In Exercises 40 and 41, use the list of the first prime numbers given below.

$$\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, \ldots\}$$

40. Show that Goldbach's Conjecture (see page 5) is true for the even numbers from 20 to 40 by writing each even number as a sum of two primes.

41. Show that the following conjecture is not true by finding a counterexample.

Conjecture: All *odd* numbers can be expressed as the sum of two primes.

FOCUS ON CAREERS

LABORATORY TECHNOLOGIST
Laboratory technologists study microscopic cells, such as bacteria. The time it takes for a population of bacteria to double (the *doubling period*) may be as short as 20 min.

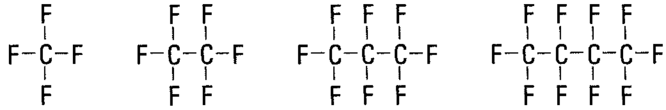
CAREER LINK
www.mcdougallittell.com

42. BACTERIA GROWTH Suppose you are studying bacteria in biology class. The table shows the number of bacteria after n doubling periods.

n (periods)	0	1	2	3	4	5
Billions of bacteria	3	6	12	24	48	96

Your teacher asks you to predict the number of bacteria after 8 doubling periods. What would your prediction be?

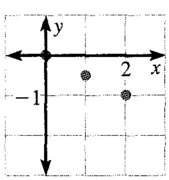
43. SCIENCE CONNECTION Diagrams and formulas for four molecular compounds are shown. Draw a diagram and write the formula for the next two compounds in the pattern.



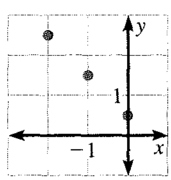
CF_4 C_2F_6 C_3F_8 C_4F_{10}

USING ALGEBRA Find a pattern in the coordinates of the points. Then use the pattern to find the y-coordinate of the point (3, ?).

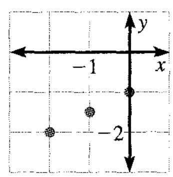
44.



45.



46.



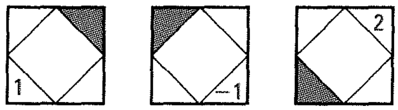
Test Preparation

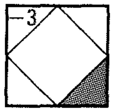
47. MULTIPLE CHOICE Which number is next in the sequence?

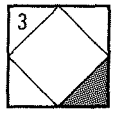
45, 90, 135, 180, . . .

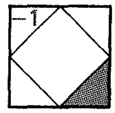
(A) 205 (B) 210 (C) 215 (D) 220 (E) 225

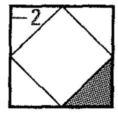
48. MULTIPLE CHOICE What is the next figure in the pattern?

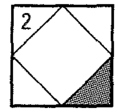


(A)


(B)


(C)


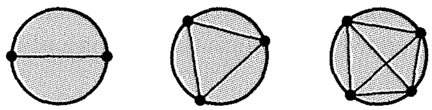
(D)


(E)


★ Challenge

DIVIDING A CIRCLE **In Exercises 49–51, use the information about regions in a circle formed by connecting points on the circle.**

If you draw points on a circle and then connect every pair of points, the circle is divided into a number of regions, as shown.



2 regions **4 regions** **?**

49. Copy and complete the table for the case of 4 and 5 points.

Number of points on circle	2	3	4	5	6
Maximum number of regions	2	4	?	?	?

50. Make a conjecture about the relationship between the number of points on the circle and number of regions in the circle.

EXTRA CHALLENGE
www.mcdougallittell.com

51. Test your conjecture for the case of 6 points. What do you notice?

MIXED REVIEW

PLOTTING POINTS **Plot in a coordinate plane.** (Skills Review, p. 792, for 1.2)

52. (5, 2) **53.** (3, −8) **54.** (−4, −6) **55.** (1, −10)

56. (−2, 7) **57.** (−3, 8) **58.** (4, −1) **59.** (−2, −6)

EVALUATING EXPRESSIONS **Evaluate the expression.** (Skills Review, p. 786)

60. 3^2 **61.** 5^2 **62.** $(-4)^2$ **63.** -7^2

64. $3^2 + 4^2$ **65.** $5^2 + 12^2$ **66.** $(-2)^2 + 2^2$ **67.** $(-10)^2 + (-5)^2$

FINDING A PATTERN **Write the next number in the sequence.** (Review 1.1)

68. 1, 5, 25, 125, . . . **69.** 4.4, 40.4, 400.4, 4000.4, . . .

70. 3, 7, 11, 15, . . . **71.** −1, +1, −2, +2, −3, . . .

GUIDED PRACTICE

Vocabulary Check ✓

1. Describe what each of these symbols means: $\overline{PQ}$, $\overrightarrow{PQ}$, $\overleftrightarrow{PQ}$, $\overrightarrow{QP}$.

Concept Check ✓

2. Sketch a line that contains point R between points S and T. Which of the following are true?

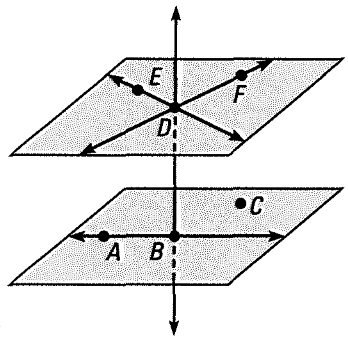
   A. $\overrightarrow{SR}$ is the same as $\overrightarrow{ST}$. **B.** $\overleftrightarrow{SR}$ is the same as $\overleftrightarrow{RT}$.

   C. $\overrightarrow{RS}$ is the same as $\overrightarrow{TS}$. **D.** $\overrightarrow{RS}$ and $\overrightarrow{RT}$ are opposite rays.

   E. $\overline{ST}$ is the same as $\overline{TS}$. **F.** $\overrightarrow{ST}$ is the same as $\overrightarrow{TS}$.

Skill Check ✓

Decide whether the statement is *true* or *false*.



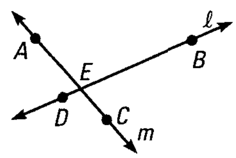
3. Points A, B, and C are collinear.
4. Points A, B, and C are coplanar.
5. Point F lies on $\overleftrightarrow{DE}$.
6. $\overleftrightarrow{DE}$ lies on plane DEF.
7. $\overleftrightarrow{BD}$ and $\overleftrightarrow{DE}$ intersect.
8. $\overleftrightarrow{BD}$ is the intersection of plane ABC and plane DEF.

PRACTICE AND APPLICATIONS

STUDENT HELP

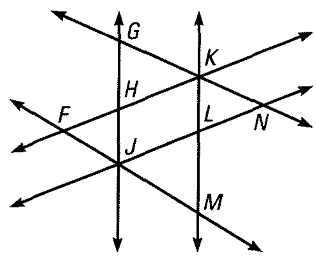
Extra Practice to help you master skills is on p. 803.

EVALUATING STATEMENTS **Decide whether the statement is *true* or *false*.**



9. Point A lies on line ℓ.
10. A, B, and C are collinear.
11. Point B lies on line ℓ.
12. A, B, and C are coplanar.
13. Point C lies on line m.
14. D, E, and B are collinear.
15. Point D lies on line m.
16. D, E, and B are coplanar.

NAMING COLLINEAR POINTS **Name a point that is collinear with the given points.**



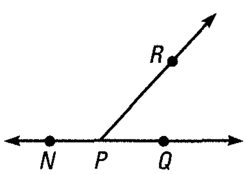
17. F and H
18. G and K
19. K and L
20. M and J
21. J and N
22. K and H
23. H and G
24. J and F

STUDENT HELP

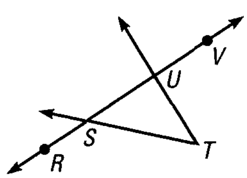
HOMEWORK HELP
Example 1: Exs. 9–43
Example 2: Exs. 44–49
Example 3: Exs. 50, 51
Example 4: Exs. 52–67

NAMING NONCOLLINEAR POINTS **Name three points in the diagram that are not collinear.**

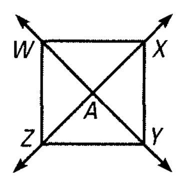
25.



26.



27.



NAMING COPLANAR POINTS Name a point that is coplanar with the given points.

28. A, B, and C

29. D, C, and F

30. G, A, and D

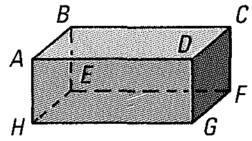
31. E, F, and G

32. A, B, and H

33. B, C, and F

34. A, B, and F

35. B, C, and G



NAMING NONCOPLANAR POINTS Name all the points that are not coplanar with the given points.

36. N, K, and L

37. S, P, and M

38. P, Q, and N

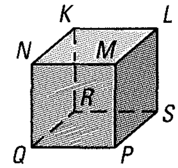
39. R, S, and L

40. P, Q, and R

41. R, K, and N

42. P, S, and K

43. Q, K, and L



COMPLETING DEFINITIONS Complete the sentence.

44. $\overline{AB}$ consists of the endpoints A and B and all the points on the line $\overleftrightarrow{AB}$ that lie __?__.

45. $\overrightarrow{CD}$ consists of the initial point C and all points on the line $\overleftrightarrow{CD}$ that lie __?__.

46. Two rays or segments are collinear if they __?__.

47. $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are opposite rays if __?__.

SKETCHING FIGURES Sketch the lines, segments, and rays.

48. Draw four points J, K, L, and M, no three of which are collinear. Then sketch $\overrightarrow{JK}$, $\overline{KL}$, $\overleftrightarrow{LM}$, and $\overrightarrow{MJ}$.

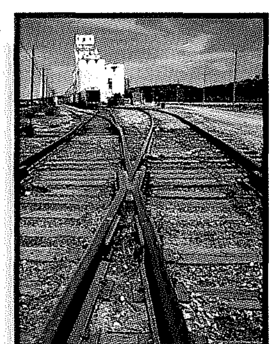
49. Draw five points P, Q, R, S, and T, no three of which are collinear. Then sketch $\overleftrightarrow{PQ}$, $\overleftrightarrow{RS}$, $\overline{QR}$, $\overline{ST}$, and $\overrightarrow{TP}$.

50. Draw two points, X and Y. Then sketch $\overleftrightarrow{XY}$. Add a point W between X and Y so that $\overrightarrow{WX}$ and $\overrightarrow{WY}$ are opposite rays.

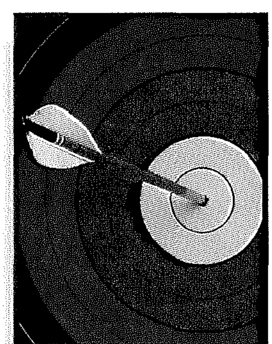
51. Draw two points, A and B. Then sketch $\overrightarrow{AB}$. Add a point C on the ray so that B is between A and C.

EVERYDAY INTERSECTIONS What kind of geometric intersection does the photograph suggest?

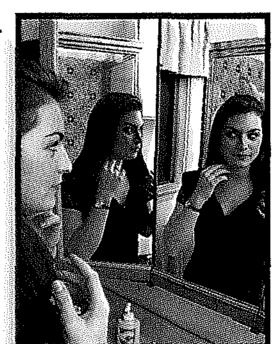
52.



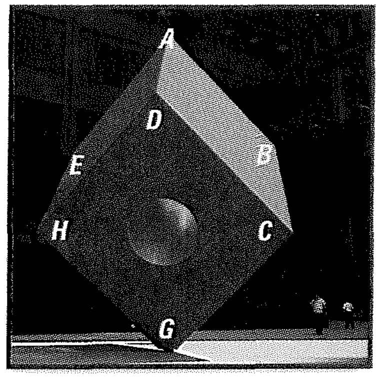
53.



54.



COMPLETING SENTENCES Fill in each blank with the appropriate response based on the points labeled in the photograph.



Red Cube, by sculptor Isamu Noguchi

55. $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ intersect at __?__.

56. $\overleftrightarrow{AD}$ and $\overleftrightarrow{AE}$ intersect at __?__.

57. $\overleftrightarrow{HG}$ and $\overleftrightarrow{DH}$ intersect at __?__.

58. Plane ABC and plane DCG intersect at __?__.

59. Plane GHD and plane DHE intersect at __?__.

60. Plane EAD and plane BCD intersect at __?__.

SKETCHING FIGURES Sketch the figure described.

61. Three points that are coplanar but not collinear.

62. Two lines that lie in a plane but do not intersect.

63. Three lines that intersect in a point and all lie in the same plane.

64. Three lines that intersect in a point but do not all lie in the same plane.

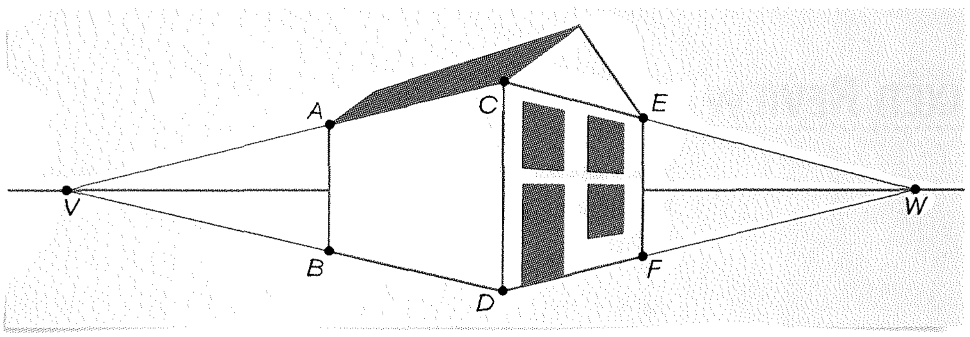
65. Two lines that intersect and another line that does not intersect either one.

