

BASICS OF GEOMETRY

▶ *How are airport runways named?*



CHAPTER

1

APPLICATION: Runways

*A*ir traffic controllers and pilots need a way to refer to runways that is consistent among different airports.

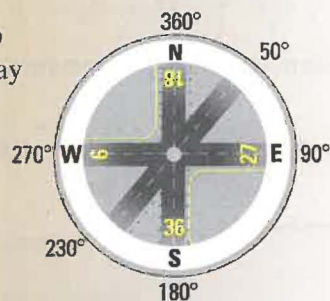
Runways are named based on the angles they form with due north, measured in a clockwise direction. These angles are called *bearings*.

The bearing of a runway is divided by 10 to find the runway number.

Approach	Bearing	Runway
from west to east	90°	9
from north to south	180°	18
from east to west	270°	27
from south to north	360°	36

Think & Discuss

- Why is it important to use a consistent runway naming scheme?
- The bearing of the unlabeled runway in the diagram is 50°. What are the missing runway numbers?



Learn More About It

You will learn more about angles of intersecting runways in Exercises 55–60 on p. 32.



APPLICATION LINK Visit www.mcdougallittell.com for more information about airport runways.

PREVIEW

What's the chapter about?

Chapter 1 is about the **basic elements of geometry**. In Chapter 1, you'll learn

- how to measure segments and angles.
- how to divide a segment or angle into two equal parts.
- relationships among special pairs of angles.

KEY VOCABULARY

► **New**

- | | | |
|------------------------------|------------------------------|-------------------------------|
| • conjecture, p. 4 | • congruent segments, p. 19 | • straight angle, p. 28 |
| • point, line, plane, p. 10 | • congruent angles, p. 26 | • segment bisector, p. 34 |
| • segment, ray, p. 11 | • measure of an angle, p. 27 | • angle bisector, p. 36 |
| • postulate, p. 17 | • acute angle, p. 28 | • vertical angles, p. 44 |
| • length of a segment, p. 17 | • right angle, p. 28 | • complementary angles, p. 46 |
| | • obtuse angle, p. 28 | • supplementary angles, p. 46 |

PREPARE

Are you ready for the chapter?

SKILL REVIEW Do these exercises to review key skills that you'll apply in this chapter. See the given **reference page** if there is something you don't understand.

Subtract the integers. (Skills Review, p. 785)

- | | | | |
|-------------|----------------|----------------|----------------|
| 1. $17 - 9$ | 2. $9 - 17$ | 3. $5 - (-3)$ | 4. $3 - (-5)$ |
| 5. $-7 - 2$ | 6. $-7 - (-2)$ | 7. $-6 - (-5)$ | 8. $-5 - (-6)$ |

Evaluate the sum. (Skills Review, p. 786)

- | | | | |
|----------------|--------------------|--------------------|--------------------|
| 9. $2^2 + 4^2$ | 10. $5^2 + (-2)^2$ | 11. $(-1)^2 + 1^2$ | 12. $(-5)^2 + 0^2$ |
|----------------|--------------------|--------------------|--------------------|

Evaluate the radical expression. Round your answer to two decimal places. (Skills Review, p. 799)

- | | | | |
|---------------------|---------------------|------------------------|--------------------|
| 13. $\sqrt{36 + 4}$ | 14. $\sqrt{1 + 49}$ | 15. $\sqrt{225 + 100}$ | 16. $\sqrt{9 + 9}$ |
|---------------------|---------------------|------------------------|--------------------|

STUDENT HELP

► **Study Tip**

"Student Help" boxes throughout the chapter give you study tips and tell you where to look for extra help in this book and on the Internet.

STUDY STRATEGY

Here's a study strategy!

Learning Vocabulary

Important words in this book are in bold type and are highlighted in yellow.

- Keep a section in your notebook for writing down definitions of new words.
- Draw and label sketches near your definitions, if this helps you.
- Use your vocabulary pages to review for quizzes and tests.

1.1

Patterns and Inductive Reasoning

What you should learn

GOAL 1 Find and describe patterns.

GOAL 2 Use inductive reasoning to make real-life conjectures, as in Ex. 42.

Why you should learn it

▼ To make predictions based on observations, such as predicting full moons in Example 6.

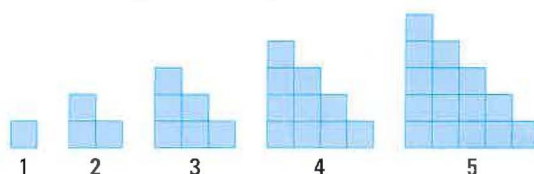


GOAL 1 FINDING AND DESCRIBING PATTERNS

Geometry, like much of mathematics and science, developed when people began recognizing and describing patterns. In this course, you will study many amazing patterns that were discovered by people throughout history and all around the world. You will also learn to recognize and describe patterns of your own. Sometimes, patterns allow you to make accurate predictions.

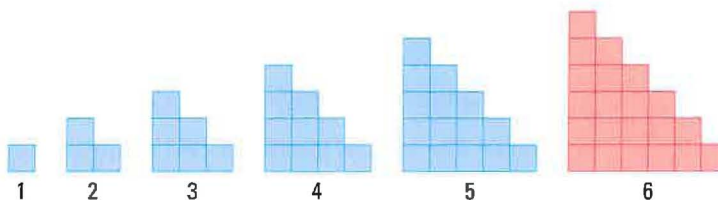
EXAMPLE 1 Describing a Visual Pattern

Sketch the next figure in the pattern.



SOLUTION

Each figure in the pattern looks like the previous figure with another row of squares added to the bottom. Each figure looks like a stairway.



► The sixth figure in the pattern has six squares in the bottom row.

EXAMPLE 2 Describing a Number Pattern

Describe a pattern in the sequence of numbers. Predict the next number.

- a. 1, 4, 16, 64, ... b. -5, -2, 4, 13, ...

SOLUTION

- a. Each number is four times the previous number. The next number is 256.
b. You add 3 to get the second number, then add 6 to get the third number, then add 9 to get the fourth number. To find the fifth number, add the next multiple of 3, which is 12.

► So, the next number is $13 + 12$, or 25.

GOAL 2 USING INDUCTIVE REASONING

Much of the reasoning in geometry consists of three stages.

- 1 **Look for a Pattern** Look at several examples. Use diagrams and tables to help discover a pattern.
- 2 **Make a Conjecture** Use the examples to make a general *conjecture*.
A **conjecture** is an unproven statement that is based on observations.
Discuss the conjecture with others. Modify the conjecture, if necessary.
- 3 **Verify the Conjecture** Use logical reasoning to verify that the conjecture is true in all cases. (You will do this in Chapter 2 and throughout this book.)

Looking for patterns and making conjectures is part of a process called **inductive reasoning**.



EXAMPLE 3 Making a Conjecture

Complete the conjecture.

Conjecture: The sum of the first n odd positive integers is $\underline{\quad ? \quad}$.

SOLUTION

List some specific examples and look for a pattern.

Examples:

first odd positive integer:

$$1 = 1^2$$

sum of first **two** odd positive integers:

$$1 + 3 = 4 = 2^2$$

sum of first **three** odd positive integers:

$$1 + 3 + 5 = 9 = 3^2$$

sum of first **four** odd positive integers:

$$1 + 3 + 5 + 7 = 16 = 4^2$$

Conjecture: The sum of the first n odd positive integers is n^2 .

.....

To prove that a conjecture is true, you need to prove it is true in *all* cases. To prove that a conjecture is false, you need to provide a single *counterexample*.

A **counterexample** is an example that shows a conjecture is false.

EXAMPLE 4 Finding a Counterexample

Show the conjecture is false by finding a counterexample.

Conjecture: For all real numbers x , the expression x^2 is greater than or equal to x .

SOLUTION

The conjecture is false. Here is a counterexample: $(0.5)^2 = 0.25$, and 0.25 is *not* greater than or equal to 0.5. In fact, any number between 0 and 1 is a counterexample.

Not every conjecture is known to be true or false. Conjectures that are not known to be true or false are called *unproven* or *undecided*.

EXAMPLE 5 *Examining an Unproven Conjecture*

In the early 1700s a Prussian mathematician named Goldbach noticed that many even numbers greater than 2 can be written as the sum of two primes.

Specific Cases:

$$\begin{array}{lll} 4 = 2 + 2 & 10 = 3 + 7 & 16 = 3 + 13 \\ 6 = 3 + 3 & 12 = 5 + 7 & 18 = 5 + 13 \\ 8 = 3 + 5 & 14 = 3 + 11 & 20 = 3 + 17 \end{array}$$

Conjecture: Every even number greater than 2 can be written as the sum of two primes.

This is called *Goldbach's Conjecture*. No one has ever proved that this conjecture is true or found a counterexample to show that it is false. As of the writing of this book, it is unknown whether this conjecture is true or false. It is known, however, that all even numbers up to 4×10^{14} confirm Goldbach's Conjecture.

EXAMPLE 6 *Using Inductive Reasoning in Real Life*

MOON CYCLES A full moon occurs when the moon is on the opposite side of Earth from the sun. During a full moon, the moon appears as a complete circle.



Use inductive reasoning and the information below to make a conjecture about how often a full moon occurs.

Specific Cases: In 2005, the first six full moons occur on January 25, February 24, March 25, April 24, May 23, and June 22.

SOLUTION

Conjecture: A full moon occurs every 29 or 30 days.

This conjecture is true. The moon revolves around Earth once approximately every 29.5 days.

.....

Inductive reasoning is important to the study of mathematics: you look for a pattern in specific cases and then you write a conjecture that you think describes the general case. Remember, though, that just because something is true for several specific cases does not *prove* that it is true in general.

FOCUS ON APPLICATIONS



CALENDARS

The earliest calendars were based on lunar or seasonal patterns. The ancient Egyptians were the first to introduce a calendar based on the solar year.

GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

Skill Check ✓

1. Explain what a *conjecture* is.
2. How can you prove that a conjecture is false?

Sketch the next figure in the pattern.

3.



4.



Describe a pattern in the sequence of numbers. Predict the next number.

5. 2, 6, 18, 54, ...

6. 0, 1, 4, 9, ...

7. 256, 64, 16, 4, ...

8. 3, 0, -3, 0, 3, 0, ...

9. 7.0, 7.5, 8.0, 8.5, ...

10. 13, 7, 1, -5, ...

11. Complete the conjecture based on the pattern you observe.

$$3 + 4 + 5 = 4 \cdot 3$$

$$6 + 7 + 8 = 7 \cdot 3$$

$$9 + 10 + 11 = 10 \cdot 3$$

$$4 + 5 + 6 = 5 \cdot 3$$

$$7 + 8 + 9 = 8 \cdot 3$$

$$10 + 11 + 12 = 11 \cdot 3$$

$$5 + 6 + 7 = 6 \cdot 3$$

$$8 + 9 + 10 = 9 \cdot 3$$

$$11 + 12 + 13 = 12 \cdot 3$$

Conjecture: The sum of any three consecutive integers is $\underline{\quad} \cdot \underline{\quad}$.

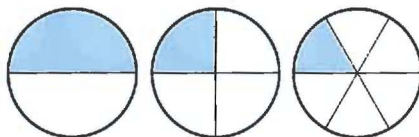
PRACTICE AND APPLICATIONS

STUDENT HELP

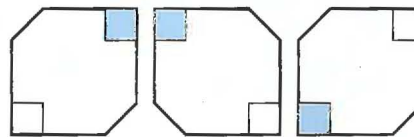
Extra Practice to help you master skills is on p. 803.

SKETCHING VISUAL PATTERNS Sketch the next figure in the pattern.

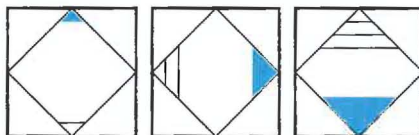
12.



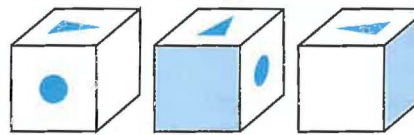
13.



14.



15.



STUDENT HELP

HOMEWORK HELP

- Example 1: Exs. 12–15, 24, 25
- Example 2: Exs. 16–23, 26–28
- Example 3: Exs. 29–33
- Example 4: Exs. 34–39
- Example 5: Exs. 40, 41
- Example 6: Exs. 42, 43

DESCRIBING NUMBER PATTERNS Describe a pattern in the sequence of numbers. Predict the next number.

16. 1, 4, 7, 10, ...

17. 10, 5, 2.5, 1.25, ...

18. 1, 11, 121, 1331, ...

19. 5, 0, -5, -10, ...

20. 7, 9, 13, 19, 27, ...

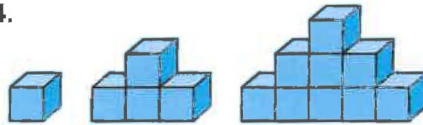
21. 1, 3, 6, 10, 15, ...

22. 256, 16, 4, 2, ...

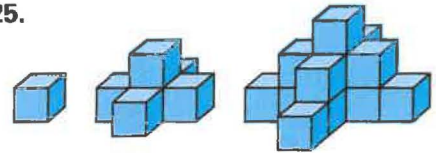
23. 1.1, 1.01, 1.001, 1.0001, ...

VISUALIZING PATTERNS The first three objects in a pattern are shown. How many blocks are in the next object?

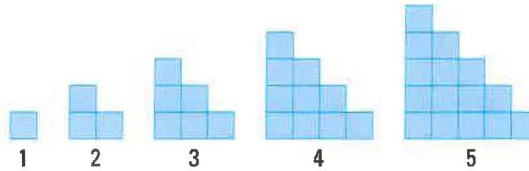
24.



25.




MAKING PREDICTIONS In Exercises 26–28, use the pattern from Example 1 shown below. Each square is 1 unit \times 1 unit.



26. Find the distance around each figure. Organize your results in a table.

27. Use your table to describe a pattern in the distances.

28. Predict the distance around the twentieth figure in this pattern.

STUDENT HELP
 **HOMEWORK HELP**
 Visit our Web site
www.mcdougallittell.com
 for help with Exs. 29–31.

MAKING CONJECTURES Complete the conjecture based on the pattern you observe in the specific cases.

29. **Conjecture:** The sum of any two odd numbers is ? .


$1 + 1 = 2$	$7 + 11 = 18$
$1 + 3 = 4$	$13 + 19 = 32$
$3 + 5 = 8$	$201 + 305 = 506$

30. **Conjecture:** The product of any two odd numbers is ? .

$1 \times 1 = 1$	$7 \times 11 = 77$
$1 \times 3 = 3$	$13 \times 19 = 247$
$3 \times 5 = 15$	$201 \times 305 = 61,305$

31. **Conjecture:** The product of a number $(n - 1)$ and the number $(n + 1)$ is always equal to ? .

$3 \cdot 5 = 4^2 - 1$	$6 \cdot 8 = 7^2 - 1$
$4 \cdot 6 = 5^2 - 1$	$7 \cdot 9 = 8^2 - 1$
$5 \cdot 7 = 6^2 - 1$	$8 \cdot 10 = 9^2 - 1$

 **CALCULATOR** Use a calculator to explore the pattern. Write a conjecture based on what you observe.

32. $101 \times 34 = \underline{\quad ? \quad}$

33. $11 \times 11 = \underline{\quad ? \quad}$

$101 \times 25 = \underline{\quad ? \quad}$

$111 \times 111 = \underline{\quad ? \quad}$

$101 \times 97 = \underline{\quad ? \quad}$

$1111 \times 1111 = \underline{\quad ? \quad}$

$101 \times 49 = \underline{\quad ? \quad}$


$11,111 \times 11,111 = \underline{\quad ? \quad}$

FINDING COUNTEREXAMPLES Show the conjecture is false by finding a counterexample.

34. All prime numbers are odd.
35. The sum of two numbers is always greater than the larger number.
36. If the product of two numbers is even, then the two numbers must be even.
37. If the product of two numbers is positive, then the two numbers must both be positive.
38. The square root of a number x is always less than x .
39. If m is a nonzero integer, then $\frac{m+1}{m}$ is always greater than 1.

GOLDBACH'S CONJECTURE In Exercises 40 and 41, use the list of the first prime numbers given below.

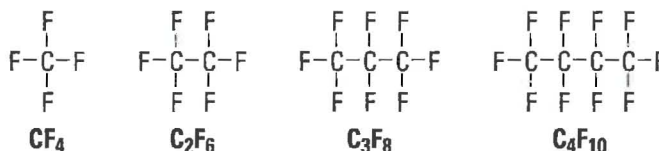
{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ...}

40. Show that Goldbach's Conjecture (see page 5) is true for the even numbers from 20 to 40 by writing each even number as a sum of two primes.
41. Show that the following conjecture is not true by finding a counterexample.
Conjecture: All *odd* numbers can be expressed as the sum of two primes.
42.  **BACTERIA GROWTH** Suppose you are studying bacteria in biology class. The table shows the number of bacteria after n doubling periods.

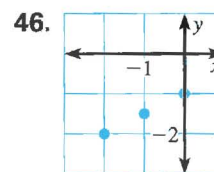
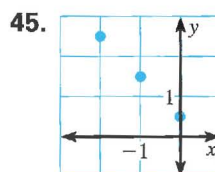
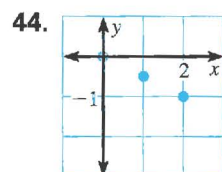
n (periods)	0	1	2	3	4	5
Billions of bacteria	3	6	12	24	48	96

Your teacher asks you to predict the number of bacteria after 8 doubling periods. What would your prediction be?

43. **SCIENCE CONNECTION** Diagrams and formulas for four molecular compounds are shown. Draw a diagram and write the formula for the next two compounds in the pattern.



44.  **USING ALGEBRA** Find a pattern in the coordinates of the points. Then use the pattern to find the y -coordinate of the point (3, ?).



FOCUS ON CAREERS



 **LABORATORY TECHNOLOGIST**

Laboratory technologists study microscopic cells, such as bacteria. The time it takes for a population of bacteria to double (the *doubling period*) may be as short as 20 min.

 **CAREER LINK**
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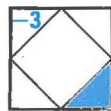


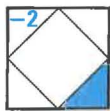

47. **MULTIPLE CHOICE** Which number is next in the sequence?

45, 90, 135, 180, ...

- (A) 205 (B) 210 (C) 215 (D) 220 (E) 225

48. **MULTIPLE CHOICE** What is the next figure in the pattern?



- (A)  (B)  (C)  (D)  (E) 

★ **Challenge**

DIVIDING A CIRCLE In Exercises 49–51, use the information about regions in a circle formed by connecting points on the circle.

If you draw points on a circle and then connect every pair of points, the circle is divided into a number of regions, as shown.



49. Copy and complete the table for the case of 4 and 5 points.

Number of points on circle	2	3	4	5	6
Maximum number of regions	2	4	?	?	?

50. Make a conjecture about the relationship between the number of points on the circle and number of regions in the circle.

51. Test your conjecture for the case of 6 points. What do you notice?

EXTRA CHALLENGE

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MIXED REVIEW

PLOTTING POINTS Plot in a coordinate plane. (Skills Review, p. 792, for 1.2)

52. (5, 2) 53. (3, -8) 54. (-4, -6) 55. (1, -10)

56. (-2, 7) 57. (-3, 8) 58. (4, -1) 59. (-2, -6)

EVALUATING EXPRESSIONS Evaluate the expression. (Skills Review, p. 786)

60. 3^2 61. 5^2 62. $(-4)^2$ 63. -7^2

64. $3^2 + 4^2$ 65. $5^2 + 12^2$ 66. $(-2)^2 + 2^2$ 67. $(-10)^2 + (-5)^2$

FINDING A PATTERN Write the next number in the sequence. (Review 1.1)

68. 1, 5, 25, 125, ... 69. 4.4, 40.4, 400.4, 4000.4, ...

70. 3, 7, 11, 15, ... 71. -1, +1, -2, +2, -3, ...

1.2

Points, Lines, and Planes

What you should learn

GOAL 1 Understand and use the basic undefined terms and defined terms of geometry.

GOAL 2 Sketch the intersections of lines and planes.

Why you should learn it

▼ To name and draw the basic elements of geometry, including lines that intersect, as in the perspective drawing in Exs. 68–72.



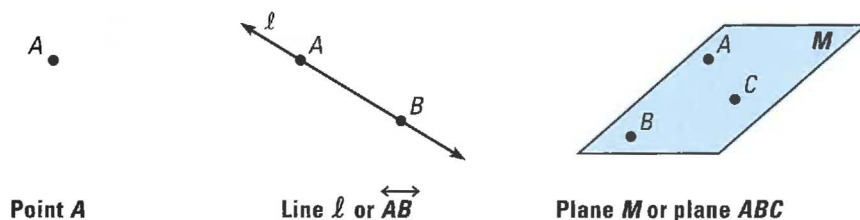
GOAL 1 USING UNDEFINED TERMS AND DEFINITIONS

A **definition** uses known words to describe a new word. In geometry, some words, such as *point*, *line*, and *plane*, are **undefined terms**. Although these words are not formally defined, it is important to have general agreement about what each word means.

A **point** has no dimension. It is usually represented by a small dot.

A **line** extends in one dimension. It is usually represented by a straight line with two arrowheads to indicate that the line extends without end in two directions. In this book, lines are always straight lines.

A **plane** extends in two dimensions. It is usually represented by a shape that looks like a tabletop or wall. You must imagine that the plane extends without end, even though the drawing of a plane appears to have edges.



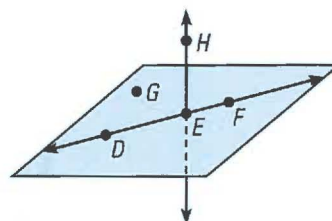
A few basic concepts in geometry must also be commonly understood without being defined. One such concept is the idea that a point *lies on* a line or a plane.

Collinear points are points that lie on the same line.

Coplanar points are points that lie on the same plane.

EXAMPLE 1 Naming Collinear and Coplanar Points

- Name three points that are collinear.
- Name four points that are coplanar.
- Name three points that are not collinear.

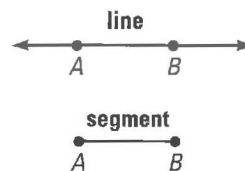


SOLUTION

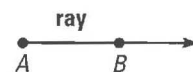
- Points D , E , and F lie on the same line, so they are collinear.
- Points D , E , F , and G lie on the same plane, so they are coplanar. Also, D , E , F , and H are coplanar, although the plane containing them is not drawn.
- There are many correct answers. For instance, points H , E , and G do not lie on the same line.

Another undefined concept in geometry is the idea that a point on a line *is between* two other points on the line. You can use this idea to define other important terms in geometry.

Consider the **line** AB (symbolized by \overleftrightarrow{AB}).
The **line segment** or **segment** AB (symbolized by \overline{AB}) consists of the **endpoints** A and B , and all points on \overleftrightarrow{AB} that are between A and B .



The **ray** AB (symbolized by \overrightarrow{AB}) consists of the **initial point** A and all points on \overleftrightarrow{AB} that lie on the same side of A as point B .



Note that \overleftrightarrow{AB} is the same as \overleftrightarrow{BA} , and \overline{AB} is the same as \overline{BA} . However, \overrightarrow{AB} and \overrightarrow{BA} are *not* the same.



They have different initial points and extend in different directions.

If C is between A and B , then \overrightarrow{CA} and \overrightarrow{CB} are **opposite rays**.

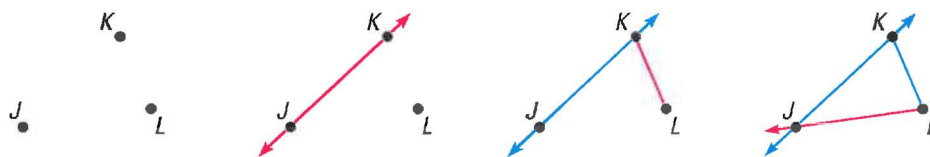


Like points, segments and rays are collinear if they lie on the same line. So, any two opposite rays are collinear. Segments, rays, and lines are coplanar if they lie on the same plane.

EXAMPLE 2 Drawing Lines, Segments, and Rays

Draw three noncollinear points, J , K , and L . Then draw \overleftrightarrow{JK} , \overline{KL} and \overrightarrow{LJ} .

SOLUTION



- 1 Draw J , K , and L .
- 2 Draw \overrightarrow{JK} .
- 3 Draw \overline{KL} .
- 4 Draw \overrightarrow{LJ} .

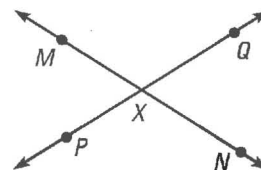
EXAMPLE 3 Drawing Opposite Rays

Draw two lines. Label points on the lines and name two pairs of opposite rays.

SOLUTION

Points M , N , and X are collinear and X is between M and N . So, \overrightarrow{XM} and \overrightarrow{XN} are opposite rays.

Points P , Q , and X are collinear and X is between P and Q . So, \overrightarrow{XP} and \overrightarrow{XQ} are opposite rays.



GOAL 2 SKETCHING INTERSECTIONS OF LINES AND PLANES

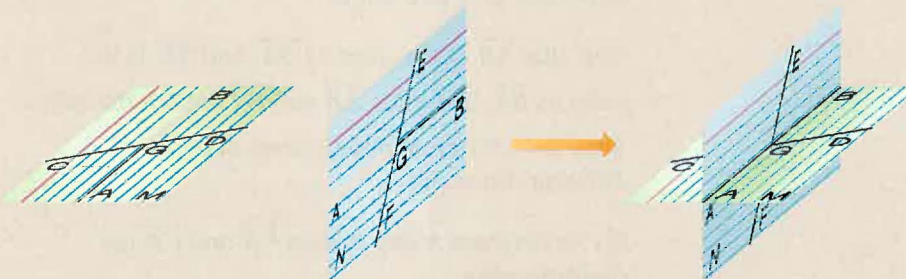
Two or more geometric figures **intersect** if they have one or more points in common. The **intersection** of the figures is the set of points the figures have in common.