66. Two planes that do not intersect.

67. Three planes that intersect in a line.

TWO-POINT PERSPECTIVE In Exercises 68–72, use the information and diagram below.

In *perspective drawing*, lines that do not intersect in real life are represented in a drawing by lines that appear to intersect at a point far away on the horizon. This point is called a *vanishing point.*



The diagram shows a drawing of a house with two vanishing points. You can use the vanishing points to draw the hidden parts of the house.

68. Name two lines that intersect at vanishing point V.

69. Name two lines that intersect at vanishing point W.

70. Trace the diagram. Draw $\overleftrightarrow{EV}$ and $\overleftrightarrow{AW}$. Label their intersection as G.

71. Draw $\overleftrightarrow{FV}$ and $\overleftrightarrow{BW}$. Label their intersection as H.

72. Draw the hidden edges of the house: $\overline{AG}$, $\overline{EG}$, $\overline{BH}$, $\overline{FH}$, and $\overline{GH}$.

GUIDED PRACTICE

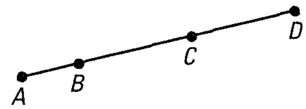
Vocabulary Check ✓

1. What is a *postulate*?

Concept Check ✓

2. Draw a sketch of three collinear points. Label them. Then write the Segment Addition Postulate for the points.

3. Use the diagram. How can you determine BD if you know BC and CD? if you know AB and AD?



Skill Check ✓

Find the distance between the two points.

4. $C(0, 0)$, $D(5, 2)$ **5.** $G(3, 0)$, $H(8, 10)$ **6.** $M(1, -3)$, $N(3, 5)$

7. $P(-8, -6)$, $Q(-3, 0)$ **8.** $S(7, 3)$, $T(1, -5)$ **9.** $V(-2, -6)$, $W(1, -2)$

Use the Distance Formula to decide whether $\overline{JK} \cong \overline{KL}$.

10. $J(3, -5)$, $K(-1, 2)$, $L(-5, -5)$

11. $J(0, -8)$, $K(4, 3)$, $L(-2, -7)$

12. $J(10, 2)$, $K(7, -3)$, $L(4, -8)$

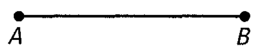
PRACTICE AND APPLICATIONS

STUDENT HELP

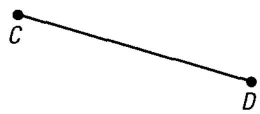
Extra Practice to help you master skills is on p. 803.

MEASUREMENT **Measure the length of the segment to the nearest millimeter.**

13.



14.

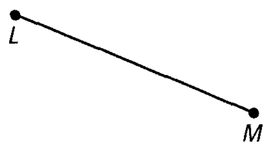


15.

16.

17.

18.



BETWEENNESS **Draw a sketch of the three collinear points. Then write the Segment Addition Postulate for the points.**

19. E is between D and F. **20.** H is between G and J.

21. M is between N and P. **22.** R is between Q and S.

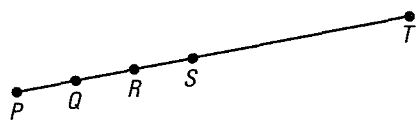
LOGICAL REASONING **In the diagram of the collinear points, $PT = 20$, $QS = 6$, and $PQ = QR = RS$. Find each length.**

23. QR **24.** RS

25. PQ **26.** ST

27. RP **28.** RT

29. SP **30.** QT



STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 13–18
Example 2: Exs. 19–33
Example 3: Exs. 34–43
Example 4: Exs. 44–54

USING ALGEBRA **Suppose M is between L and N. Use the Segment Addition Postulate to solve for the variable. Then find the lengths of $\overline{LM}$, $\overline{MN}$, and $\overline{LN}$.**

31. $LM = 3x + 8$
$MN = 2x - 5$
$LN = 23$

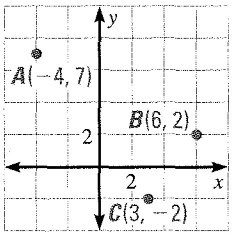
32. $LM = 7y + 9$
$MN = 3y + 4$
$LN = 143$

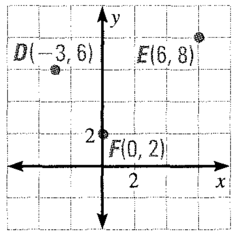
33. $LM = \frac{1}{2}z + 2$
$MN = 3z + \frac{3}{2}$
$LN = 5z + 2$

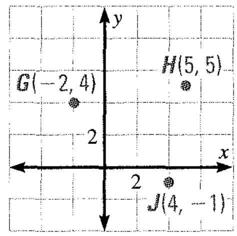
STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 34–36.

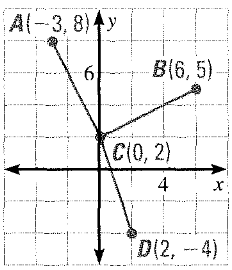
DISTANCE FORMULA **Find the distance between each pair of points.**

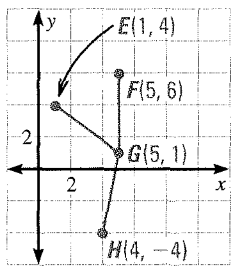
34.


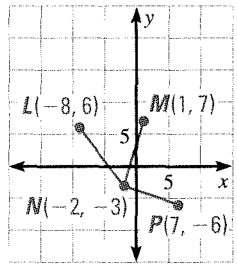
35.


36.


DISTANCE FORMULA **Find the lengths of the segments. Tell whether any of the segments have the same length.**

37.


38.


39.


CONGRUENCE **Use the Distance Formula to decide whether $\overline{PQ} \cong \overline{QR}$.**

40. $P(4, -4)$
$Q(1, -6)$
$R(-1, -3)$

41. $P(-1, -6)$
$Q(-8, 5)$
$R(3, -2)$

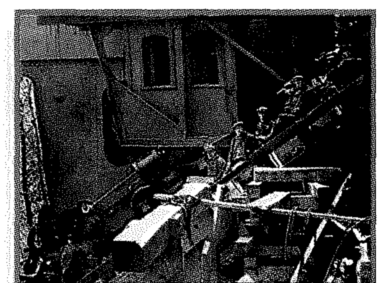
42. $P(5, 1)$
$Q(-5, -7)$
$R(-3, 6)$

43. $P(-2, 0)$
$Q(10, -14)$
$R(-4, -2)$

CAMBRIA INCLINE **In Exercises 44 and 45, use the information about the incline railway given below.**
In the days before automobiles were available, railways called "inclines" brought people up and down hills in many cities. In Johnstown, Pennsylvania, the Cambria Incline was reputedly the steepest in the world when it was completed in 1893. It rises about 514 feet vertically as it moves 734 feet horizontally.

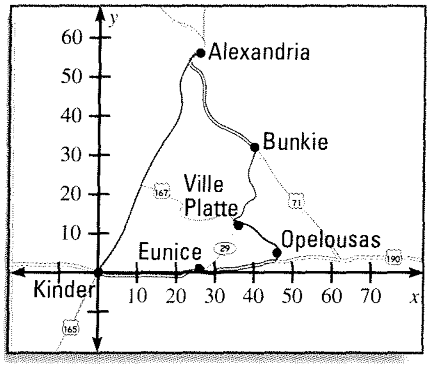
44. On graph paper, draw a coordinate plane and mark the axes using a scale that allows you to plot (0, 0) and (734, 514). Plot the points and connect them with a segment to represent the incline track.

45. Use the Distance Formula to estimate the length of the track.


Workers constructing the Cambria Incline

DRIVING DISTANCES In Exercises 46 and 47, use the map of cities in Louisiana shown below. Coordinates on the map are given in miles.

The coordinates of Alexandria, Kinder, Eunice, Opelousas, Ville Platte, and Bunkie are $A(26, 56)$, $K(0, 0)$, $E(26, 1)$, $O(46, 5)$, $V(36, 12)$, and $B(40, 32)$.



46. What is the shortest flying distance between Eunice and Alexandria?

47. Using only roads shown on the map, what is the approximate shortest driving distance between Eunice and Alexandria?

LONG-DISTANCE RATES In Exercises 48–52, find the distance between the two cities using the information given in the table, which is from a coordinate system used for calculating long-distance telephone rates.

Buffalo, NY	(5075, 2326)	Omaha, NE	(6687, 4595)
Chicago, IL	(5986, 3426)	Providence, RI	(4550, 1219)
Dallas, TX	(8436, 4034)	San Diego, CA	(9468, 7629)
Miami, FL	(8351, 527)	Seattle, WA	(6336, 8896)

48. Buffalo and Dallas

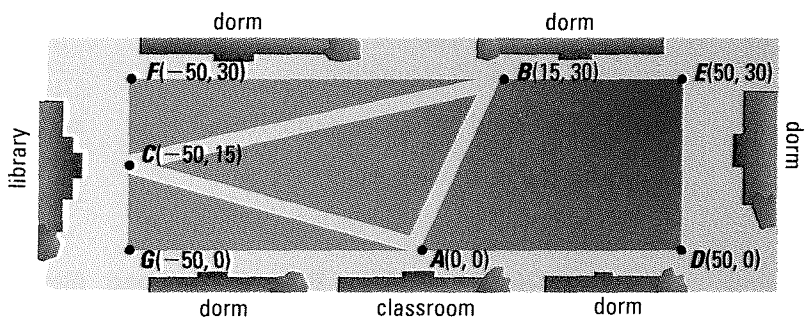
49. Chicago and Seattle

50. Miami and Omaha

51. Providence and San Diego

52. The long-distance coordinate system is measured in units of $\sqrt{0.1}$ mile. Convert the distances you found in Exs. 48–51 to miles.

CAMPUS PATHWAYS In Exercises 53 and 54, use the campus map below. Sidewalks around the edge of a campus quadrangle connect the buildings. Students sometimes take shortcuts by walking across the grass along the pathways shown. The coordinate system shown is measured in yards.



53. Find the distances from A to B, from B to C, and from C to A if you have to walk around the quadrangle along the sidewalks.

54. Find the distances from A to B, from B to C, and from C to A if you are able to walk across the grass along the pathways.

Test Preparation

55. **MULTIPLE CHOICE** Points K and L are on $\overline{AB}$. If $AK > BL$, then which statement must be true?

   (A) $AK < KB$ (B) $AL < LB$ (C) $AL > BK$

   (D) $KL < LB$ (E) $AL + BK > AB$

56. **MULTIPLE CHOICE** Suppose point M lies on $\overline{CD}$, $CM = 2 \cdot MD$, and $CD = 18$. What is the length of MD?

   (A) 3 (B) 6 (C) 9 (D) 12 (E) 36

★ Challenge

THREE-DIMENSIONAL DISTANCE In Exercises 57–59, use the following information to find the distance between the pair of points.

In a three-dimensional coordinate system, the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

EXTRA CHALLENGE
www.mcdougallittell.com

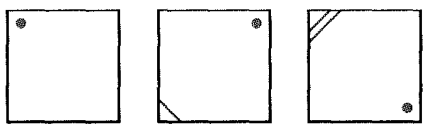
57. $P(0, 20, -32)$
    $Q(2, -10, -20)$

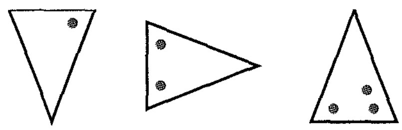
58. $A(-8, 15, -4)$
    $B(10, 1, -6)$

59. $F(4, -42, 60)$
    $G(-7, -11, 38)$

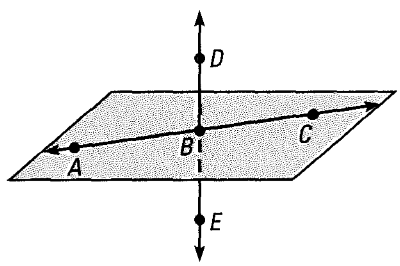
MIXED REVIEW

SKETCHING VISUAL PATTERNS Sketch the next figure in the pattern. (Review 1.1)

60. 

61. 

EVALUATING STATEMENTS Determine if the statement is *true* or *false*. (Review 1.2)



62. E lies on $\overleftrightarrow{BD}$.

63. E lies on $\overrightarrow{BD}$.

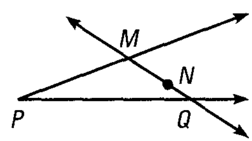
64. A, B, and D are collinear.

65. $\overrightarrow{BD}$ and $\overrightarrow{BE}$ are opposite rays.

66. B lies in plane ADC.

67. The intersection of $\overleftrightarrow{DE}$ and $\overleftrightarrow{AC}$ is B.

NAMING RAYS Name the ray described. (Review 1.2 for 1.4)



68. Name a ray that contains M.

69. Name a ray that has N as an endpoint.

70. Name two rays that intersect at P.

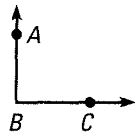
71. Name a pair of opposite rays.

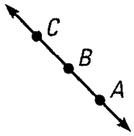
GUIDED PRACTICE

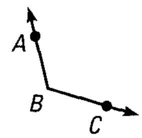
Vocabulary Check ✓

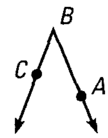
Match the angle with its classification.

A. acute **B.** obtuse **C.** right **D.** straight

1. 

2. 

3. 

4. 

Concept Check ✓

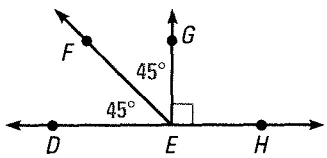
Use the diagram at the right to answer the questions. Explain your answers.

5. Is $\angle DEF \cong \angle FEG$?

6. Is $\angle DEG \cong \angle HEG$?

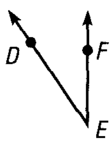
7. Are $\angle DEF$ and $\angle FEH$ adjacent?

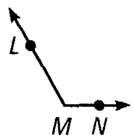
8. Are $\angle GED$ and $\angle DEF$ adjacent?

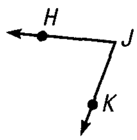


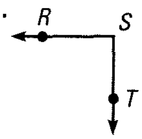
Skill Check ✓

Name the vertex and sides of the angle. Then estimate its measure.

9. 

10. 

11. 

12. 

Classify the angle as *acute, obtuse, right,* or *straight*.

13. $m\angle A = 180°$ **14.** $m\angle B = 90°$

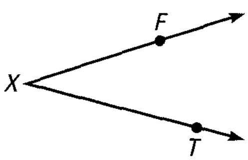
15. $m\angle C = 100°$ **16.** $m\angle D = 45°$

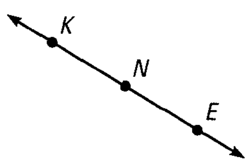
PRACTICE AND APPLICATIONS

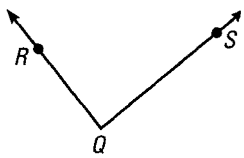
STUDENT HELP

Extra Practice to help you master skills is on pp. 803 and 804.

NAMING PARTS **Name the vertex and sides of the angle.**

17. 

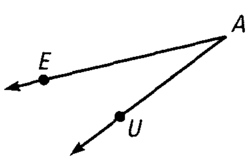
18. 

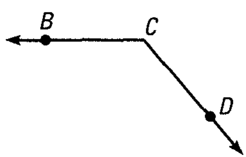
19. 

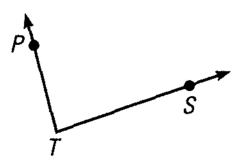
STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 17–22
Example 2: Exs. 23–34
Example 3: Exs. 35–43
Example 4: Exs. 38, 39

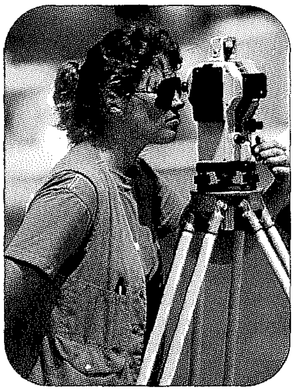
NAMING ANGLES **Write two names for the angle.**

20. 

21. 

22. 

FOCUS ON CAREERS

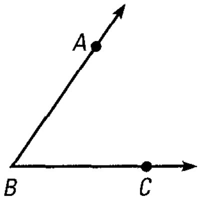


REAL LIFE **SURVEYOR**
Surveyors use a tool called a theodolite, which can measure angles to the nearest 1/3600 of a degree.

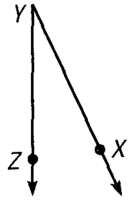
INTERNET **CAREER LINK**
www.mcdougallittell.com

MEASURING ANGLES **Copy the angle, extend its sides, and use a protractor to measure it to the nearest degree.**

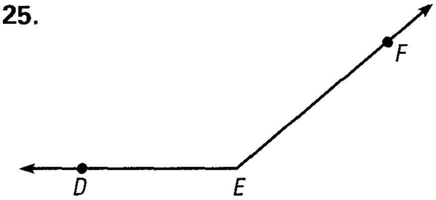
23.



24.

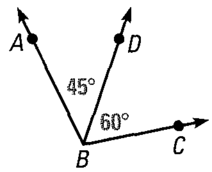


25.

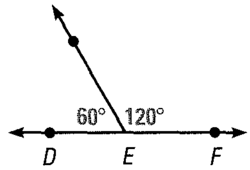


ANGLE ADDITION **Use the Angle Addition Postulate to find the measure of the unknown angle.**

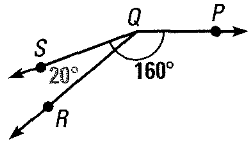
26. $m\angle ABC = \underline{?}$



27. $m\angle DEF = \underline{?}$



28. $m\angle PQR = \underline{?}$



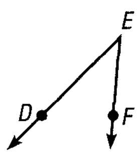
LOGICAL REASONING **Draw a sketch that uses all of the following information.**

D is in the interior of $\angle BAE$.	$m\angle BAC = 130°$
E is in the interior of $\angle DAF$.	$m\angle EAC = 100°$
F is in the interior of $\angle EAC$.	$m\angle BAD = m\angle EAF = m\angle FAC$

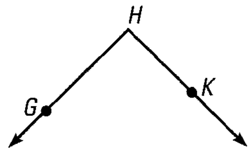
29. Find $m\angle FAC$.
30. Find $m\angle BAD$.
31. Find $m\angle FAB$.
32. Find $m\angle DAE$.
33. Find $m\angle FAD$.
34. Find $m\angle BAE$.

CLASSIFYING ANGLES **State whether the angle appears to be *acute, right, obtuse,* or *straight*. Then estimate its measure.**

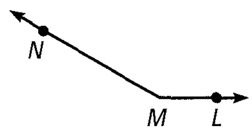
35.



36.



37.



LOGICAL REASONING **Draw five points, *A, B, C, D,* and *E* so that all three statements are true.**

38. $\angle DBE$ is a straight angle.
$\angle DBA$ is a right angle.
$\angle ABC$ is a straight angle.

39. C is in the interior of $\angle ADE$.
$m\angle ADC + m\angle CDE = 120°$.
$\angle CDB$ is a straight angle.

USING ALGEBRA **In a coordinate plane, plot the points and sketch $\angle ABC$. Classify the angle. Write the coordinates of a point that lies in the interior of the angle and the coordinates of a point that lies in the exterior of the angle.**

40. $A(3, -2)$, $B(5, -1)$, $C(4, -4)$

41. $A(5, -1)$, $B(3, -2)$, $C(4, -4)$

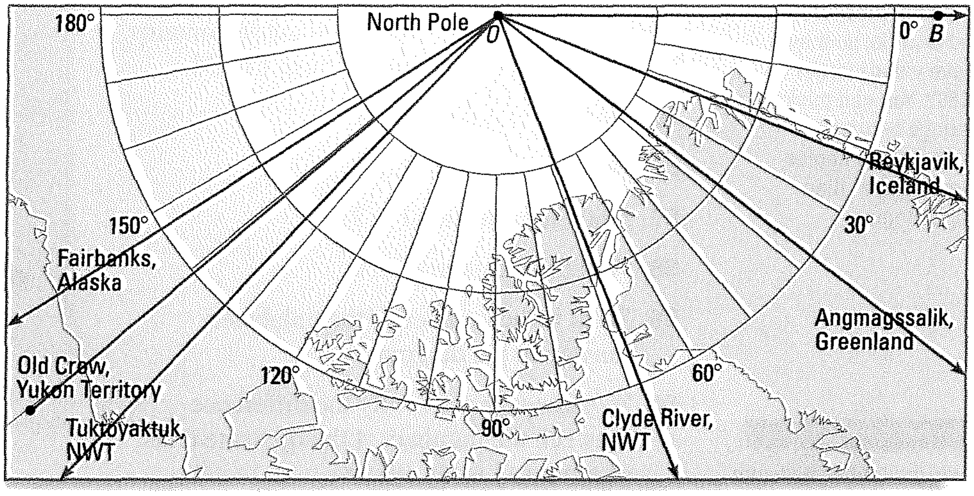
42. $A(5, -1)$, $B(3, -2)$, $C(0, -1)$

43. $A(-3, 1)$, $B(-2, 2)$, $C(-1, 4)$

GEOGRAPHY For each city on the polar map, estimate the measure of $\angle BOA$, where B is on the Prime Meridian (0° longitude), O is the North Pole, and A is the city.

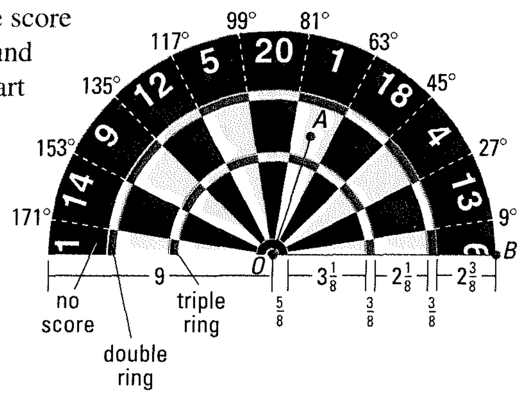
44. Clyde River, Canada **45.** Fairbanks, Alaska **46.** Angmagssalik, Greenland

47. Old Crow, Canada **48.** Reykjavik, Iceland **49.** Tuktoyaktuk, Canada



PLAYING DARTS In Exercises 50–53, use the following information to find the score for the indicated dart toss landing at point A.

A dartboard is 18 inches across. It is divided into twenty wedges of equal size. The score of a toss is indicated by numbers around the board. The score is doubled if a dart lands in the *double ring* and tripled if it lands in the *triple ring*. Only the top half of the dart board is shown.



50. $m\angle BOA = 160°; AO = 3$ in.

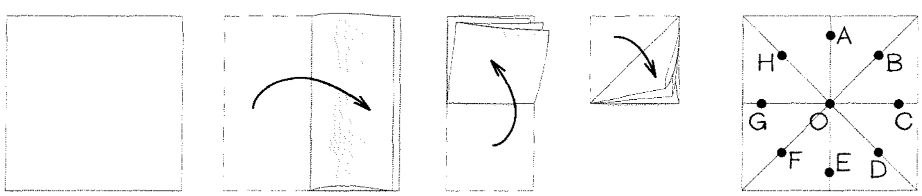
51. $m\angle BOA = 35°; AO = 4$ in.

52. $m\angle BOA = 60°; AO = 5$ in.

53. $m\angle BOA = 90°; AO = 6.5$ in.

Test Preparation

54. MULTI-STEP PROBLEM Use a piece of paper folded in half three times and labeled as shown.



a. Name eight congruent acute angles.

b. Name eight right angles.

c. Name eight congruent obtuse angles.

d. Name two adjacent angles that combine to form a straight angle.

★ **Challenge**

AIRPORT RUNWAYS In Exercises 55–60, use the diagram of Ronald Reagan Washington National Airport and the information about runway numbering on page 1.

An airport runway is named by dividing its *bearing* (the angle measured clockwise from due north) by 10. Because a full circle contains 360°, runway numbers range from 1 to 36.

STUDENT HELP

HOMEWORK HELP
Bearings are measured around a circle, so they can have values larger than 180°. You can think of bearings between 180° and 360° as angles that are "bigger" than a straight angle.

55. Find the measure of $\angle 1$.

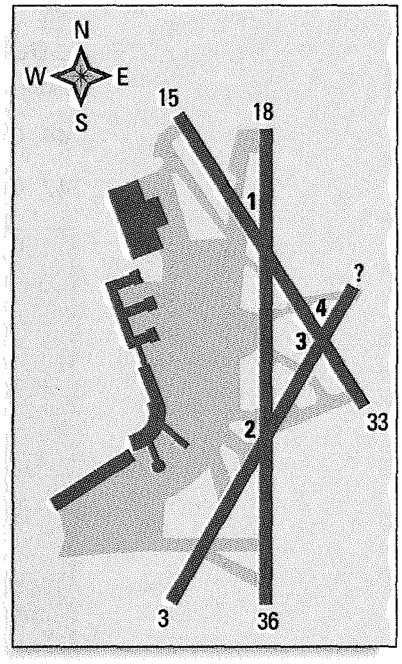
56. Find the measure of $\angle 2$.

57. Find the measure of $\angle 3$.

58. Find the measure of $\angle 4$.

59. What is the number of the unlabeled runway in the diagram?

60. *Writing* Explain why the difference between the numbers at the opposite ends of a runway is always 18.



EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

STUDENT HELP

Skills Review
For help solving equations, see p.790.