ACTIVITY

Developing
Concepts

Modeling Intersections

Use two index cards. Label them as shown and cut slots halfway along each card.



1. What is the intersection of \overline{AB} and \overline{CD} ? of \overline{AB} and \overline{EF} ?
2. Slide the cards together. What is the intersection of \overline{CD} and \overline{EF} ?
3. What is the intersection of planes M and N ?
4. Are \overline{CD} and \overline{EF} coplanar? Explain.

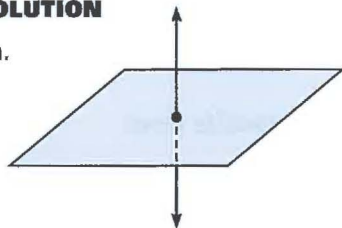
EXAMPLE 4 Sketching Intersections

Sketch the figure described.

- a. a line that intersects a plane in one point
- b. two planes that intersect in a line

SOLUTION

a.

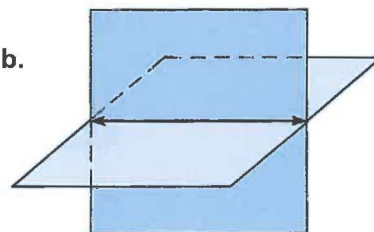


Draw a plane and a line.

Emphasize the point where they meet.

Dashes indicate where the line is hidden by the plane.

b.



Draw two planes.

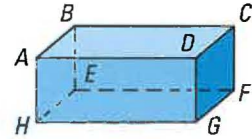
Emphasize the line where they meet.

Dashes indicate where one plane is hidden by the other plane.

STUDENT HELP
INTERNET **HOMEWORK HELP**
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for extra examples.

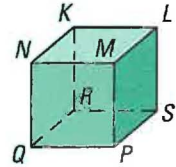
NAMING COPLANAR POINTS Name a point that is coplanar with the given points.

28. A , B , and C 29. D , C , and F
 30. G , A , and D 31. E , F , and G
 32. A , B , and H 33. B , C , and F
 34. A , B , and F 35. B , C , and G



NAMING NONCOPLANAR POINTS Name all the points that are not coplanar with the given points.

36. N , K , and L 37. S , P , and M
 38. P , Q , and N 39. R , S , and L
 40. P , Q , and R 41. R , K , and N
 42. P , S , and K 43. Q , K , and L



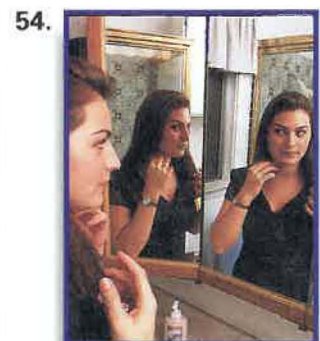
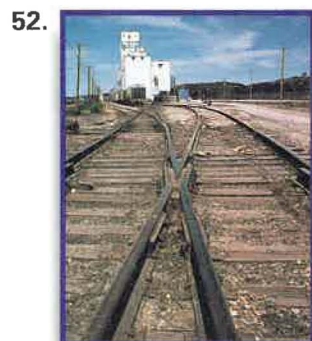
COMPLETING DEFINITIONS Complete the sentence.

44. \overline{AB} consists of the endpoints A and B and all the points on the line \overleftrightarrow{AB} that lie ____? ____.
 45. \overrightarrow{CD} consists of the initial point C and all points on the line \overleftrightarrow{CD} that lie ____? ____.
 46. Two rays or segments are collinear if they ____? ____.
 47. \overrightarrow{CA} and \overrightarrow{CB} are opposite rays if ____? ____.

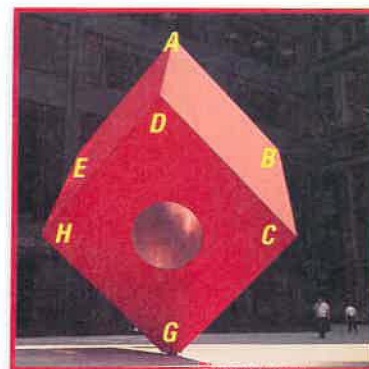
SKETCHING FIGURES Sketch the lines, segments, and rays.

48. Draw four points J , K , L , and M , no three of which are collinear. Then sketch \overleftrightarrow{JK} , \overline{KL} , \overleftarrow{LM} , and \overrightarrow{MJ} .
 49. Draw five points P , Q , R , S , and T , no three of which are collinear. Then sketch \overleftrightarrow{PQ} , \overleftarrow{RS} , \overline{QR} , \overline{ST} , and \overrightarrow{TP} .
 50. Draw two points, X and Y . Then sketch \overleftrightarrow{XY} . Add a point W between X and Y so that \overrightarrow{WX} and \overrightarrow{WY} are opposite rays.
 51. Draw two points, A and B . Then sketch \overrightarrow{AB} . Add a point C on the ray so that B is between A and C .

EVERYDAY INTERSECTIONS What kind of geometric intersection does the photograph suggest?



COMPLETING SENTENCES Fill in each blank with the appropriate response based on the points labeled in the photograph.



Red Cube, by sculptor Isamu Noguchi

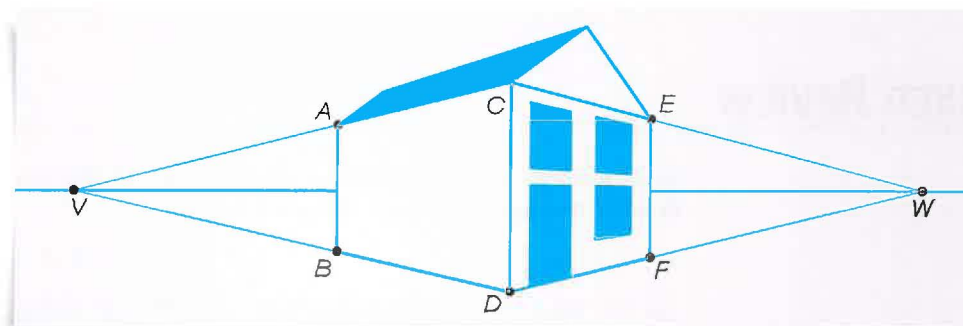
55. \overleftrightarrow{AB} and \overleftrightarrow{BC} intersect at ? .
56. \overleftrightarrow{AD} and \overleftrightarrow{AE} intersect at ? .
57. \overleftrightarrow{HG} and \overleftrightarrow{DH} intersect at ? .
58. Plane ABC and plane DCG intersect at ? .
59. Plane GHD and plane DHE intersect at ? .
60. Plane EAD and plane BCD intersect at ? .

SKETCHING FIGURES Sketch the figure described.

61. Three points that are coplanar but not collinear.
62. Two lines that lie in a plane but do not intersect.
63. Three lines that intersect in a point and all lie in the same plane.
64. Three lines that intersect in a point but do not all lie in the same plane.
65. Two lines that intersect and another line that does not intersect either one.
66. Two planes that do not intersect.
67. Three planes that intersect in a line.

 **TWO-POINT PERSPECTIVE** In Exercises 68–72, use the information and diagram below.

In *perspective drawing*, lines that do not intersect in real life are represented in a drawing by lines that appear to intersect at a point far away on the horizon. This point is called a *vanishing point*.

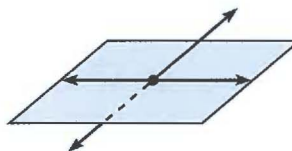


The diagram shows a drawing of a house with two vanishing points. You can use the vanishing points to draw the hidden parts of the house.

68. Name two lines that intersect at vanishing point V .
69. Name two lines that intersect at vanishing point W .
70. Trace the diagram. Draw \overleftrightarrow{EV} and \overleftrightarrow{AW} . Label their intersection as G .
71. Draw \overleftrightarrow{FV} and \overleftrightarrow{BW} . Label their intersection as H .
72. Draw the hidden edges of the house: \overline{AG} , \overline{EG} , \overline{BH} , \overline{FH} , and \overline{GH} .

73. MULTIPLE CHOICE Which statement(s) are true about the two lines shown in the drawing to the right?

- I. The lines intersect in one point.
- II. The lines do not intersect.
- III. The lines are coplanar.



- (A) I only (B) I and II only (C) I and III only
- (D) II and III only (E) I, II, and III

74. MULTIPLE CHOICE What is the intersection of \overrightarrow{PQ} and \overrightarrow{QP} ?

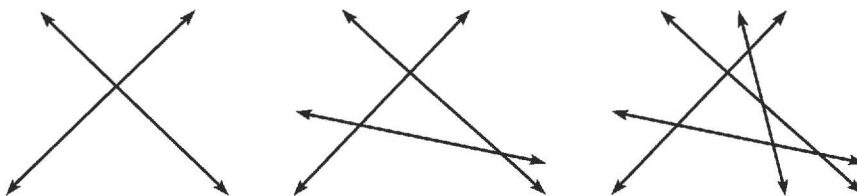
- (A) \overrightarrow{PQ} (B) \overrightarrow{PQ} (C) P and Q (D) P only (E) Q only

75. MULTIPLE CHOICE Points K , L , M , and N are not coplanar. What is the intersection of plane KLM and plane KLN ?

- (A) K and L (B) M and N (C) \overline{KL} (D) \overleftrightarrow{KL}
- (E) The planes do not intersect.

★ Challenge

76. INTERSECTING LINES In each diagram below, every line intersects all the other lines, but only two lines pass through each intersection point.



Can you draw 5 lines that intersect in this way? 6 lines? Is there a pattern to the number of intersection points?

EXTRA CHALLENGE

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MIXED REVIEW

DESCRIBING PATTERNS Describe a pattern in the sequence of numbers. Predict the next number. (Review 1.1)

- 77. 1, 6, 36, 216, ... 78. 2, -2, 2, -2, 2, ...
- 79. 8.1, 88.11, 888.111, 8888.1111, ... 80. 0, 3, 9, 18, 30, ...

OPERATIONS WITH INTEGERS Simplify the expression. (Skills Review, p.785)

- 81. $0 - 2$ 82. $3 - 9$ 83. $9 - (-4)$ 84. $-5 - (-2)$
- 85. $5 - 0$ 86. $4 - 7$ 87. $3 - (-8)$ 88. $-7 - (-5)$

RADICAL EXPRESSIONS Simplify the expression. Round your answer to two decimal places. (Skills Review, p. 799, for 1.3)

- 89. $\sqrt{21 + 100}$ 90. $\sqrt{40 + 60}$ 91. $\sqrt{25 + 144}$ 92. $\sqrt{9 + 16}$
- 93. $\sqrt{5^2 + 7^2}$ 94. $\sqrt{3^2 + (-2)^2}$ 95. $\sqrt{(-3)^2 + 3^2}$ 96. $\sqrt{(-5)^2 + 10^2}$

1.3

Segments and Their Measures

What you should learn

GOAL 1 Use segment postulates.

GOAL 2 Use the Distance Formula to measure distances, as applied in Exs. 45–54.

Why you should learn it

▼ To solve **real-life** problems, such as finding distances along a diagonal city street in Example 4.



GOAL 1 USING SEGMENT POSTULATES

In geometry, rules that are accepted without proof are called **postulates** or **axioms**. Rules that are proved are called *theorems*. In this lesson, you will study two postulates about the lengths of segments.

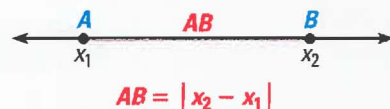
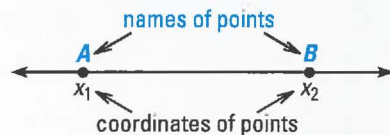
POSTULATE

POSTULATE 1 *Ruler Postulate*

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the **coordinate** of the point.

The **distance** between points A and B , written as AB , is the absolute value of the difference between the coordinates of A and B .

AB is also called the **length** of \overline{AB} .



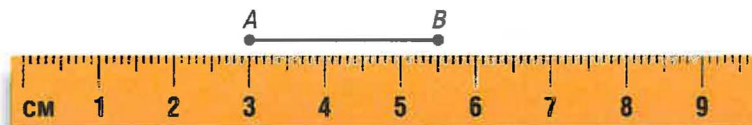
EXAMPLE 1 Finding the Distance Between Two Points

Measure the length of the segment to the nearest millimeter.



SOLUTION

Use a metric ruler. Align one mark of the ruler with A . Then estimate the coordinate of B . For example, if you align A with 3, B appears to align with 5.5.



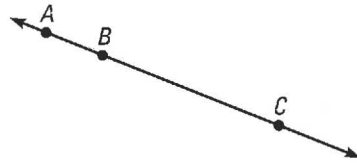
$$AB = |5.5 - 3| = |2.5| = 2.5$$

► The distance between A and B is about 2.5 cm.