USING ALGEBRA Solve for *x*. (Skills Review, p. 790, for 1.5)

61. $\frac{x+3}{2} = 3$ **62.** $\frac{5+x}{2} = 5$ **63.** $\frac{x+4}{2} = -4$

64. $\frac{-8+x}{2} = 12$ **65.** $\frac{x+7}{2} = -10$ **66.** $\frac{-9+x}{2} = -7$

67. $\frac{x+(-1)}{2} = 7$ **68.** $\frac{8+x}{2} = -1$ **69.** $\frac{x+(-3)}{2} = -4$

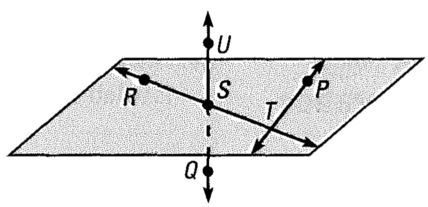
EVALUATING STATEMENTS Decide whether the statement is *true* or *false*. (Review 1.2)

70. U, S, and Q are collinear.

71. T, Q, S, and P are coplanar.

72. $\overleftrightarrow{UQ}$ and $\overleftrightarrow{PT}$ intersect.

73. $\overrightarrow{SR}$ and $\overrightarrow{TS}$ are opposite rays.



DISTANCE FORMULA Find the distance between the two points. (Review 1.3 for 1.5)

74. $A(3, 10)$, $B(-2, -2)$ **75.** $C(0, 8)$, $D(-8, 3)$

76. $E(-3, 11)$, $F(4, 4)$ **77.** $G(10, -2)$, $H(0, 9)$

78. $J(5, 7)$, $K(7, 5)$ **79.** $L(0, -3)$, $M(-3, 0)$

Folding Bisectors

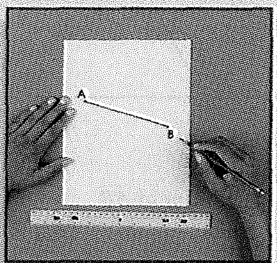
GROUP ACTIVITY
Work with a partner.

MATERIALS
- rulers
- paper
- protractor
- pencils

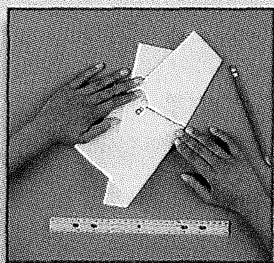
QUESTION **How can you divide a segment or an angle into two equal parts?**

You can fold a piece of paper so that one half of a segment or angle lies exactly on the other half.

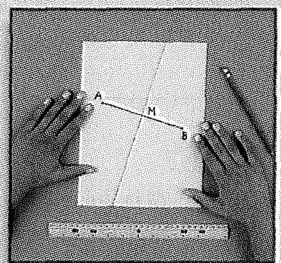
EXPLORING THE CONCEPT: SEGMENT BISECTOR



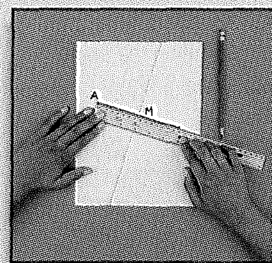
1. On a piece of paper, draw $\overline{AB}$.



2. Fold the paper so that B is on top of A.

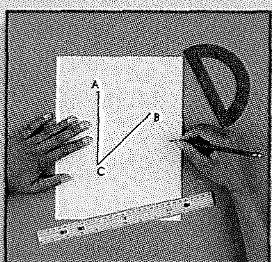


3. Label the point where the fold intersects $\overline{AB}$ as point M.

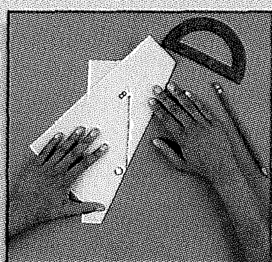


4. Use a ruler to measure $\overline{AM}$ and $\overline{MB}$.

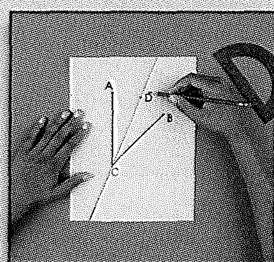
EXPLORING THE CONCEPT: ANGLE BISECTOR



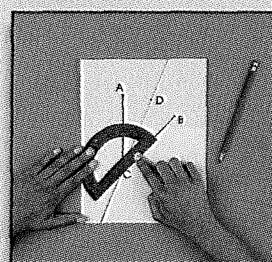
5. On a piece of paper, draw $\angle ACB$.



6. Fold the paper so $\overrightarrow{CB}$ is on top of $\overrightarrow{CA}$.



7. Draw any point on the fold and label the point D.



8. Use a protractor to measure $\angle ACD$ and $\angle BCD$.

DRAWING CONCLUSIONS

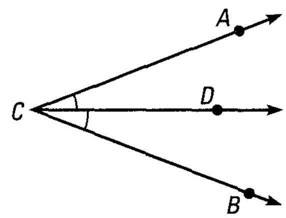
1. What do you notice about the segments you measured in Step 4?
2. What do you notice about the angles you measured in Step 8?

EXTENSION

CRITICAL THINKING Is it possible to fold congruent angles from a straight angle if you are given the vertex of the angle? Explain.

GOAL 2 BISECTING AN ANGLE

An **angle bisector** is a ray that divides an angle into two adjacent angles that are congruent. In the diagram at the right, the ray $\overrightarrow{CD}$ bisects $\angle ABC$ because it divides the angle into two congruent angles, $\angle ACD$ and $\angle BCD$.



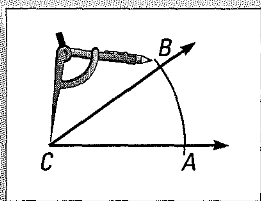
$m\angle ACD = m\angle BCD$

In this book, matching *congruence arcs* identify congruent angles in diagrams.

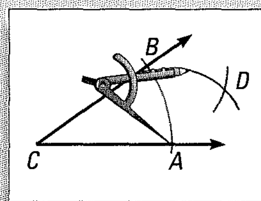
ACTIVITY
Construction

Angle Bisector

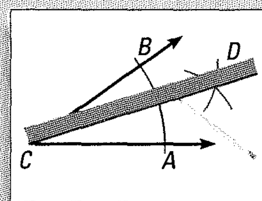
Use the following steps to construct an angle bisector of $\angle C$.



1. Place the compass point at C. Draw an arc that intersects both sides of the angle. Label the intersections A and B.



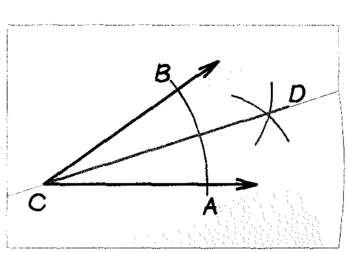
2. Place the compass point at A. Draw an arc. Then place the compass point at B. Using the same compass setting, draw another arc.



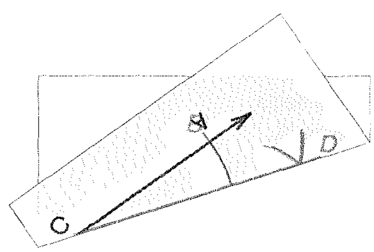
3. Label the intersection D. Use a straightedge to draw a ray through C and D. This is the angle bisector.

After you have constructed an angle bisector, you should check that it divides the original angle into two congruent angles. One way to do this is to use a protractor to check that the angles have the same measure.

Another way is to fold the piece of paper along the angle bisector. When you hold the paper up to a light, you should be able to see that the sides of the two angles line up, which implies that the angles are congruent.



Fold on $\overrightarrow{CD}$.


The sides of angles $\angle BCD$ and $\angle ACD$ line up.

GUIDED PRACTICE

Vocabulary Check ✓

1. What kind of geometric figure is an *angle bisector*?

Concept Check ✓

2. How do you indicate congruent segments in a diagram? How do you indicate congruent angles in a diagram?

3. What is the simplified form of the Midpoint Formula if one of the endpoints of a segment is $(0, 0)$ and the other is (x, y)?

Skill Check ✓

Find the coordinates of the midpoint of a segment with the given endpoints.

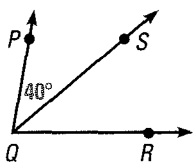
4. $A(5, 4)$, $B(-3, 2)$ **5.** $A(-1, -9)$, $B(11, -5)$ **6.** $A(6, -4)$, $B(1, 8)$

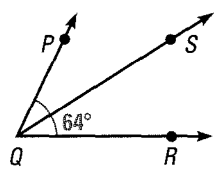
Find the coordinates of the other endpoint of a segment with the given endpoint and midpoint *M*.

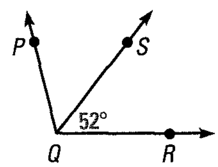
7. $C(3, 0)$, $M(3, 4)$ **8.** $D(5, 2)$, $M(7, 6)$ **9.** $E(-4, 2)$, $M(-3, -2)$

10. Suppose $m\angle JKL$ is 90°. If the ray $\overrightarrow{KM}$ bisects $\angle JKL$, what are the measures of $\angle JKM$ and $\angle LKM$?

$\overrightarrow{QS}$ is the angle bisector of $\angle PQR$. Find the two angle measures not given in the diagram.

11.


12.


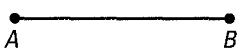
13.


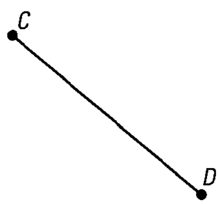
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 804.

CONSTRUCTION **Use a ruler to measure and redraw the line segment on a piece of paper. Then use construction tools to construct a segment bisector.**

14.


15.


16.


STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 17–24
Example 2: Exs. 25–30
Example 3: Exs. 37–42
Example 4: Exs. 37–42
Example 5: Exs. 44–49

FINDING THE MIDPOINT **Find the coordinates of the midpoint of a segment with the given endpoints.**

17. $A(0, 0)$, $B(-8, 6)$ **18.** $J(-1, 7)$, $K(3, -3)$ **19.** $C(10, 8)$, $D(-2, 5)$ **20.** $P(-12, -9)$, $Q(2, 10)$

21. $S(0, -8)$, $T(-6, 14)$ **22.** $E(4, 4)$, $F(4, -18)$ **23.** $V(-1.5, 8)$, $W(0.25, -1)$ **24.** $G(-5.5, -6.1)$, $H(-0.5, 9.1)$

USING ALGEBRA Find the coordinates of the other endpoint of a segment with the given endpoint and midpoint *M*.

25. $R(2, 6)$
$M(-1, 1)$

26. $T(-8, -1)$
$M(0, 3)$

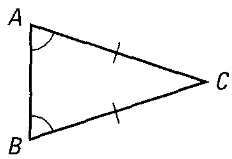
27. $W(3, -12)$
$M(2, -1)$

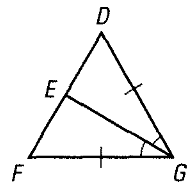
28. $Q(-5, 9)$
$M(-8, -2)$

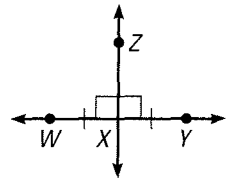
29. $A(6, 7)$
$M(10, -7)$

30. $D(-3.5, -6)$
$M(1.5, 4.5)$

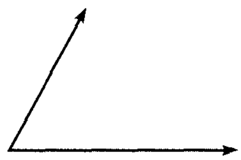
RECOGNIZING CONGRUENCE Use the marks on the diagram to name the congruent segments and congruent angles.

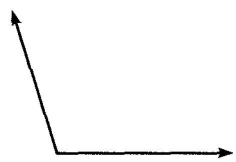
31. 

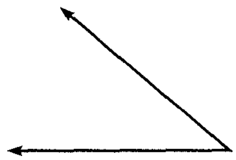
32. 

33. 

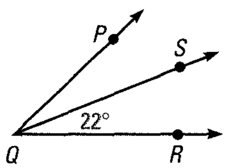
CONSTRUCTION Use a protractor to measure and redraw the angle on a piece of paper. Then use construction tools to find the angle bisector.

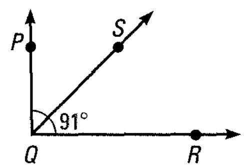
34. 

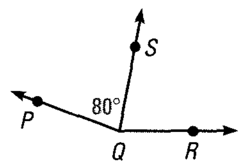
35. 

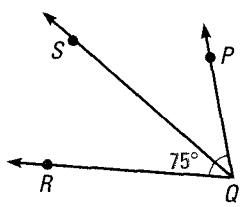
36. 

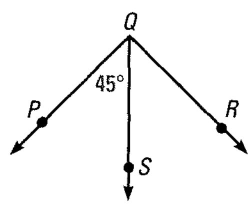
ANALYZING ANGLE BISECTORS $\overrightarrow{QS}$ is the angle bisector of $\angle PQR$. Find the two angle measures not given in the diagram.

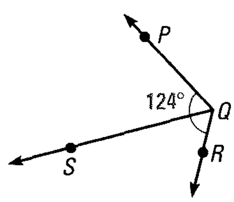
37. 

38. 

39. 

40. 

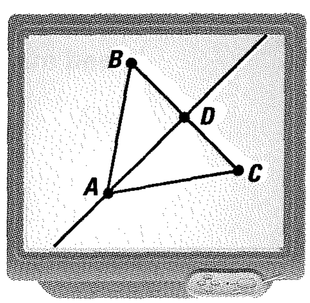
41. 

42. 

STUDENT HELP

INTERNET SOFTWARE HELP
Visit our Web site www.mcdougallittell.com to see instructions for several software applications.

43. TECHNOLOGY Use geometry software to draw a triangle. Construct the angle bisector of one angle. Then find the midpoint of the opposite side of the triangle. Change your triangle and observe what happens.



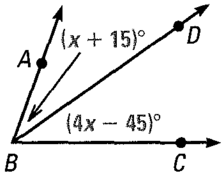
Does the angle bisector *always* pass through the midpoint of the opposite side? Does it *ever* pass through the midpoint?

STUDENT HELP

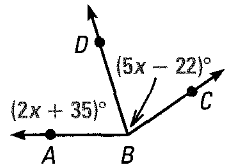
INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with Ex. 44–49.

USING ALGEBRA $\overrightarrow{BD}$ **bisects** $\angle ABC$. **Find the value of** x.

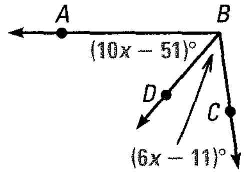
44.



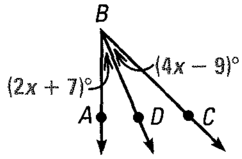
45.



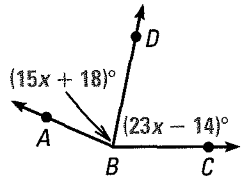
46.



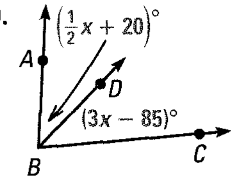
47.



48.



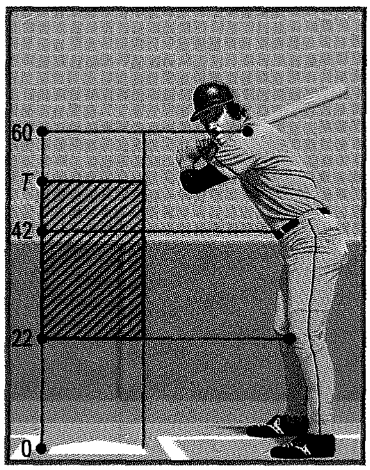
49.



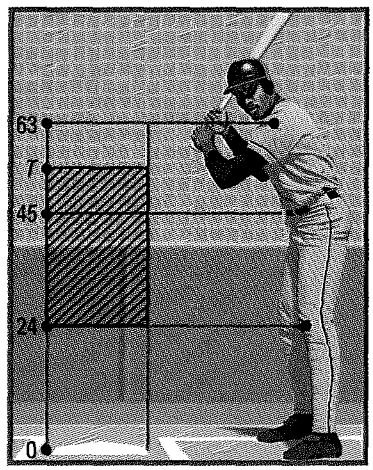
STRIKE ZONE In Exercises 50 and 51, use the information below. For each player, find the coordinate of *T*, a point on the top of the strike zone. In baseball, the "strike zone" is the region a baseball needs to pass through in order for an umpire to declare it a strike if it is not hit. The *top of the strike zone* is a horizontal plane passing through the midpoint between the top of the hitter's shoulders and the top of the uniform pants when the player is in a batting stance.

▶ Source: Major League Baseball

50.

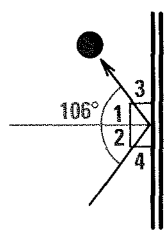


51.

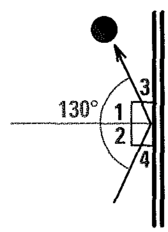


AIR HOCKEY When an air hockey puck is hit into the sideboards, it bounces off so that $\angle 1$ and $\angle 2$ are congruent. Find $m\angle 1$, $m\angle 2$, $m\angle 3$, and $m\angle 4$.

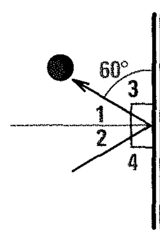
52.



53.

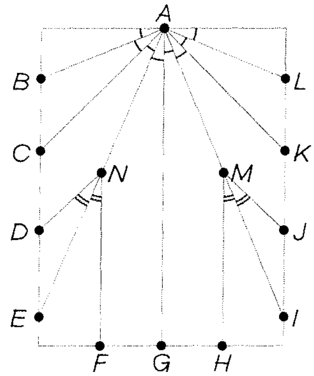


54.



55. **PAPER AIRPLANES** The diagram represents an unfolded piece of paper used to make a paper airplane. The segments represent where the paper was folded to make the airplane.

Using the diagram, name as many pairs of congruent segments and as many congruent angles as you can.



56. *Writing* Explain, in your own words, how you would divide a line segment into four congruent segments using a compass and straightedge. Then explain how you could do it using the Midpoint Formula.

57. **MIDPOINT FORMULA REVISITED** Another version of the Midpoint Formula, for $A(x_1, y_1)$ and $B(x_2, y_2)$, is

$$M\left[x_1 + \frac{1}{2}(x_2 - x_1), y_1 + \frac{1}{2}(y_2 - y_1)\right].$$

Redo Exercises 17–24 using this version of the Midpoint Formula. Do you get the same answers as before? Use algebra to explain why the formula above is equivalent to the one in the lesson.

Test Preparation

58. **MULTI-STEP PROBLEM** Sketch a triangle with three sides of different lengths.

a. Using construction tools, find the midpoints of all three sides and the angle bisectors of all three angles of your triangle.

b. Determine whether or not the angle bisectors pass through the midpoints.

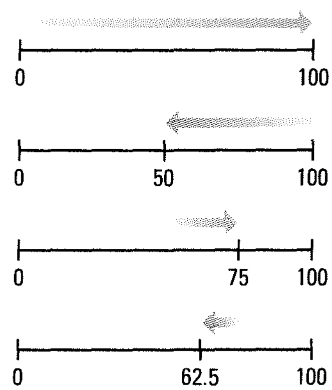
c. *Writing* Write a brief paragraph explaining your results. Determine if your results would be different if you used a different kind of triangle.

★ Challenge

INFINITE SERIES A football team practices running back and forth on the field in a special way. First they run from one end of the 100 yd field to the other. Then they turn around and run half the previous distance. Then they turn around again and run half the previous distance, and so on.

59. Suppose the athletes continue the running drill with smaller and smaller distances. What is the coordinate of the point that they approach?

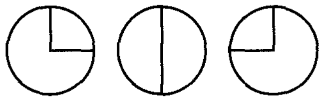
60. What is the total distance that the athletes cover?

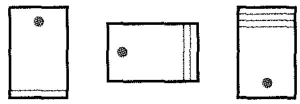


EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

SKETCHING VISUAL PATTERNS Sketch the next figure in the pattern. (Review 1.1)

61. 

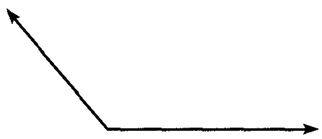
62. 

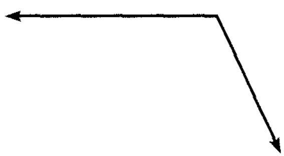
DISTANCE FORMULA Find the distance between the two points. (Review 1.3)

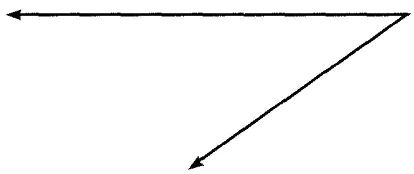
63. $A(3, 12), B(-5, -1)$
64. $C(-6, 9), D(-2, -7)$
65. $E(8, -8), F(2, 14)$
66. $G(3, -8), H(0, -2)$
67. $J(-4, -5), K(5, -1)$
68. $L(-10, 1), M(-4, 9)$

MEASURING ANGLES Use a protractor to find the measure of the angle. (Review 1.4 for 1.6)

69. 

70. 

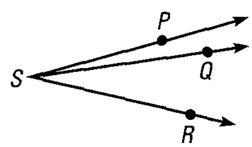
71. 

72. 

QUIZ 2

Self-Test for Lessons 1.4 and 1.5

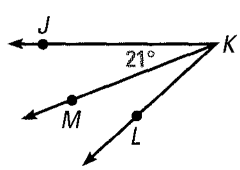
1. State the Angle Addition Postulate for the three angles shown at the right. (Lesson 1.4)



In a coordinate plane, plot the points and sketch $\angle DEF$. Classify the angle. Write the coordinates of a point that lies in the interior of the angle and the coordinates of a point that lies in the exterior of the angle. (Lesson 1.4)

2. $D(-2, 3)$ $E(4, -3)$ $F(2, 6)$
3. $D(-6, -3)$ $E(0, -5)$ $F(8, -5)$
4. $D(-1, 8)$ $E(-4, 0)$ $F(4, 0)$
5. $D(1, 10)$ $E(1, 1)$ $F(8, 1)$

6. In the diagram, $\overrightarrow{KM}$ is the angle bisector of $\angle JKL$. Find $m\angle MKL$ and $m\angle JKL$. (Lesson 1.5)



GUIDED PRACTICE

Vocabulary Check ✓

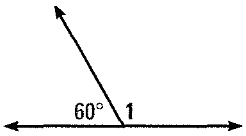
1. Explain the difference between *complementary angles* and *supplementary angles.*

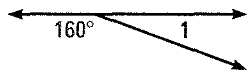
Concept Check ✓

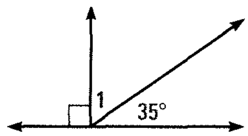
2. Sketch examples of acute vertical angles and obtuse vertical angles.
3. Sketch examples of adjacent congruent complementary angles and adjacent congruent supplementary angles.

Skill Check ✓

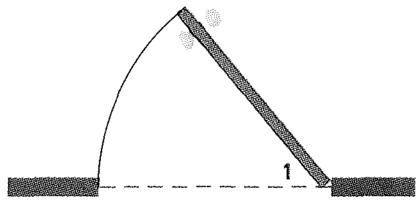
FINDING ANGLE MEASURES **Find the measure of ∠1.**

4. 

5. 

6. 

7. OPENING A DOOR The figure shows a doorway viewed from above. If you open the door so that the measure of ∠1 is 50°, how many more degrees would you have to open the door so that the angle between the wall and the door is 90°?

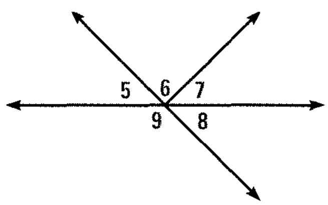


PRACTICE AND APPLICATIONS

STUDENT HELP

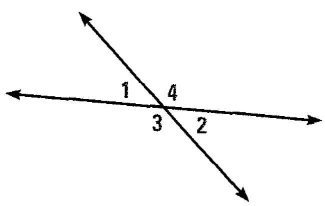
Extra Practice to help you master skills is on p. 804.

IDENTIFYING ANGLE PAIRS **Use the figure at the right.**



8. Are ∠5 and ∠6 a linear pair?
9. Are ∠5 and ∠9 a linear pair?
10. Are ∠5 and ∠8 a linear pair?
11. Are ∠5 and ∠8 vertical angles?
12. Are ∠5 and ∠7 vertical angles?
13. Are ∠9 and ∠6 vertical angles?

EVALUATING STATEMENTS **Decide whether the statement is *always, sometimes,* or *never* true.**



14. If $m\angle 1 = 40°$, then $m\angle 2 = 140°$.
15. If $m\angle 4 = 130°$, then $m\angle 2 = 50°$.
16. ∠1 and ∠4 are congruent.
17. $m\angle 2 + m\angle 3 = m\angle 1 + m\angle 4$
18. $\angle 2 \cong \angle 1$
19. $m\angle 2 = 90° - m\angle 3$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 8–13
Example 2: Exs. 14–27
Example 3: Exs. 28–36
Example 4: Exs. 37–40
Example 5: Exs. 41, 42
Example 6: Exs. 43, 44

FINDING ANGLE MEASURES **Use the figure at the right.**

20. If $m\angle 6 = 72°$, then $m\angle 7 =$ __?__.

21. If $m\angle 8 = 80°$, then $m\angle 6 =$ __?__.

22. If $m\angle 9 = 110°$, then $m\angle 8 =$ __?__.

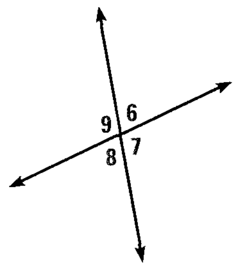
23. If $m\angle 9 = 123°$, then $m\angle 7 =$ __?__.

24. If $m\angle 7 = 142°$, then $m\angle 8 =$ __?__.

25. If $m\angle 6 = 13°$, then $m\angle 9 =$ __?__.

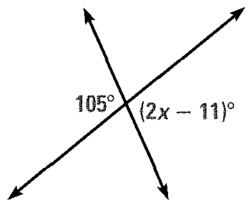
26. If $m\angle 9 = 170°$, then $m\angle 6 =$ __?__.