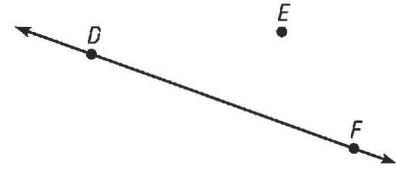
.....

It doesn't matter how you place the ruler. For example, if the ruler in Example 1 is placed so that A is aligned with 4, then B aligns with 6.5. The difference in the coordinates is the same.

When three points lie on a line, you can say that one of them is **between** the other two. This concept applies to collinear points only. For instance, in the figures below, point B is between points A and C , but point E is not between points D and F .



Point B is between points A and C .



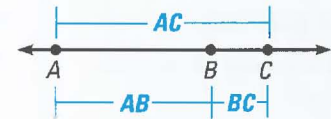
Point E is not between points D and F .

POSTULATE

POSTULATE 2 Segment Addition Postulate

If B is between A and C , then $AB + BC = AC$.

If $AB + BC = AC$, then B is between A and C .



EXAMPLE 2 Finding Distances on a Map



MAP READING Use the map to find the distances between the three cities that lie on a line.

SOLUTION

Using the scale on the map, you can estimate that the distance between Athens and Macon is

$$AM = 80 \text{ miles.}$$

The distance between Macon and Albany is

$$MB = 90 \text{ miles.}$$

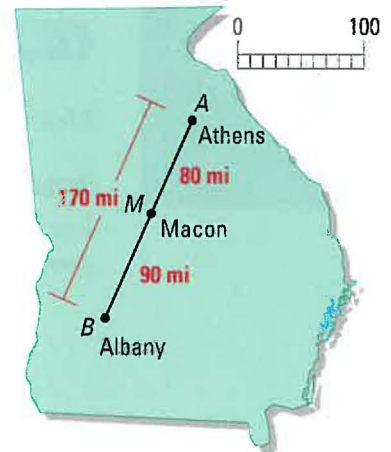
Knowing that Athens, Macon, and Albany lie on the same line, you can use the Segment Addition Postulate to conclude that the distance between Athens and Albany is

$$AB = AM + MB = 80 + 90 = 170 \text{ miles.}$$

.....

The Segment Addition Postulate can be generalized to three or more segments, as long as the segments lie on a line. If P , Q , R , and S lie on a line as shown, then

$$PS = PQ + QR + RS.$$



GOAL 2 USING THE DISTANCE FORMULA

STUDENT HELP

Study Tip

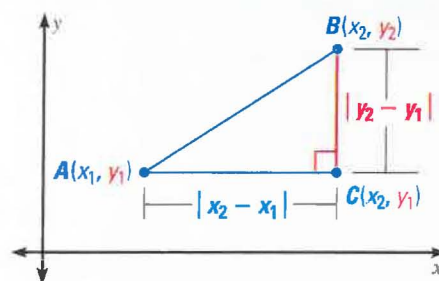
The small numbers in x_1 and x_2 are called *subscripts*. You read them as “x sub 1” and “x sub 2.”

The **Distance Formula** is a formula for computing the distance between two points in a *coordinate plane*.

THE DISTANCE FORMULA

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



EXAMPLE 3 Using the Distance Formula

Find the lengths of the segments. Tell whether any of the segments have the same length.

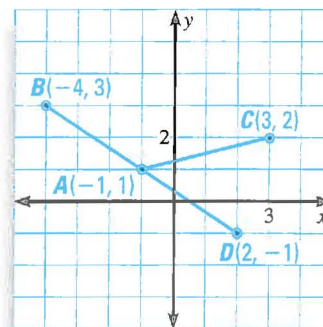
SOLUTION

Use the Distance Formula.

$$\begin{aligned} AB &= \sqrt{[(-4) - (-1)]^2 + (3 - 1)^2} \\ &= \sqrt{(-3)^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{[3 - (-1)]^2 + (2 - 1)^2} \\ &= \sqrt{4^2 + 1^2} = \sqrt{16 + 1} = \sqrt{17} \end{aligned}$$

$$\begin{aligned} AD &= \sqrt{[2 - (-1)]^2 + (-1 - 1)^2} \\ &= \sqrt{3^2 + (-2)^2} = \sqrt{9 + 4} = \sqrt{13} \end{aligned}$$



► So, \overline{AB} and \overline{AD} have the same length, but \overline{AC} has a different length.

Segments that have the same length are called **congruent segments**. For instance, in Example 3, \overline{AB} and \overline{AD} are congruent because each has a length of $\sqrt{13}$. There is a special symbol, \cong , for indicating *congruence*.

LENGTHS ARE EQUAL.

$$AB = AD$$

“is equal to”

SEGMENTS ARE CONGRUENT.

$$\overline{AB} \cong \overline{AD}$$

“is congruent to”

The Distance Formula is based on the *Pythagorean Theorem*, which you will see again when you work with right triangles in Chapter 9.

STUDENT HELP

Study Tip

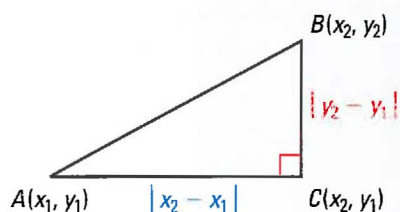
The red mark at one corner of each triangle indicates a right angle.

CONCEPT SUMMARY

DISTANCE FORMULA AND PYTHAGOREAN THEOREM

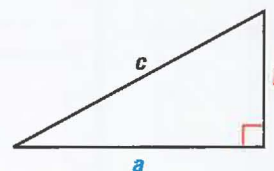
DISTANCE FORMULA

$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$



PYTHAGOREAN THEOREM

$$c^2 = a^2 + b^2$$

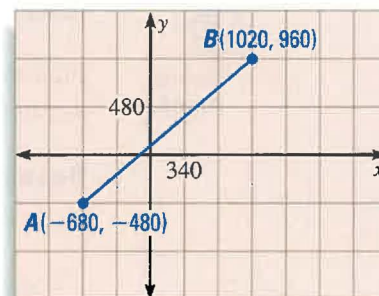


EXAMPLE 4 Finding Distances on a City Map



MAP READING On the map, the city blocks are 340 feet apart east-west and 480 feet apart north-south.

- Find the walking distance between A and B.
- What would the distance be if a diagonal street existed between the two points?



SOLUTION

- To walk from A to B, you would have to walk five blocks east and three blocks north.

$$5 \text{ blocks} \cdot 340 \frac{\text{feet}}{\text{block}} = 1700 \text{ feet}$$

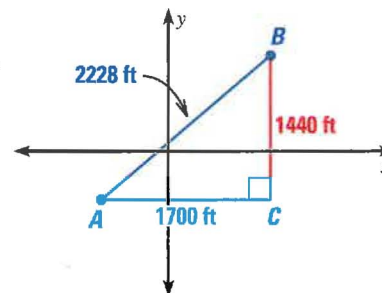
$$3 \text{ blocks} \cdot 480 \frac{\text{feet}}{\text{block}} = 1440 \text{ feet}$$

- ▶ So, the walking distance is **1700 + 1440**, which is a total of **3140** feet.

- To find the diagonal distance between A and B, use the Distance Formula.

$$\begin{aligned} AB &= \sqrt{[1020 - (-680)]^2 + [960 - (-480)]^2} \\ &= \sqrt{1700^2 + 1440^2} \\ &= \sqrt{4,963,600} \approx \mathbf{2228} \text{ feet} \end{aligned}$$

- ▶ So, the diagonal distance would be about 2228 feet, which is 912 feet less than the walking distance.



STUDENT HELP

Study Tip

If you use a calculator to compute distances, use the parenthesis keys to group what needs to be squared.

GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

1. What is a *postulate*?
2. Draw a sketch of three collinear points. Label them. Then write the Segment Addition Postulate for the points.
3. Use the diagram. How can you determine BD if you know BC and CD ? if you know AB and AD ?



Skill Check ✓

Find the distance between the two points.

- | | | |
|--------------------------|------------------------|--------------------------|
| 4. $C(0, 0), D(5, 2)$ | 5. $G(3, 0), H(8, 10)$ | 6. $M(1, -3), N(3, 5)$ |
| 7. $P(-8, -6), Q(-3, 0)$ | 8. $S(7, 3), T(1, -5)$ | 9. $V(-2, -6), W(1, -2)$ |

Use the Distance Formula to decide whether $\overline{JK} \cong \overline{KL}$.

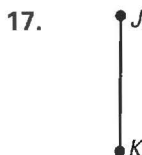
- | | | |
|---|--|--|
| 10. $J(3, -5)$
$K(-1, 2)$
$L(-5, -5)$ | 11. $J(0, -8)$
$K(4, 3)$
$L(-2, -7)$ | 12. $J(10, 2)$
$K(7, -3)$
$L(4, -8)$ |
|---|--|--|

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 803.

MEASUREMENT Measure the length of the segment to the nearest millimeter.

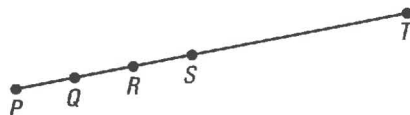


BETWEENNESS Draw a sketch of the three collinear points. Then write the Segment Addition Postulate for the points.

- | | |
|----------------------------------|----------------------------------|
| 19. E is between D and F . | 20. H is between G and J . |
| 21. M is between N and P . | 22. R is between Q and S . |

LOGICAL REASONING In the diagram of the collinear points, $PT = 20$, $QS = 6$, and $PQ = QR = RS$. Find each length.

- | | |
|----------|----------|
| 23. QR | 24. RS |
| 25. PQ | 26. ST |
| 27. RP | 28. RT |
| 29. SP | 30. QT |



STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 13–18
Example 2: Exs. 19–33
Example 3: Exs. 34–43
Example 4: Exs. 44–54

xy USING ALGEBRA Suppose M is between L and N . Use the Segment Addition Postulate to solve for the variable. Then find the lengths of \overline{LM} , \overline{MN} , and \overline{LN} .

31. $LM = 3x + 8$

$MN = 2x - 5$

$LN = 23$

32. $LM = 7y + 9$

$MN = 3y + 4$

$LN = 143$

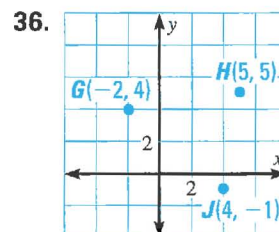
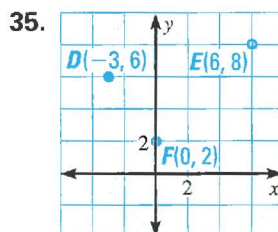
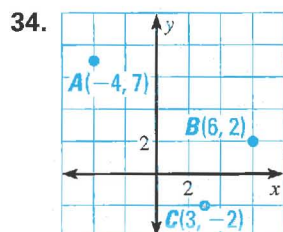
33. $LM = \frac{1}{2}z + 2$

$MN = 3z + \frac{3}{2}$

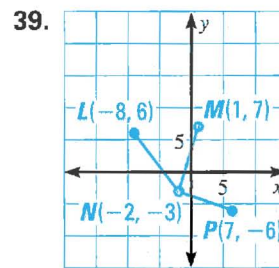
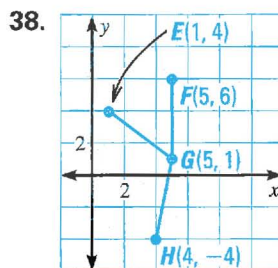
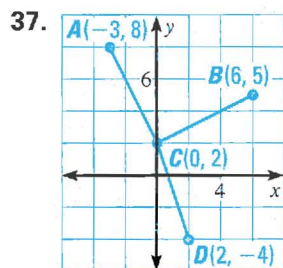
$LN = 5z + 2$

STUDENT HELP
INTERNET **HOMEWORK HELP**
 Visit our Web site
www.mcdougallittell.com
 for help with Exs. 34–36.

DISTANCE FORMULA Find the distance between each pair of points.



DISTANCE FORMULA Find the lengths of the segments. Tell whether any of the segments have the same length.



CONGRUENCE Use the Distance Formula to decide whether $\overline{PQ} \cong \overline{QR}$.

40. $P(4, -4)$

$Q(1, -6)$

$R(-1, -3)$

41. $P(-1, -6)$

$Q(-8, 5)$

$R(3, -2)$

42. $P(5, 1)$

$Q(-5, -7)$

$R(-3, 6)$

43. $P(-2, 0)$

$Q(10, -14)$

$R(-4, -2)$

CAMBRIA INCLINE In Exercises 44 and 45, use the information about the incline railway given below.

In the days before automobiles were available, railways called “inclines” brought people up and down hills in many cities. In Johnstown, Pennsylvania, the Cambria Incline was reputedly the steepest in the world when it was completed in 1893. It rises about 514 feet vertically as it moves 734 feet horizontally.

44. On graph paper, draw a coordinate plane and mark the axes using a scale that allows you to plot $(0, 0)$ and $(734, 514)$. Plot the points and connect them with a segment to represent the incline track.
45. Use the Distance Formula to estimate the length of the track.