27. If $m\angle 8 = 26°$, then $m\angle 7 =$ __?__.

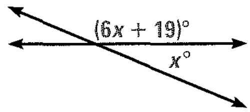


USING ALGEBRA **Find the value(s) of the variable(s).**

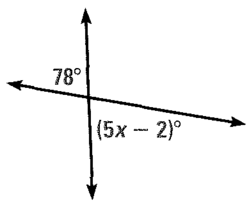
28.



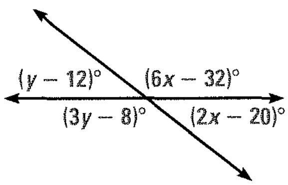
29.



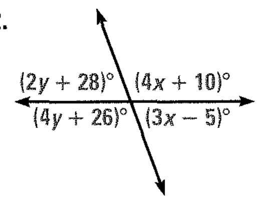
30.



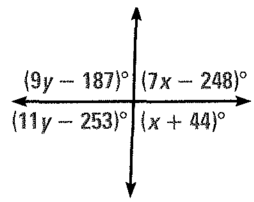
31.



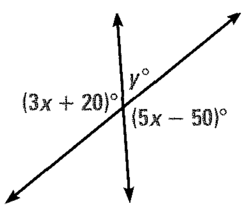
32.



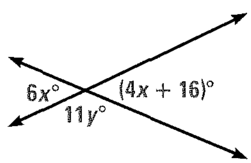
33.



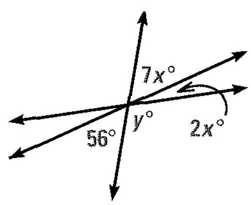
34.



35.

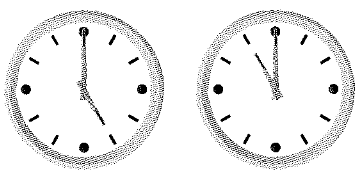


36.

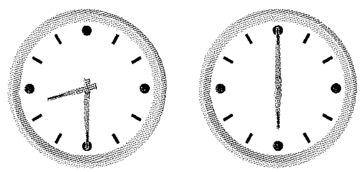


IDENTIFYING ANGLES **State whether the two angles shown are *complementary, supplementary,* or *neither*.**

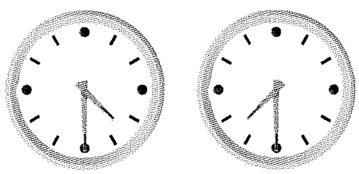
37.



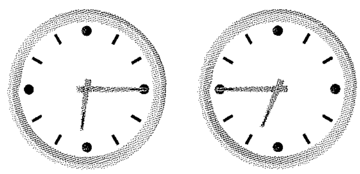
38.



39.



40.



41. **FINDING COMPLEMENTS** In the table, assume that $\angle 1$ and $\angle 2$ are complementary. Copy and complete the table.

$m\angle 1$	2°	10°	25°	33°	40°	49°	55°	62°	76°	86°
$m\angle 2$	?	?	?	?	?	?	?	?	?	?

42. **FINDING SUPPLEMENTS** In the table, assume that $\angle 1$ and $\angle 2$ are supplementary. Copy and complete the table.

$m\angle 1$	4°	16°	48°	72°	90°	99°	120°	152°	169°	178°
$m\angle 2$	?	?	?	?	?	?	?	?	?	?

43. **USING ALGEBRA** $\angle A$ and $\angle B$ are complementary. The measure of $\angle B$ is three times the measure of $\angle A$. Find $m\angle A$ and $m\angle B$.

44. **USING ALGEBRA** $\angle C$ and $\angle D$ are supplementary. The measure of $\angle D$ is eight times the measure of $\angle C$. Find $m\angle C$ and $m\angle D$.

FINDING ANGLES **$\angle A$ and $\angle B$ are complementary. Find $m\angle A$ and $m\angle B$.**

45. $m\angle A = 5x + 8$
$m\angle B = x + 4$

46. $m\angle A = 3x - 7$
$m\angle B = 11x - 1$

47. $m\angle A = 8x - 7$
$m\angle B = x - 11$

48. $m\angle A = \frac{3}{4}x - 13$
$m\angle B = 3x - 17$

FINDING ANGLES **$\angle A$ and $\angle B$ are supplementary. Find $m\angle A$ and $m\angle B$.**

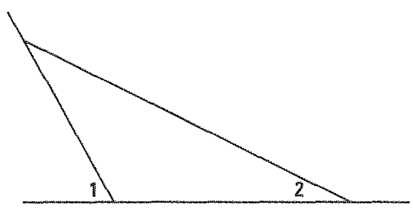
49. $m\angle A = 3x$
$m\angle B = x + 8$

50. $m\angle A = 6x - 1$
$m\angle B = 5x - 17$

51. $m\angle A = 12x + 1$
$m\angle B = x + 10$

52. $m\angle A = \frac{3}{8}x + 50$
$m\angle B = x + 31$

53. **BRIDGES** The Alamillo Bridge in Seville, Spain, was designed by Santiago Calatrava. In the bridge, $m\angle 1 = 58°$ and $m\angle 2 = 24°$. Find the supplements of both $\angle 1$ and $\angle 2$.



54. **BASEBALL** The foul lines of a baseball field intersect at home plate to form a right angle. Suppose you hit a baseball whose path forms an angle of 34° with the third base foul line. What is the angle between the first base foul line and the path of the baseball?

FOCUS ON PEOPLE

SANTIAGO CALATRAVA, a Spanish born architect, has developed designs for bridges, train stations, stadiums, and art museums.

APPLICATION LINK
www.mcdougallittell.com

55. **PLANTING TREES** To support a young tree, you attach wires from the trunk to the ground. The obtuse angle the wire makes with the ground is supplementary to the acute angle the wire makes, and it is three times as large. Find the measures of the angles.

56. ***Writing*** Give an example of an angle that *does not* have a complement. In general, what is true about an angle that has a complement?

Test Preparation

57. **MULTIPLE CHOICE** In the diagram shown at the right, what are the values of x and y?

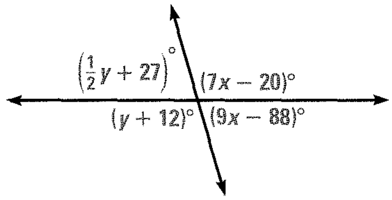
(A) $x = 74, y = 106$

(B) $x = 16, y = 88$

(C) $x = 74, y = 16$

(D) $x = 18, y = 118$

(E) $x = 18, y = 94$

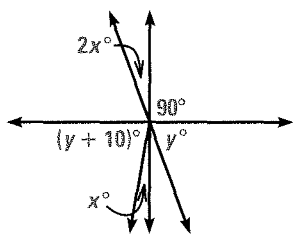


58. **MULTIPLE CHOICE** $\angle F$ and $\angle G$ are supplementary. The measure of $\angle G$ is six and one half times the measure of $\angle F$. What is $m\angle F$?

(A) 20° (B) 24° (C) 24.5° (D) 26.5° (E) 156°

★ Challenge

59. **USING ALGEBRA** Find the values of x and y in the diagram shown at the right.



MIXED REVIEW

SOLVING EQUATIONS **Solve the equation.** (Skills Review, p. 802, for 1.7)

60. $3x = 96$

61. $\frac{1}{2} \cdot 5 \cdot h = 20$

62. $\frac{1}{2} \cdot b \cdot 6 = 15$

63. $s^2 = 200$

64. $2 \cdot 3.14 \cdot r = 40$

65. $3.14 \cdot r^2 = 314$

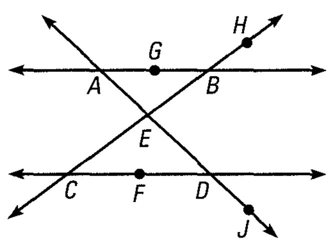
FINDING COLLINEAR POINTS **Use the diagram to find a third point that is collinear with the given points.** (Review 1.2)

66. A and J

67. D and F

68. H and E

69. B and G



FINDING THE MIDPOINT **Find the coordinates of the midpoint of a segment with the given endpoints.** (Review 1.5)

70. $A(0, 0), B(-6, -4)$

71. $F(2, 5), G(-10, 7)$

72. $K(8, -6), L(-2, -2)$

73. $M(-14, -9), N(0, 11)$

74. $P(-1.5, 4), Q(5, -9)$

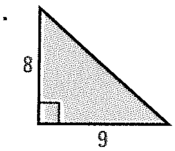
75. $S(-2.4, 5), T(7.6, 9)$

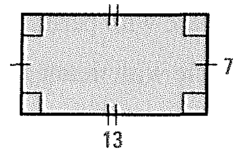
GUIDED PRACTICE

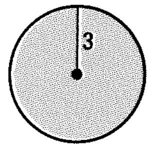
Vocabulary Check ✓ **1.** The perimeter of a circle is called its ___?___.

Concept Check ✓ **2.** Explain how to find the perimeter of a rectangle.

Skill Check ✓ **In Exercises 3–5, find the area of the figure. (Where necessary, use $\pi \approx 3.14$.)**

3.


4.


5.


6. The perimeter of a square is 12 meters. What is the length of a side of the square?

7. The radius of a circle is 4 inches. What is the circumference of the circle? (Use $\pi \approx 3.14$.)

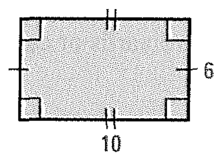
8. FENCING You are putting a fence around a rectangular garden with length 15 feet and width 8 feet. What is the length of the fence that you will need?

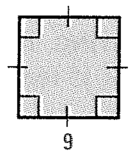
PRACTICE AND APPLICATIONS

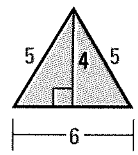
STUDENT HELP

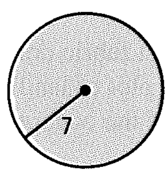
Extra Practice to help you master skills is on p. 804.

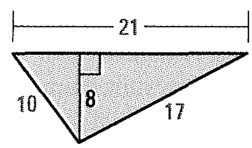
FINDING PERIMETER, CIRCUMFERENCE, AND AREA **Find the perimeter (or circumference) and area of the figure. (Where necessary, use $\pi \approx 3.14$.)**

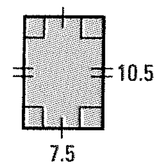
9.


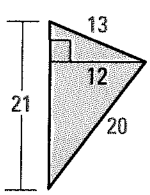
10.


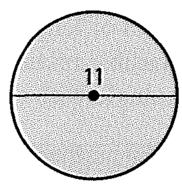
11.


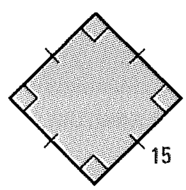
12.


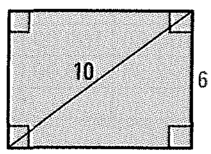
13.


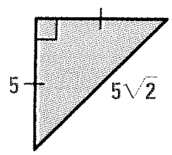
14.


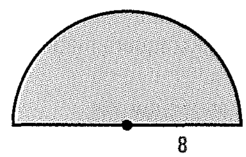
15.


16.


17.


18.


19.


20.


STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 9–26
Example 2: Exs. 9–26
Example 3: Exs. 27–33
Example 4: Exs. 34–40
Example 5: Exs. 34–40
Example 6: Exs. 41–48

STUDENT HELP

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 21–26.

FINDING AREA **Find the area of the figure described.**

21. Triangle with height 6 cm and base 5 cm

22. Rectangle with length 12 yd and width 9 yd

23. Square with side length 8 ft

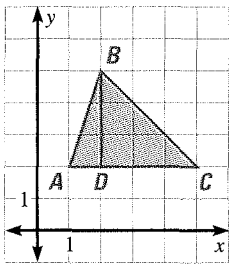
24. Circle with radius 10 m (Use $\pi \approx 3.14$.)

25. Square with perimeter 24 m

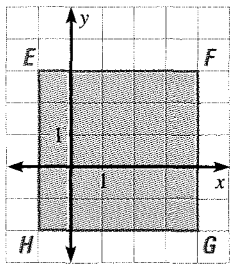
26. Circle with diameter 100 ft (Use $\pi \approx 3.14$.)

FINDING AREA **Find the area of the figure.**

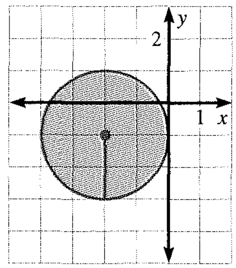
27.



28.



29.



FINDING AREA **Draw the figure in a coordinate plane and find its area.**

30. Triangle defined by $A(3, 4)$, $B(7, 4)$, and $C(5, 7)$

31. Triangle defined by $R(-2, -3)$, $S(6, -3)$, and $T(5, 4)$

32. Rectangle defined by $L(-2, -4)$, $M(-2, 1)$, $N(7, 1)$, and $P(7, -4)$

33. Square defined by $W(5, 0)$, $X(0, 5)$, $Y(-5, 0)$, and $Z(0, -5)$

34. **CARPETING** How many square yards of carpet are needed to carpet a room that is 15 feet by 25 feet?