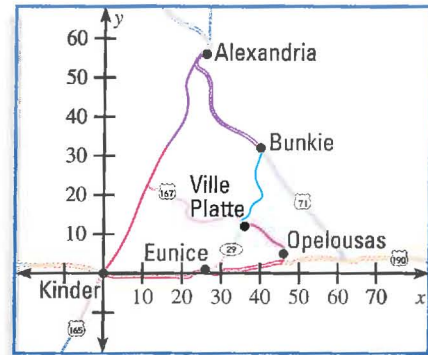


Workers constructing the Cambria Incline

DRIVING DISTANCES In Exercises 46 and 47, use the map of cities in Louisiana shown below. Coordinates on the map are given in miles.

The coordinates of Alexandria, Kinder, Eunice, Opelousas, Ville Platte, and Bunkie are $A(26, 56)$, $K(0, 0)$, $E(26, 1)$, $O(46, 5)$, $V(36, 12)$, and $B(40, 32)$.

46. What is the shortest flying distance between Eunice and Alexandria?
47. Using only roads shown on the map, what is the approximate shortest driving distance between Eunice and Alexandria?

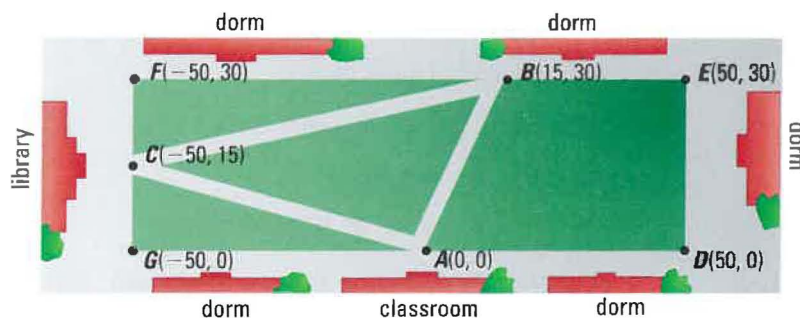


LONG-DISTANCE RATES In Exercises 48–52, find the distance between the two cities using the information given in the table, which is from a coordinate system used for calculating long-distance telephone rates.

Buffalo, NY	(5075, 2326)	Omaha, NE	(6687, 4595)
Chicago, IL	(5986, 3426)	Providence, RI	(4550, 1219)
Dallas, TX	(8436, 4034)	San Diego, CA	(9468, 7629)
Miami, FL	(8351, 527)	Seattle, WA	(6336, 8896)

48. Buffalo and Dallas
49. Chicago and Seattle
50. Miami and Omaha
51. Providence and San Diego
52. The long-distance coordinate system is measured in units of $\sqrt{0.1}$ mile. Convert the distances you found in Exs. 48–51 to miles.

CAMPUS PATHWAYS In Exercises 53 and 54, use the campus map below. Sidewalks around the edge of a campus quadrangle connect the buildings. Students sometimes take shortcuts by walking across the grass along the pathways shown. The coordinate system shown is measured in yards.



53. Find the distances from A to B , from B to C , and from C to A if you have to walk around the quadrangle along the sidewalks.
54. Find the distances from A to B , from B to C , and from C to A if you are able to walk across the grass along the pathways.

55. **MULTIPLE CHOICE** Points K and L are on \overline{AB} . If $AK > BL$, then which statement must be true?
 (A) $AK < KB$ (B) $AL < LB$ (C) $AL > BK$
 (D) $KL < LB$ (E) $AL + BK > AB$
56. **MULTIPLE CHOICE** Suppose point M lies on \overline{CD} , $CM = 2 \cdot MD$, and $CD = 18$. What is the length of MD ?
 (A) 3 (B) 6 (C) 9 (D) 12 (E) 36

★ Challenge

THREE-DIMENSIONAL DISTANCE In Exercises 57–59, use the following information to find the distance between the pair of points.

In a three-dimensional coordinate system, the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is


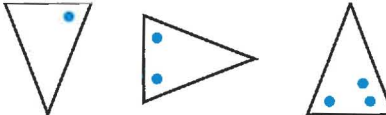
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

EXTRA CHALLENGE
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57. $P(0, 20, -32)$ 58. $A(-8, 15, -4)$ 59. $F(4, -42, 60)$
 $Q(2, -10, -20)$ $B(10, 1, -6)$ $G(-7, -11, 38)$

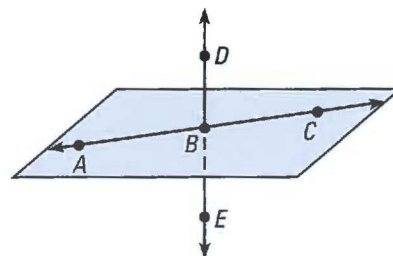
MIXED REVIEW

SKETCHING VISUAL PATTERNS Sketch the next figure in the pattern. (Review 1.1)

60. 
61. 

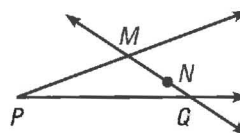
EVALUATING STATEMENTS Determine if the statement is *true* or *false*. (Review 1.2)

62. E lies on \overleftrightarrow{BD} .
 63. E lies on \overrightarrow{BD} .
 64. A , B , and D are collinear.
 65. \overrightarrow{BD} and \overrightarrow{BE} are opposite rays.
 66. B lies in plane ADC .
 67. The intersection of \overleftrightarrow{DE} and \overleftrightarrow{AC} is B .



NAMING RAYS Name the ray described. (Review 1.2 for 1.4)

68. Name a ray that contains M .
 69. Name a ray that has N as an endpoint.
 70. Name two rays that intersect at P .
 71. Name a pair of opposite rays.




Write the next number in the sequence. (Lesson 1.1)

1. 10, 9.5, 9, 8.5, ...

2. 0, 2, -2, 4, -4, ...

Sketch the figure described. (Lesson 1.2)

3. Two segments that do not intersect.
4. Two lines that do not intersect, and a third line that intersects each of them.
5. Two lines that intersect a plane at the same point.
6. Three planes that do not intersect.
7.  **MINIATURE GOLF** At a miniature golf course, a water hazard blocks the direct shot from the tee at $T(0, 0)$ to the cup at $C(-1, 7)$. If you hit the ball so it bounces off an angled wall at $B(3, 4)$, it will go into the cup. The coordinate system is measured in feet. Draw a diagram of the situation. Find TB and BC . (Lesson 1.3)

MATH & History

Geometric Constructions



APPLICATION LINK

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THEN

MORE THAN 2000 YEARS AGO, the Greek mathematician Euclid published a 13 volume work called *The Elements*. In his systematic approach, figures are *constructed* using only a *compass* and a *straightedge* (a ruler without measuring marks).

NOW

TODAY, geometry software may be used to construct geometric figures. Programs allow you to perform constructions as if you have only a compass and straightedge. They also let you make measurements of lengths, angles, and areas.

1. Draw two points and use a *straightedge* to construct the line that passes through them.
2. With the points as centers, use a *compass* to draw two circles of different sizes so that the circles intersect in two points. Mark the two points of intersection and construct the line through them.
3. Connect the four points you constructed. What are the properties of the shape formed?



An early printed edition of *The Elements*



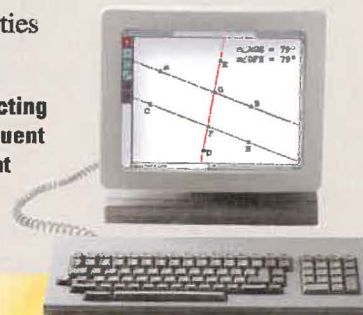
Euclid develops *The Elements*.

c. 300 B.C.



Gauss proves constructing a shape with 17 congruent sides and 17 congruent angles is possible.

1796



1990s

Geometry software duplicates the tools for construction on screen.

1.4

Angles and Their Measures

What you should learn

GOAL 1 Use angle postulates.

GOAL 2 Classify angles as acute, right, obtuse, or straight.

Why you should learn it

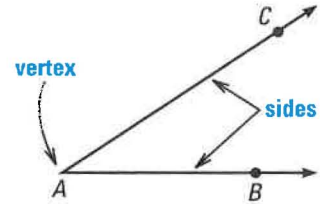
▼ To solve **real-life** problems about angles, such as the field of vision of a horse wearing blinkers in **Example 2**.



GOAL 1 USING ANGLE POSTULATES

An **angle** consists of two different rays that have the same initial point. The rays are the **sides** of the angle. The initial point is the **vertex** of the angle.

The angle that has sides \overrightarrow{AB} and \overrightarrow{AC} is denoted by $\angle BAC$, $\angle CAB$, or $\angle A$. The point A is the vertex of the angle.



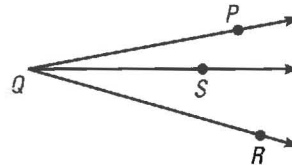
EXAMPLE 1 Naming Angles

Name the angles in the figure.

SOLUTION

There are three different angles.

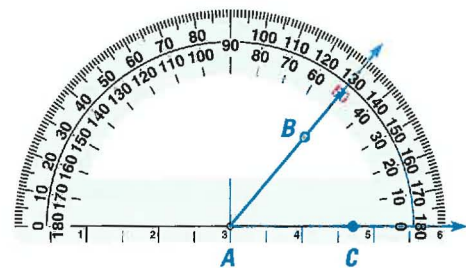
- $\angle PQS$ or $\angle SQP$
- $\angle SQR$ or $\angle RQS$
- $\angle PQR$ or $\angle RQP$



You should not name any of these angles as $\angle Q$ because all three angles have Q as their vertex. The name $\angle Q$ would not distinguish one angle from the others.

The *measure* of $\angle A$ is denoted by $m\angle A$. The measure of an angle can be approximated with a protractor, using units called *degrees* ($^\circ$). For instance, $\angle BAC$ has a measure of 50° , which can be written as

$$m\angle BAC = 50^\circ.$$



Angles that have the same measure are called **congruent angles**. For instance, $\angle BAC$ and $\angle DEF$ each have a measure of 50° , so they are congruent.

MEASURES ARE EQUAL.

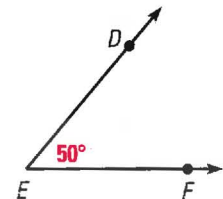
$$m\angle BAC = m\angle DEF$$

“is equal to”

ANGLES ARE CONGRUENT.

$$\angle BAC \cong \angle DEF$$

“is congruent to”



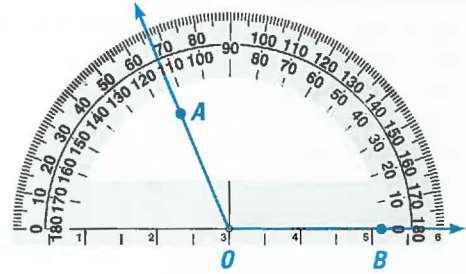


POSTULATE

POSTULATE 3 Protractor Postulate

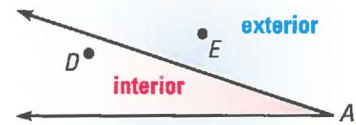
Consider a point A on one side of \overleftrightarrow{OB} . The rays of the form \overrightarrow{OA} can be matched one to one with the real numbers from 0 to 180.

The **measure** of $\angle AOB$ is equal to the absolute value of the difference between the real numbers for \overrightarrow{OA} and \overrightarrow{OB} .



A point is in the **interior** of an angle if it is between points that lie on each side of the angle.

A point is in the **exterior** of an angle if it is not on the angle or in its interior.

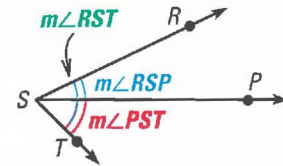


POSTULATE

POSTULATE 4 Angle Addition Postulate

If P is in the interior of $\angle RST$, then

$$m\angle RSP + m\angle PST = m\angle RST.$$

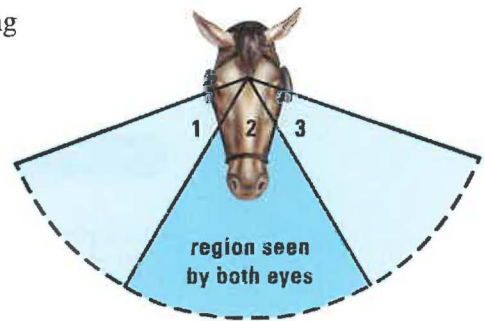


EXAMPLE 2 Calculating Angle Measures

STUDENT HELP
Study Tip
 As shown in Example 2, it is sometimes easier to label angles with numbers instead of letters.

REAL LIFE VISION Each eye of a horse wearing blinkers has an angle of vision that measures 100° . The angle of vision that is seen by both eyes measures 60° .

Find the angle of vision seen by the left eye alone.



SOLUTION

You can use the Angle Addition Postulate.