35. **WINDOWS** A rectangular pane of glass measuring 12 inches by 18 inches is surrounded by a wooden frame that is 2 inches wide. What is the area of the window, including the frame?

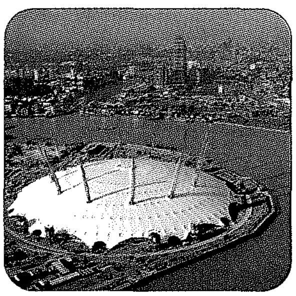
36. **MILLENNIUM DOME** The largest fabric dome in the world, the Millennium Dome covers a circular plot of land with a diameter of 320 meters. What is the circumference of the covered land? What is its area? (Use $\pi \approx 3.14$.)

37. **SPREADSHEET** Use a spreadsheet to show many different possible values of length and width for a rectangle with an area of 100 m^2. For each possible rectangle, calculate the perimeter. What are the dimensions of the rectangle with the smallest perimeter?

Perimeter of Rectangle

	A	B	C	D	E	F	G	H
1	Length	1.00	2.00	3.00	4.00	5.00	6.00	...
2	Width	100.00	50.00	33.33	25.00	20.00	16.67	...
3	Area	100.00	100.00	100.00	100.00	100.00	100.00	...
4	Perimeter	202.00	104.00	72.67	58.00	50.00	45.33	...
5								

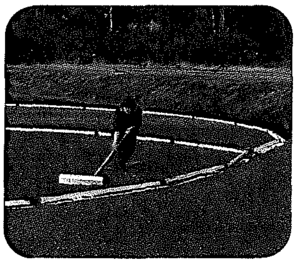
FOCUS ON APPLICATIONS



MILLENNIUM DOME
Built for the year 2000, this dome in Greenwich, England, is over 50 m tall and is covered by more than 100,000 square meters of fabric.

APPLICATION LINK www.mcdougallittell.com

FOCUS ON APPLICATIONS

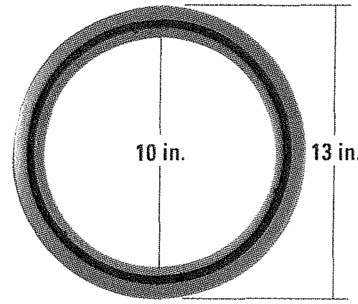


REAL LIFE **CRANBERRIES** Cranberries were once called "bounceberries" because they bounce when they are ripe.

38. CRANBERRY HARVEST To harvest cranberries, the field is flooded so that the berries float. The berries are gathered with an inflatable boom. What area of cranberries can be gathered into a circular region with a radius of 5.5 meters? (Use $\pi \approx 3.14$.)

39. BICYCLES How many times does a bicycle tire that has a radius of 21 inches rotate when it travels 420 inches? (Use $\pi \approx 3.14$.)

40. FLYING DISC A plastic flying disc is circular and has a circular hole in the middle. If the diameter of the outer edge of the ring is 13 inches and the diameter of the inner edge of the ring is 10 inches, what is the area of plastic in the ring? (Use $\pi \approx 3.14$.)



LOGICAL REASONING **Use the given measurements to find the unknown measurement. (Where necessary, use $\pi \approx 3.14$.)**

41. A rectangle has an area of 36 in.^2 and a length of 9 in. Find its perimeter.

42. A square has an area of 10,000 m^2. Find its perimeter.

43. A triangle has an area of 48 ft^2 and a base of 16 ft. Find its height.

44. A triangle has an area of 52 yd^2 and a height of 13 yd. Find its base.

45. A circle has an area of 200π cm^2. Find its radius.

46. A circle has an area of 1 m^2. Find its diameter.

47. A circle has a circumference of 100 yd. Find its area.

48. A right triangle has sides of length 4.5 cm, 6 cm, and 7.5 cm. Find its area.

Test Preparation

49. MULTI-STEP PROBLEM **Use the following information.**
Earth has a radius of about 3960 miles at the equator. Because there are 5280 feet in one mile, the radius of Earth is about 20,908,800 feet.

a. Suppose you could wrap a cable around Earth to form a circle that is snug against the ground. Find the length of the cable in feet by finding the circumference of Earth. (Assume that Earth is perfectly round. Use $\pi \approx 3.14$.)

b. Suppose you add 6 feet to the cable length in part (a). Use this length as the circumference of a new circle. Find the radius of the larger circle.

c. Use your results from parts (a) and (b) to find how high off of the ground the longer cable would be if it was evenly spaced around Earth.

d. Would the answer to part (c) be different on a planet with a different radius? Explain.

★ Challenge

50. DOUBLING A RECTANGLE'S SIDES The length and width of a rectangle are doubled. How do the perimeter and area of the new rectangle compare with the perimeter and area of the original rectangle? Illustrate your answer.

MIXED REVIEW

SKETCHING FIGURES Sketch the points, lines, segments, and rays. (Review 1.2 for 2.1)

51. Draw opposite rays using the points A, B, and C, with B as the initial point for both rays.

52. Draw four noncollinear points, W, X, Y, and Z, no three of which are collinear. Then sketch $\overleftrightarrow{XY}$, $\overrightarrow{YW}$, $\overline{XZ}$ and $\overleftrightarrow{ZY}$.

USING ALGEBRA Plot the points in a coordinate plane and sketch $\angle DEF$. Classify the angle. Write the coordinates of one point in the interior of the angle and one point in the exterior of the angle. (Review 1.4)

53. $D(2, -2)$ $E(4, -3)$ $F(6, -2)$

54. $D(0, 0)$ $E(-3, 0)$ $F(0, -2)$

55. $D(0, 1)$ $E(2, 3)$ $F(4, 1)$

56. $D(-3, -2)$ $E(3, -4)$ $F(1, 3)$

FINDING THE MIDPOINT Find the coordinates of the midpoint of a segment with the given endpoints. (Review 1.5)

57. $A(0, 0)$, $B(5, 3)$

58. $C(2, -3)$, $D(4, 4)$

59. $E(-3, 4)$, $F(-2, -1)$

60. $G(-2, 0)$, $H(-7, -6)$

61. $J(0, 5)$, $K(14, 1)$

62. $M(-44, 9)$, $N(6, -7)$

QUIZ 3

Self-Test for Lessons 1.6 and 1.7

In Exercises 1–4, find the measure of the angle. (Lesson 1.6)

1. Complement of $\angle A$; $m\angle A = 41°$

2. Supplement of $\angle B$; $m\angle B = 127°$

3. Supplement of $\angle C$; $m\angle C = 22°$

4. Complement of $\angle D$; $m\angle D = 35°$

5. $\angle A$ and $\angle B$ are complementary. The measure of $\angle A$ is five times the measure of $\angle B$. Find $m\angle A$ and $m\angle B$. (Lesson 1.6)

In Exercises 6–9, use the given information to find the unknown measurement. (Lesson 1.7)

6. Find the area and circumference of a circle with a radius of 18 meters. (Use $\pi \approx 3.14$.)

7. Find the area of a triangle with a base of 13 inches and a height of 11 inches.

8. Find the area and perimeter of a rectangle with a length of 10 centimeters and a width of 4.6 centimeters.

9. Find the area of a triangle defined by $P(-3, 4)$, $Q(7, 4)$, and $R(-1, 12)$.

10. WALLPAPER You are buying rolls of wallpaper to paper the walls of a rectangular room. The room measures 12 feet by 24 feet and the walls are 8 feet high. A roll of wallpaper contains 28 ft^2. About how many rolls of wallpaper will you need? (Lesson 1.7)

CHAPTER 1 Chapter Review

VOCABULARY

- conjecture, p. 4
- inductive reasoning, p. 4
- counterexample, p. 4
- definition, undefined, p. 10
- point, line, plane, p. 10
- collinear, coplanar, p. 10
- line segment, p. 11
- endpoints, p. 11
- ray, p. 11
- initial point, p. 11
- opposite rays, p. 11
- intersect, intersection, p. 12
- postulates, or axioms, p. 17
- coordinate, p. 17
- distance, length, p. 17
- between, p. 18
- Distance Formula, p. 19
- congruent segments, p. 19
- angle, p. 26
- sides, vertex of an angle, p. 26
- congruent angles, p. 26
- measure of an angle, p. 27
- interior of an angle, p. 27
- exterior of an angle, p. 27
- acute, obtuse angles, p. 28
- right, straight angles, p. 28
- adjacent angles, p. 28
- midpoint, p. 34
- bisect, p. 34
- segment bisector, p. 34
- compass, straightedge, p. 34
- construct, construction, p. 34
- Midpoint Formula, p. 35
- angle bisector, p. 36
- vertical angles, p. 44
- linear pair, p. 44
- complementary angles, p. 46
- complement of an angle, p. 46
- supplementary angles, p. 46
- supplement of an angle, p. 46

1.1 PATTERNS AND INDUCTIVE REASONING

Examples on pp. 3–5

EXAMPLE Make a conjecture based on the results shown.

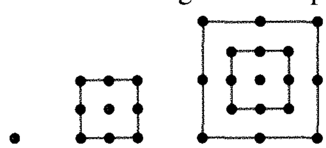
Conjecture: Given a 3-digit number, form a 6-digit number by repeating the digits. Divide the number by 7, then 11, then 13. The result is the original number.

$456{,}456 \div 7 \div 11 \div 13 = 456$
$562{,}562 \div 7 \div 11 \div 13 = 562$
$109{,}109 \div 7 \div 11 \div 13 = 109$

In Exercises 1–3, describe a pattern in the sequence of numbers.

1. 5, 12, 19, 26, 33, . . . **2.** 0, 2, 6, 14, 30, . . . **3.** 4, 12, 36, 108, 324, . . .

4. Sketch the next figure in the pattern.



5. Make a conjecture based on the results.

$4 \cdot 5 \cdot 6 \cdot 7 + 1 = 29 \cdot 29$
$5 \cdot 6 \cdot 7 \cdot 8 + 1 = 41 \cdot 41$
$6 \cdot 7 \cdot 8 \cdot 9 + 1 = 55 \cdot 55$

6. Show the conjecture is false by finding a counterexample:

Conjecture: *The cube of a number is always greater than the number.*

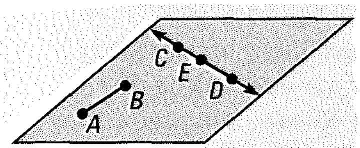
1.2 POINTS, LINES, AND PLANES

Examples on pp. 10–12

EXAMPLE

C, E, and *D* are collinear. $\overleftrightarrow{CD}$ is a line. $\overline{AB}$ is a segment.

A, B, C, D, and *E* are coplanar. $\overrightarrow{EC}$ and $\overrightarrow{ED}$ are opposite rays.



7. Draw five coplanar points, A, B, C, D, and E so that $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays, and $\overline{DE}$ intersects $\overleftrightarrow{AC}$ at B.

8. Sketch three planes that do not intersect.

9. Sketch two lines that are not coplanar and do not intersect.

1.3 SEGMENTS AND THEIR MEASURES

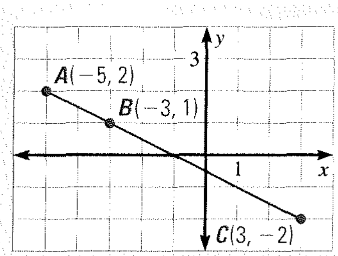
Examples on pp. 17–20

EXAMPLE B is between A and C, so $AB + BC = AC$. Use the Distance Formula to find AB and BC.

$$AB = \sqrt{[-3 - (-5)]^2 + (1 - 2)^2} = \sqrt{2^2 + (-1)^2} = \sqrt{5}$$

$$BC = \sqrt{[3 - (-3)]^2 + (-2 - 1)^2} = \sqrt{6^2 + (-3)^2} = \sqrt{45}$$

Because $AB \neq BC$, $\overline{AB}$ and $\overline{BC}$ are *not* congruent segments.



10. Q is between P and S. R is between Q and S. S is between Q and T. $PT = 30$, $QS = 16$, and $PQ = QR = RS$. Find PQ, ST, and RP.

Use the Distance Formula to decide whether $\overline{PQ} \cong \overline{QR}$.

11. $P(-4, 3)$, $Q(-2, 1)$, $R(0, -1)$

12. $P(-3, 5)$, $Q(1, 3)$, $R(4, 1)$

13. $P(-2, -2)$, $Q(0, 1)$, $R(1, 4)$

1.4 ANGLES AND THEIR MEASURES

Examples on pp. 26–28

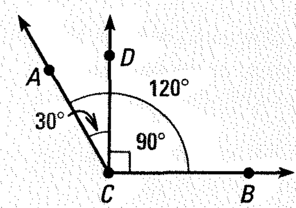
EXAMPLE

$m\angle ACD + m\angle DCB = m\angle ACB$

$\angle ACD$ is an acute angle: $m\angle ACD < 90°$.

$\angle DCB$ is a right angle: $m\angle DCB = 90°$.

$\angle ACB$ is an obtuse angle: $m\angle ACB > 90°$.



Classify the angle as *acute, right, obtuse,* or *straight*. Sketch the angle. Then use a protractor to check your results.