- $m\angle 2 + m\angle 3 = 100^\circ$ **Total vision for left eye is 100° .**
- $m\angle 3 = 100^\circ - m\angle 2$ **Subtract $m\angle 2$ from each side.**
- $m\angle 3 = 100^\circ - 60^\circ$ **Substitute 60° for $m\angle 2$.**
- $m\angle 3 = 40^\circ$ **Subtract.**

► So, the vision for the left eye alone measures 40° .

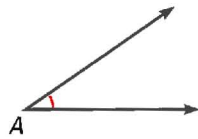
GOAL 2 CLASSIFYING ANGLES

STUDENT HELP

Study Tip

The mark used to indicate a right angle resembles the corner of a square, which has four right angles.

Angles are classified as **acute**, **right**, **obtuse**, and **straight**, according to their measures. Angles have measures greater than 0° and less than or equal to 180° .



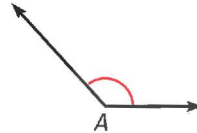
Acute angle

$$0^\circ < m\angle A < 90^\circ$$



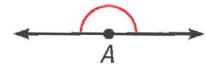
Right angle

$$m\angle A = 90^\circ$$



Obtuse angle

$$90^\circ < m\angle A < 180^\circ$$



Straight angle

$$m\angle A = 180^\circ$$

EXAMPLE 3 Classifying Angles in a Coordinate Plane

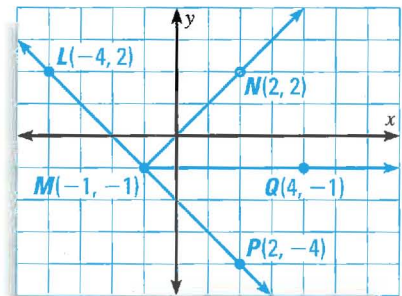
Plot the points $L(-4, 2)$, $M(-1, -1)$, $N(2, 2)$, $Q(4, -1)$, and $P(2, -4)$. Then measure and classify the following angles as acute, right, obtuse, or straight.

- a. $\angle LMN$ b. $\angle LMP$ c. $\angle NMQ$ d. $\angle LMQ$

SOLUTION

Begin by plotting the points. Then use a protractor to measure each angle.

MEASURE	CLASSIFICATION
a. $m\angle LMN = 90^\circ$	right angle
b. $m\angle LMP = 180^\circ$	straight angle
c. $m\angle NMQ = 45^\circ$	acute angle
d. $m\angle LMQ = 135^\circ$	obtuse angle

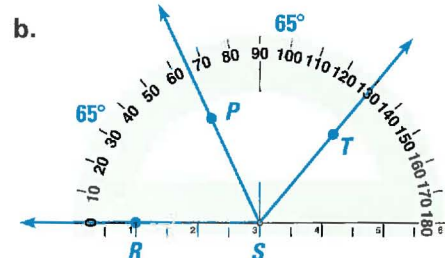
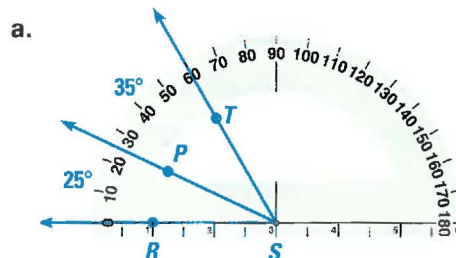


Two angles are **adjacent angles** if they share a common vertex and side, but have no common interior points.

EXAMPLE 4 Drawing Adjacent Angles

Use a protractor to draw two adjacent acute angles $\angle RSP$ and $\angle PST$ so that $\angle RST$ is (a) acute and (b) obtuse.

SOLUTION



STUDENT HELP



HOMEWORK HELP

Visit our Web site
www.mcdougallittell.com
 for extra examples.

GUIDED PRACTICE

Vocabulary Check ✓

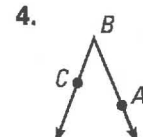
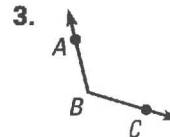
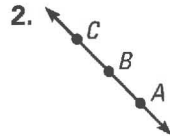
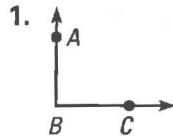
Match the angle with its classification.

A. acute

B. obtuse

C. right

D. straight



Concept Check ✓

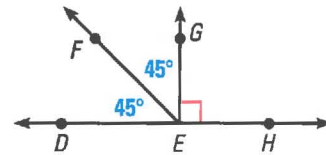
Use the diagram at the right to answer the questions. Explain your answers.

5. Is $\angle DEF \cong \angle FEG$?

6. Is $\angle DEG \cong \angle HEG$?

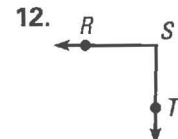
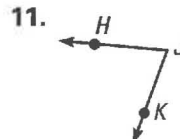
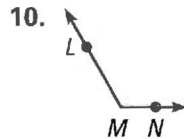
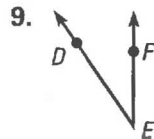
7. Are $\angle DEF$ and $\angle FEH$ adjacent?

8. Are $\angle GED$ and $\angle DEF$ adjacent?



Skill Check ✓

Name the vertex and sides of the angle. Then estimate its measure.



Classify the angle as *acute*, *obtuse*, *right*, or *straight*.

13. $m\angle A = 180^\circ$

14. $m\angle B = 90^\circ$

15. $m\angle C = 100^\circ$

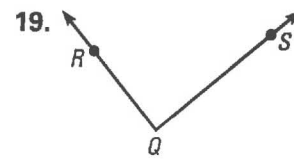
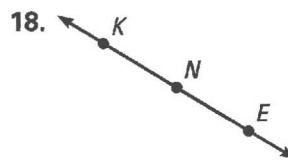
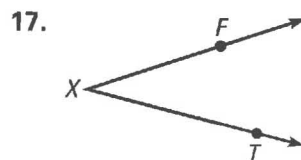
16. $m\angle D = 45^\circ$

PRACTICE AND APPLICATIONS

STUDENT HELP

→ **Extra Practice** to help you master skills is on pp. 803 and 804.

NAMING PARTS Name the vertex and sides of the angle.



STUDENT HELP

→ HOMEWORK HELP

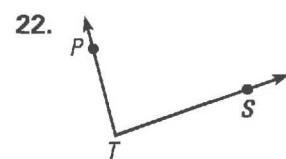
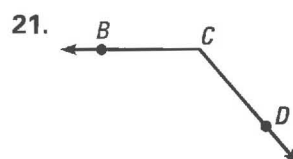
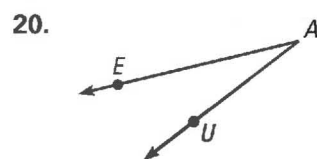
Example 1: Exs. 17–22

Example 2: Exs. 23–34

Example 3: Exs. 35–43

Example 4: Exs. 38, 39

NAMING ANGLES Write two names for the angle.

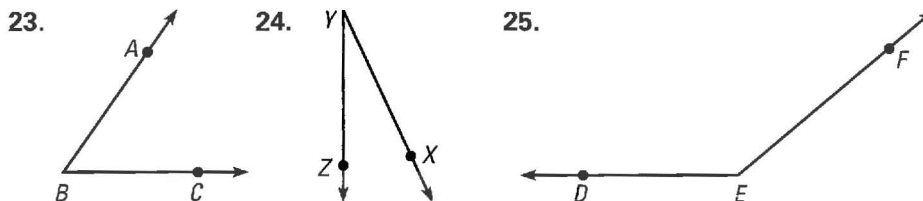




REAL LIFE SURVEYOR
 Surveyors use a tool called a theodolite, which can measure angles to the nearest $1/3600$ of a degree.

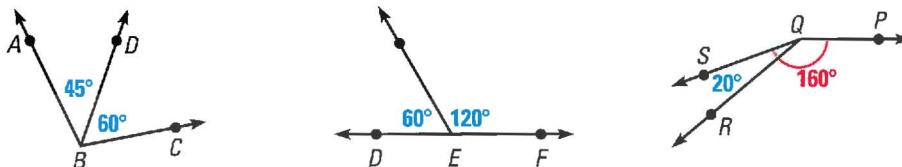
CAREER LINK
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MEASURING ANGLES Copy the angle, extend its sides, and use a protractor to measure it to the nearest degree.



ANGLE ADDITION Use the Angle Addition Postulate to find the measure of the unknown angle.

26. $m\angle ABC = ?$ 27. $m\angle DEF = ?$ 28. $m\angle PQR = ?$

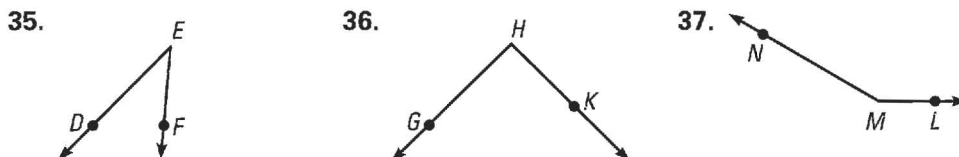


LOGICAL REASONING Draw a sketch that uses all of the following information.

D is in the interior of $\angle BAE$. $m\angle BAC = 130^\circ$
 E is in the interior of $\angle DAF$. $m\angle EAC = 100^\circ$
 F is in the interior of $\angle EAC$. $m\angle BAD = m\angle EAF = m\angle FAC$

29. Find $m\angle FAC$. 30. Find $m\angle BAD$. 31. Find $m\angle FAB$.
 32. Find $m\angle DAE$. 33. Find $m\angle FAD$. 34. Find $m\angle BAE$.

CLASSIFYING ANGLES State whether the angle appears to be *acute*, *right*, *obtuse*, or *straight*. Then estimate its measure.



LOGICAL REASONING Draw five points, A , B , C , D , and E so that all three statements are true.

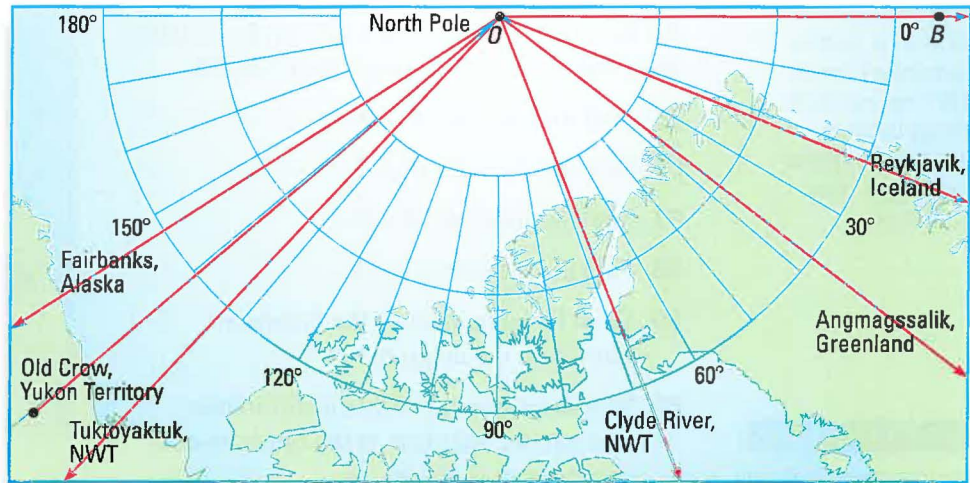
38. $\angle DBE$ is a straight angle.
 $\angle DBA$ is a right angle.
 $\angle ABC$ is a straight angle.
39. C is in the interior of $\angle ADE$.
 $m\angle ADC + m\angle CDE = 120^\circ$.
 $\angle CDB$ is a straight angle.

USING ALGEBRA In a coordinate plane, plot the points and sketch $\angle ABC$. Classify the angle. Write the coordinates of a point that lies in the interior of the angle and the coordinates of a point that lies in the exterior of the angle.

40. $A(3, -2)$ 41. $A(5, -1)$ 42. $A(5, -1)$ 43. $A(-3, 1)$
 $B(5, -1)$ $B(3, -2)$ $B(3, -2)$ $B(-2, 2)$
 $C(4, -4)$ $C(4, -4)$ $C(0, -1)$ $C(-1, 4)$

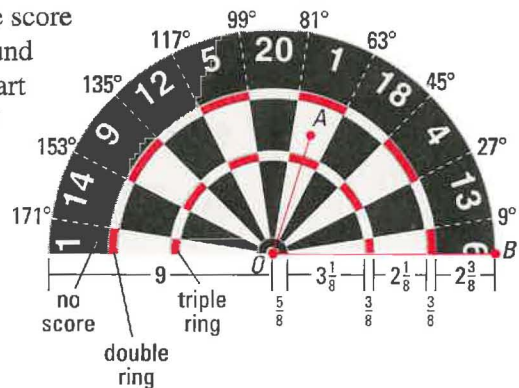
GEOGRAPHY For each city on the polar map, estimate the measure of $\angle BOA$, where B is on the Prime Meridian (0° longitude), O is the North Pole, and A is the city.

44. Clyde River, Canada 45. Fairbanks, Alaska 46. Angmagssalik, Greenland
 47. Old Crow, Canada 48. Reykjavik, Iceland 49. Tuktoyaktuk, Canada



PLAYING DARTS In Exercises 50–53, use the following information to find the score for the indicated dart toss landing at point A .

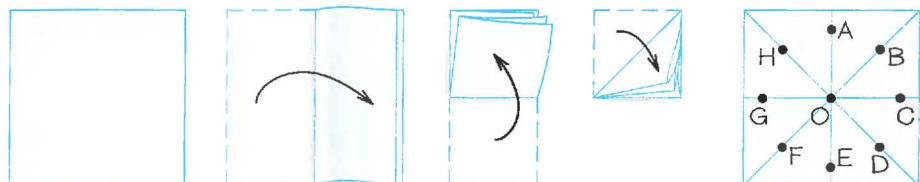
A dartboard is 18 inches across. It is divided into twenty wedges of equal size. The score of a toss is indicated by numbers around the board. The score is doubled if a dart lands in the *double ring* and tripled if it lands in the *triple ring*. Only the top half of the dart board is shown.



50. $m\angle BOA = 160^\circ$; $AO = 3$ in.
 51. $m\angle BOA = 35^\circ$; $AO = 4$ in.
 52. $m\angle BOA = 60^\circ$; $AO = 5$ in.
 53. $m\angle BOA = 90^\circ$; $AO = 6.5$ in.

54. **MULTI-STEP PROBLEM** Use a piece of paper folded in half three times and labeled as shown.

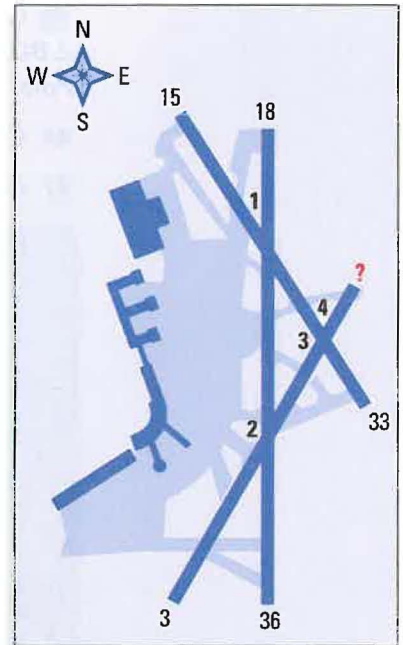
Test Preparation



- Name eight congruent acute angles.
- Name eight right angles.
- Name eight congruent obtuse angles.
- Name two adjacent angles that combine to form a straight angle.

★ Challenge

AIRPORT RUNWAYS In Exercises 55–60, use the diagram of Ronald Reagan Washington National Airport and the information about runway numbering on page 1.



STUDENT HELP

HOMEWORK HELP

Bearings are measured around a circle, so they can have values larger than 180° . You can think of bearings between 180° and 360° as angles that are “bigger” than a straight angle.

EXTRA CHALLENGE

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An airport runway is named by dividing its *bearing* (the angle measured clockwise from due north) by 10. Because a full circle contains 360° , runway numbers range from 1 to 36.

55. Find the measure of $\angle 1$.
56. Find the measure of $\angle 2$.
57. Find the measure of $\angle 3$.
58. Find the measure of $\angle 4$.
59. What is the number of the unlabeled runway in the diagram?
60. **Writing** Explain why the difference between the numbers at the opposite ends of a runway is always 18.

MIXED REVIEW

STUDENT HELP

Skills Review

For help solving equations, see p.790.

USING ALGEBRA Solve for x . (Skills Review, p. 790, for 1.5)

61. $\frac{x+3}{2} = 3$

62. $\frac{5+x}{2} = 5$

63. $\frac{x+4}{2} = -4$

64. $\frac{-8+x}{2} = 12$

65. $\frac{x+7}{2} = -10$

66. $\frac{-9+x}{2} = -7$

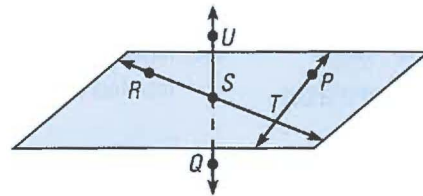
67. $\frac{x+(-1)}{2} = 7$

68. $\frac{8+x}{2} = -1$

69. $\frac{x+(-3)}{2} = -4$

EVALUATING STATEMENTS Decide whether the statement is *true* or *false*. (Review 1.2)

70. U , S , and Q are collinear.
71. T , Q , S , and P are coplanar.
72. \vec{UQ} and \vec{PT} intersect.
73. \vec{SR} and \vec{TS} are opposite rays.



DISTANCE FORMULA Find the distance between the two points. (Review 1.3 for 1.5)

74. $A(3, 10)$, $B(-2, -2)$

75. $C(0, 8)$, $D(-8, 3)$

76. $E(-3, 11)$, $F(4, 4)$

77. $G(10, -2)$, $H(0, 9)$

78. $J(5, 7)$, $K(7, 5)$

79. $L(0, -3)$, $M(-3, 0)$

Folding Bisectors

GROUP ACTIVITY

Work with a partner.

MATERIALS

- rulers
- paper
- protractor
- pencils

QUESTION How can you divide a segment or an angle into two equal parts?

You can fold a piece of paper so that one half of a segment or angle lies exactly on the other half.

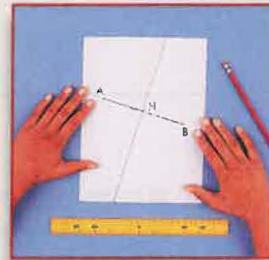
EXPLORING THE CONCEPT: SEGMENT BISECTOR



1 On a piece of paper, draw \overline{AB} .



2 Fold the paper so that B is on top of A .

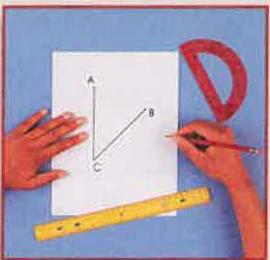


3 Label the point where the fold intersects \overline{AB} as point M .



4 Use a ruler to measure \overline{AM} and \overline{MB} .

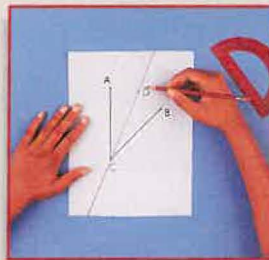
EXPLORING THE CONCEPT: ANGLE BISECTOR



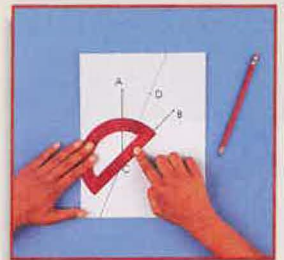
5 On a piece of paper, draw $\angle ACB$.



6 Fold the paper so \overrightarrow{CB} is on top of \overrightarrow{CA} .



7 Draw any point on the fold and label the point D .



8 Use a protractor to measure $\angle ACD$ and $\angle BCD$.

DRAWING CONCLUSIONS

1. What do you notice about the segments you measured in Step 4?
2. What do you notice about the angles you measured in Step 8?

EXTENSION

CRITICAL THINKING Is it possible to fold congruent angles from a straight angle if you are given the vertex of the angle? Explain.

1.5

Segment and Angle Bisectors

What you should learn

GOAL 1 Bisect a segment.

GOAL 2 Bisect an angle, as applied in Exs. 50–55.

Why you should learn it

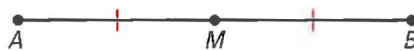
▼ To solve **real-life** problems, such as finding the angle measures of a kite in **Example 4**.



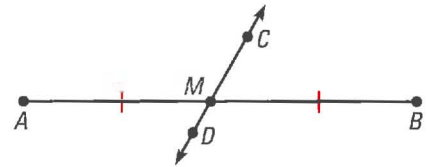
GOAL 1 BISECTING A SEGMENT

The **midpoint** of a segment is the point that divides, or **bisects**, the segment into two congruent segments. In this book, matching red *congruence marks* identify congruent segments in diagrams.

A **segment bisector** is a segment, ray, line, or plane that intersects a segment at its midpoint.



M is the midpoint of \overline{AB} if
 M is on \overline{AB} and $AM = MB$.



\overleftrightarrow{CD} is a bisector of \overline{AB} .

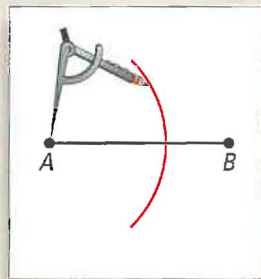
You can use a **compass** and a **straightedge** (a ruler without marks) to **construct** a segment bisector and midpoint of \overline{AB} . A **construction** is a geometric drawing that uses a limited set of tools, usually a compass and a straightedge.

ACTIVITY

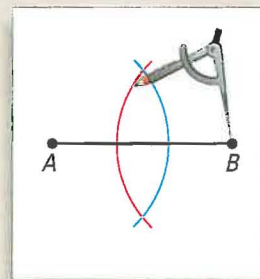
Construction

Segment Bisector and Midpoint

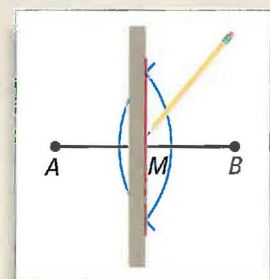
Use the following steps to construct a bisector of \overline{AB} and find the midpoint M of \overline{AB} .



- 1 Place the compass point at A . Use a compass setting greater than half the length of \overline{AB} . Draw an arc.



- 2 Keep the same compass setting. Place the compass point at B . Draw an arc. It should intersect the other arc in two places.



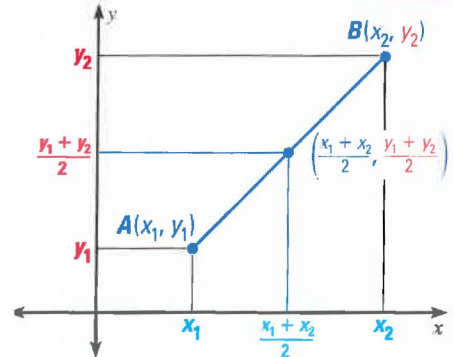
- 3 Use a straightedge to draw a segment through the points of intersection. This segment bisects \overline{AB} at M , the midpoint of \overline{AB} .

If you know the coordinates of the endpoints of a segment, you can calculate the coordinates of the midpoint. You simply take the mean, or average, of the x -coordinates and of the y -coordinates. This method is summarized as the **Midpoint Formula**.

THE MIDPOINT FORMULA

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint of \overline{AB} has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$



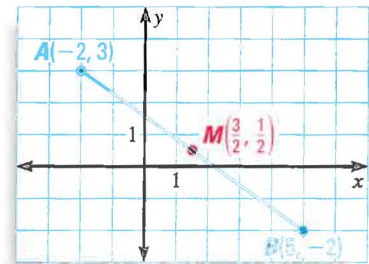
EXAMPLE 1 Finding the Coordinates of the Midpoint of a Segment

Find the coordinates of the midpoint of \overline{AB} with endpoints $A(-2, 3)$ and $B(5, -2)$.

SOLUTION

Use the Midpoint Formula as follows.

$$\begin{aligned} M &= \left(\frac{-2 + 5}{2}, \frac{3 + (-2)}{2} \right) \\ &= \left(\frac{3}{2}, \frac{1}{2} \right) \end{aligned}$$



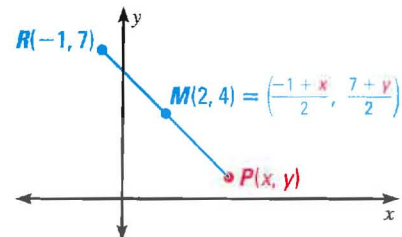
EXAMPLE 2 Finding the Coordinates of an Endpoint of a Segment

The midpoint of \overline{RP} is $M(2, 4)$. One endpoint is $R(-1, 7)$. Find the coordinates of the other endpoint.

SOLUTION

Let (x, y) be the coordinates of P . Use the Midpoint Formula to write equations involving x and y .

$$\begin{aligned} \frac{-1 + x}{2} &= 2 & \frac{7 + y}{2} &= 4 \\ -1 + x &= 4 & 7 + y &= 8 \\ x &= 5 & y &= 1 \end{aligned}$$



► So, the other endpoint of the segment is $P(5, 1)$.

STUDENT HELP

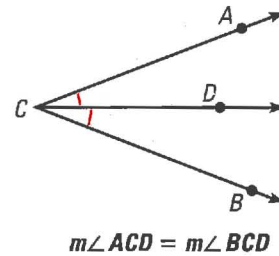
Study Tip

Sketching the points in a coordinate plane helps you check your work. You should sketch a drawing of a problem even if the directions don't ask for a sketch.

GOAL 2 BISECTING AN ANGLE

An **angle bisector** is a ray that divides an angle into two adjacent angles that are congruent. In the diagram at the right, the ray \overrightarrow{CD} bisects $\angle ABC$ because it divides the angle into two congruent angles, $\angle ACD$ and $\angle BCD$.