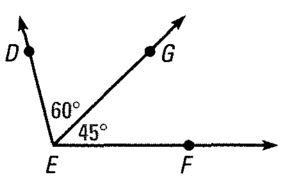
14. $m\angle KLM = 180°$

15. $m\angle A = 150°$

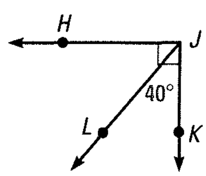
16. $m\angle Y = 45°$

Use the Angle Addition Postulate to find the measure of the unknown angle.

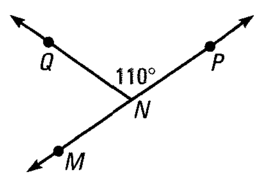
17. $m\angle DEF$



18. $m\angle HJL$



19. $m\angle QNM$

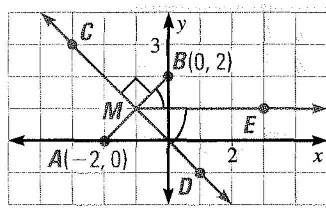


1.5 SEGMENT AND ANGLE BISECTORS

Examples on pp. 35–37

EXAMPLE If $\overleftrightarrow{CD}$ is a bisector of $\overline{AB}$, then $\overleftrightarrow{CD}$ intersects $\overline{AB}$ at its midpoint M: $M = \left(\frac{-2+0}{2}, \frac{0+2}{2}\right) = (-1, 1)$.

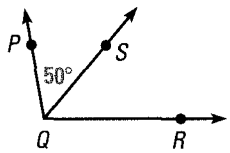
$\overrightarrow{ME}$ bisects $\angle BMD$, so $m\angle BME = m\angle EMD = 45°$.

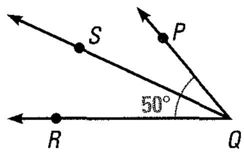


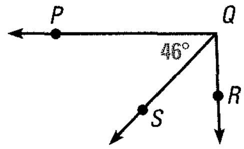
Find the coordinates of the midpoint of a segment with the given endpoints.

20. $A(0, 0)$, $B(-8, 6)$ **21.** $J(-1, 7)$, $K(3, -3)$ **22.** $P(-12, -9)$, $Q(2, 10)$

$\overrightarrow{QS}$ is the bisector of $\angle PQR$. Find any angle measures not given in the diagram.

23. 

24. 

25. 

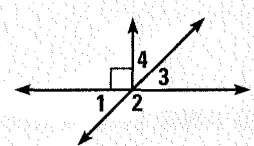
1.6 ANGLE PAIR RELATIONSHIPS

Examples on pp. 44–46

EXAMPLE $\angle 1$ and $\angle 3$ are vertical angles.

$\angle 1$ and $\angle 2$ are a linear pair and are supplementary angles.

$\angle 3$ and $\angle 4$ are complementary angles.



Use the diagram above to decide whether the statement is *always*, *sometimes*, or *never* true.

26. If $m\angle 2 = 115°$, then $m\angle 3 = 65°$.

27. $\angle 3$ and $\angle 4$ are congruent.

28. If $m\angle 1 = 40°$, then $m\angle 3 = 50°$.

29. $\angle 1$ and $\angle 4$ are complements.

1.7 INTRODUCTION TO PERIMETER, CIRCUMFERENCE, AND AREA

Examples on pp. 51–54

EXAMPLES A circle has diameter 24 ft.
Its circumference is $C = 2\pi r \approx 2(3.14)(12) = 75.36$ feet.
Its area is $A = \pi r^2 \approx 3.14(12^2) = 452.16$ square feet.

Find the perimeter (or circumference) and area of the figure described.

30. Rectangle with length 10 cm and width 4.5 cm

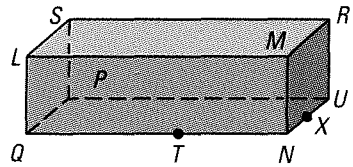
31. Circle with radius 9 in. (Use $\pi \approx 3.14$.)

32. Triangle defined by $A(-6, 0)$, $B(2, 0)$, and $C(-2, -3)$

33. A square garden has sides of length 14 ft. What is its perimeter?

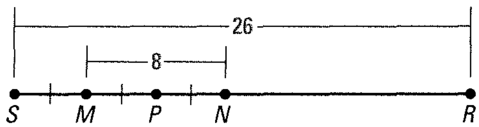
Chapter Test

Use the diagram to name the figures.



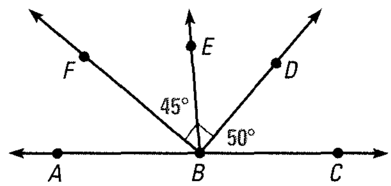
1. Three collinear points
2. Four noncoplanar points
3. Two opposite rays
4. Two intersecting lines
5. The intersection of plane *LMN* and plane *QLS*

Find the length of the segment.

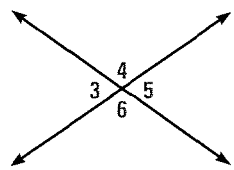


6. $\overline{MP}$
7. $\overline{SM}$
8. $\overline{NR}$
9. $\overline{MR}$

Find the measure of the angle.

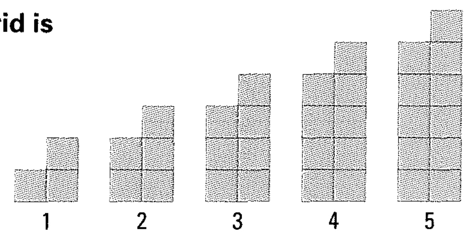


10. $\angle DBE$
11. $\angle FBC$
12. $\angle ABF$
13. $\angle DBA$



14. Refer to the diagram for Exercises 10–13. Name an obtuse angle, an acute angle, a right angle, and two complementary angles.
15. Q is between P and R. $PQ = 2w - 3$, $QR = 4 + w$, and $PR = 34$. Find the value of w. Then find the lengths of $\overline{PQ}$ and $\overline{QR}$.
16. $\overline{RT}$ has endpoints $R(-3, 8)$ and $T(3, 6)$. Find the coordinates of the midpoint, S, of $\overline{RT}$. Then use the Distance Formula to verify that $RS = ST$.
17. Use the diagram. If $m\angle 3 = 68°$, find the measures of $\angle 5$ and $\angle 4$.
18. Suppose $m\angle PQR = 130°$. If $\overrightarrow{QT}$ bisects $\angle PQR$, what is the measure of $\angle PQT$?

The first five figures in a pattern are shown. Each square in the grid is 1 unit × 1 unit.



19. Make a table that shows the distance around each figure at each stage.
20. Describe the pattern of the distances and use it to predict the distance around the figure at stage 20.

A center pivot irrigation system uses a fixed water supply to water a circular region of a field. The radius of the watering system is 560 feet long. (Use $\pi \approx 3.14$.)

21. If some workers walked around the circumference of the watered region, how far would they have to walk? Round to the nearest foot.
22. Find the area of the region watered. Round to the nearest square foot.

CHAPTER 1

Chapter Standardized Test

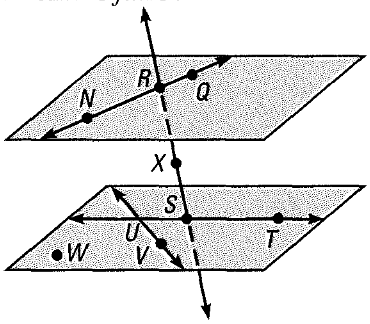
TEST-TAKING STRATEGY Work as quickly as you can through the easier sections, but avoid making careless errors on easy questions.

1. **MULTIPLE CHOICE** What is the next number in the sequence?

   4488; 44,088; 440,088; 4,400,088; . . .

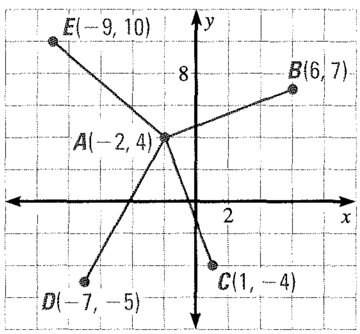
   (A) 400,008 (B) 40,000,088
   (C) 44,000,088 (D) 440,000,088
   (E) 44,000,008

2. **MULTIPLE CHOICE** Which of the following statements is *false*?



   (A) S, T, V, and W are coplanar.
   (B) X, T, S, and U are coplanar.
   (C) Q, N, and R are collinear.
   (D) S, R, and X are collinear.
   (E) $\overrightarrow{TS}$ and $\overrightarrow{TU}$ are opposite rays.

3. **MULTIPLE CHOICE** Which of the line segments shown in the coordinate plane are congruent?



   (A) $\overline{AC}$ and $\overline{AE}$ (B) $\overline{AB}$ and $\overline{AE}$
   (C) $\overline{AD}$ and $\overline{AC}$ (D) $\overline{AD}$ and $\overline{AB}$
   (E) $\overline{AB}$ and $\overline{AC}$

4. **MULTIPLE CHOICE** B is between A and C, D is between B and C, and C is between B and E. $AE = 28$, $BC = 10$, and $AB = DB = DC$. What is the length of $\overline{CE}$?

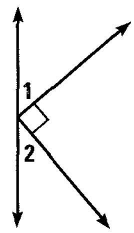
   (A) 5 (B) 10 (C) 12
   (D) 13 (E) 15

5. **MULTIPLE CHOICE** If $\angle 4$ and $\angle 5$ are complementary and $m\angle 4 = 19°$, find $m\angle 5$.

   (A) 19° (B) 71° (C) 109°
   (D) 161° (E) cannot be determined

6. **MULTIPLE CHOICE** $\angle 1$ and $\angle 2$ in the diagram are ___?___.

   (A) complementary
   (B) supplementary
   (C) congruent
   (D) vertical angles
   (E) a linear pair

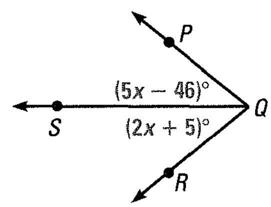


7. **MULTIPLE CHOICE** The midpoint of $\overline{BC}$ is $M(-10, -16)$. One endpoint is $B(-1, 8)$. What are the coordinates of C?

   (A) (−21, −40) (B) (−20, −40)
   (C) (−19, −40) (D) (−21, −24)
   (E) (8, 32)

8. **MULTIPLE CHOICE** If $\overrightarrow{QS}$ bisects $\angle PQR$, find the measure of $\angle PQR$.

   (A) 17°
   (B) 56°
   (C) 21°
   (D) 39°
   (E) 78°



9. **MULTIPLE CHOICE** Two angles are complementary and one angle has a measure that is 9 times the measure of the other angle. What is the angle measure of the larger angle?

   (A) 9°
   (B) 18°
   (C) 81°
   (D) 90°
   (E) 162°

10. **QUANTITATIVE COMPARISON** Consider the areas of the two triangles that are described below.

COLUMN A	COLUMN B
The area of a triangle defined by $A(-6, 7)$, $B(-6, -1)$, and $C(-3, 2)$	The area of a triangle defined by $D(0, 4)$, $E(6, 4)$, and $F(6, 0)$

Choose the statement that is true.

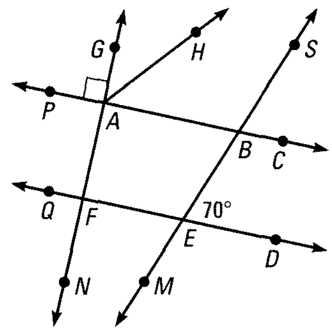
Ⓐ The quantity in column A is greater.

Ⓑ The quantity in column B is greater.

Ⓒ The two quantities are equal.

Ⓓ The relationship cannot be determined from the information given.

MULTI-STEP PROBLEM In Exercises 11–14, use the figure at the right.



11. Name an angle that is (a) acute, (b) obtuse, (c) straight, and (d) right.

12. Classify each pair of angles as *complementary*, *supplementary*, or *vertical angles*.

a. $\angle ABS$ and $\angle SBC$

b. $\angle BAH$ and $\angle GAH$

c. $\angle BEF$ and $\angle FEM$

d. $\angle ABS$ and $\angle EBC$

13. If $\overrightarrow{AH}$ bisects $\angle GAB$, find the measures of $\angle GAH$ and $\angle BAH$.

14. If $m\angle QFN = x°$, express the measures of $\angle QFA$, $\angle AFE$, and $\angle EFN$ in terms of x.

MULTI-STEP PROBLEM Consider some rectangles with a perimeter of 24 inches.

15. Copy and complete the table below.

Width (in.)	Perimeter (in.)	Length (in.)	Area (in.^{2})
1	?	?	?
2	?	?	?
3	?	?	?
4	?	?	?
5	?	?	?
6	?	?	?
7	?	?	?

16. Which rectangle in the table has the greatest area?

17. Look at the entries in the table. Describe a pattern in the widths and lengths. Use the pattern to predict the length of a rectangle with a width of 3.5 inches.

18. Make a conjecture about the dimensions of a rectangle with greatest area if the perimeter of the rectangle is known. Describe a way to test your conjecture.