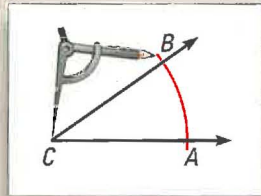
In this book, matching *congruence arcs* identify congruent angles in diagrams.



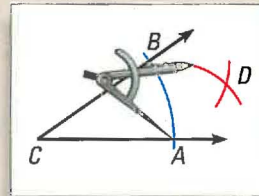
ACTIVITY

Construction Angle Bisector

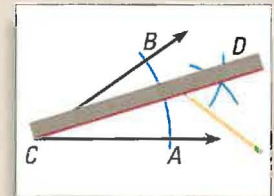
Use the following steps to construct an angle bisector of $\angle C$.



- 1 Place the compass point at C . Draw an arc that intersects both sides of the angle. Label the intersections A and B .



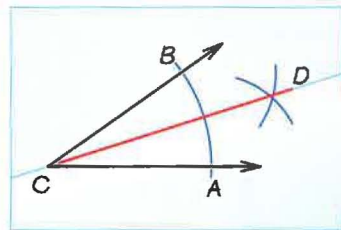
- 2 Place the compass point at A . Draw an arc. Then place the compass point at B . Using the same compass setting, draw another arc.



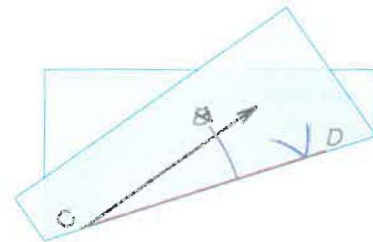
- 3 Label the intersection D . Use a straightedge to draw a ray through C and D . This is the angle bisector.

After you have constructed an angle bisector, you should check that it divides the original angle into two congruent angles. One way to do this is to use a protractor to check that the angles have the same measure.

Another way is to fold the piece of paper along the angle bisector. When you hold the paper up to a light, you should be able to see that the sides of the two angles line up, which implies that the angles are congruent.



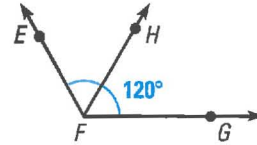
Fold on \overrightarrow{CD} .



The sides of angles $\angle BCD$ and $\angle ACD$ line up.

EXAMPLE 3 Dividing an Angle Measure in Half

The ray \overrightarrow{FH} bisects the angle $\angle EFG$.
Given that $m\angle EFG = 120^\circ$, what are the measures of $\angle EFH$ and $\angle HFG$?



SOLUTION

An angle bisector divides an angle into two congruent angles, each of which has half the measure of the original angle. So,

$$m\angle EFH = m\angle HFG = \frac{120^\circ}{2} = 60^\circ.$$

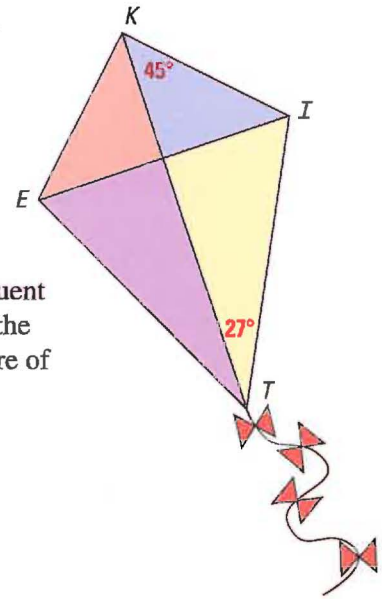
EXAMPLE 4 Doubling an Angle Measure

KITE DESIGN In the kite, two angles are bisected.

$\angle EKI$ is bisected by \overrightarrow{KT} .

$\angle ITE$ is bisected by \overrightarrow{TK} .

Find the measures of the two angles.



SOLUTION

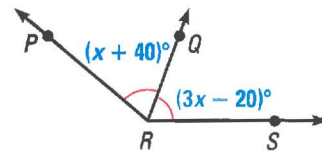
You are given the measure of one of the two congruent angles that make up the larger angle. You can find the measure of the larger angle by doubling the measure of the smaller angle.

$$m\angle EKI = 2m\angle TKI = 2(45^\circ) = 90^\circ$$

$$m\angle ITE = 2m\angle KTI = 2(27^\circ) = 54^\circ$$

EXAMPLE 5 Finding the Measure of an Angle

In the diagram, \overrightarrow{RQ} bisects $\angle PRS$. The measures of the two congruent angles are $(x + 40)^\circ$ and $(3x - 20)^\circ$. Solve for x .



SOLUTION

$$m\angle PRQ = m\angle QRS$$

$$(x + 40)^\circ = (3x - 20)^\circ$$

$$x + 60 = 3x$$

$$60 = 2x$$

$$30 = x$$

Congruent angles have equal measures.

Substitute given measures.

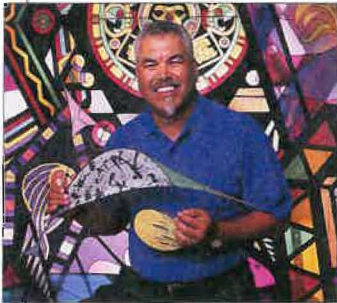
Add 20° to each side.

Subtract x from each side.

Divide each side by 2.

► So, $x = 30$. You can check by substituting to see that each of the congruent angles has a measure of 70° .

FOCUS ON PEOPLE



JOSÉ SAÍNZ, a San Diego kite designer, uses colorful patterns in his kites. The struts of his kites often bisect the angles they support.



GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

1. What kind of geometric figure is an *angle bisector*?
2. How do you indicate congruent segments in a diagram? How do you indicate congruent angles in a diagram?
3. What is the simplified form of the Midpoint Formula if one of the endpoints of a segment is $(0, 0)$ and the other is (x, y) ?

Skill Check ✓

Find the coordinates of the midpoint of a segment with the given endpoints.

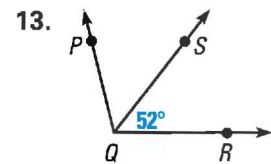
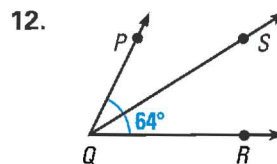
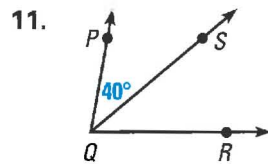
4. $A(5, 4), B(-3, 2)$ 5. $A(-1, -9), B(11, -5)$ 6. $A(6, -4), B(1, 8)$

Find the coordinates of the other endpoint of a segment with the given endpoint and midpoint M .

7. $C(3, 0)$ 8. $D(5, 2)$ 9. $E(-4, 2)$
 $M(3, 4)$ $M(7, 6)$ $M(-3, -2)$

10. Suppose $m\angle JKL$ is 90° . If the ray \overrightarrow{KM} bisects $\angle JKL$, what are the measures of $\angle JKM$ and $\angle LKM$?

\overrightarrow{QS} is the angle bisector of $\angle PQR$. Find the two angle measures not given in the diagram.



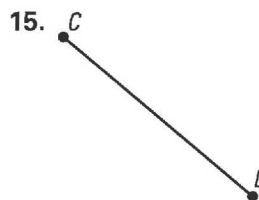
PRACTICE AND APPLICATIONS

STUDENT HELP

→ Extra Practice to help you master skills is on p. 804.



CONSTRUCTION Use a ruler to measure and redraw the line segment on a piece of paper. Then use construction tools to construct a segment bisector.



STUDENT HELP

→ HOMEWORK HELP

- Example 1:** Exs. 17–24
Example 2: Exs. 25–30
Example 3: Exs. 37–42
Example 4: Exs. 37–42
Example 5: Exs. 44–49

FINDING THE MIDPOINT Find the coordinates of the midpoint of a segment with the given endpoints.

17. $A(0, 0)$ 18. $J(-1, 7)$ 19. $C(10, 8)$ 20. $P(-12, -9)$
 $B(-8, 6)$ $K(3, -3)$ $D(-2, 5)$ $Q(2, 10)$
21. $S(0, -8)$ 22. $E(4, 4)$ 23. $V(-1.5, 8)$ 24. $G(-5.5, -6.1)$
 $T(-6, 14)$ $F(4, -18)$ $W(0.25, -1)$ $H(-0.5, 9.1)$

xy USING ALGEBRA Find the coordinates of the other endpoint of a segment with the given endpoint and midpoint M .

25. $R(2, 6)$
 $M(-1, 1)$

26. $T(-8, -1)$
 $M(0, 3)$

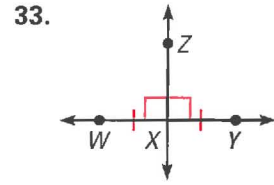
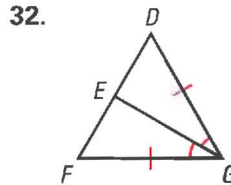
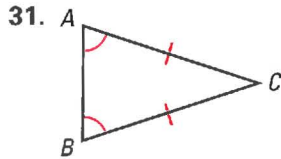
27. $W(3, -12)$
 $M(2, -1)$

28. $Q(-5, 9)$
 $M(-8, -2)$

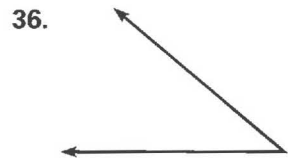
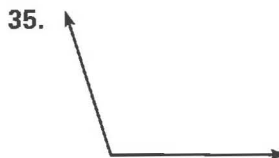
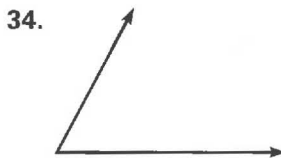
29. $A(6, 7)$
 $M(10, -7)$

30. $D(-3.5, -6)$
 $M(1.5, 4.5)$

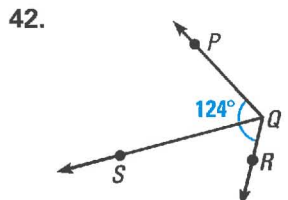
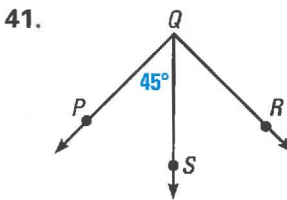
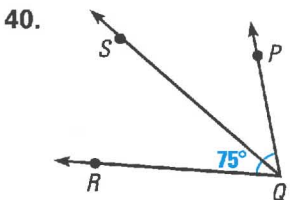
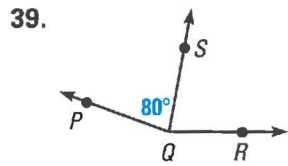
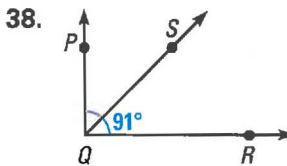
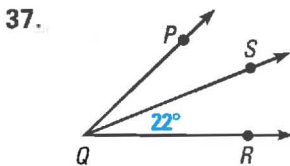
RECOGNIZING CONGRUENCE Use the marks on the diagram to name the congruent segments and congruent angles.



CONSTRUCTION Use a protractor to measure and redraw the angle on a piece of paper. Then use construction tools to find the angle bisector.

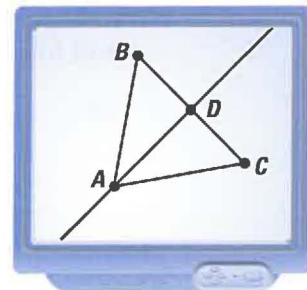


ANALYZING ANGLE BISECTORS \overrightarrow{QS} is the angle bisector of $\angle PQR$. Find the two angle measures not given in the diagram.



STUDENT HELP
SOFTWARE HELP
Visit our Web site
www.mcdougallittell.com
to see instructions for
several software
applications.

43. **TECHNOLOGY** Use geometry software to draw a triangle. Construct the angle bisector of one angle. Then find the midpoint of the opposite side of the triangle. Change your triangle and observe what happens.

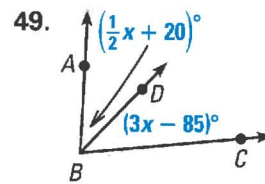
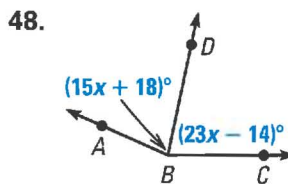
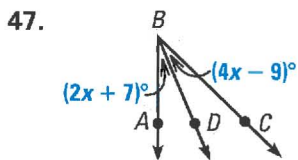
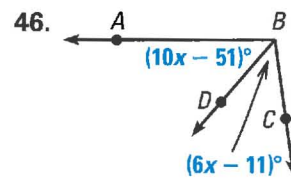
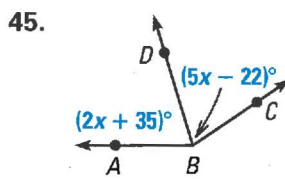
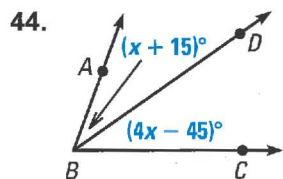


Does the angle bisector *always* pass through the midpoint of the opposite side? Does it *ever* pass through the midpoint?



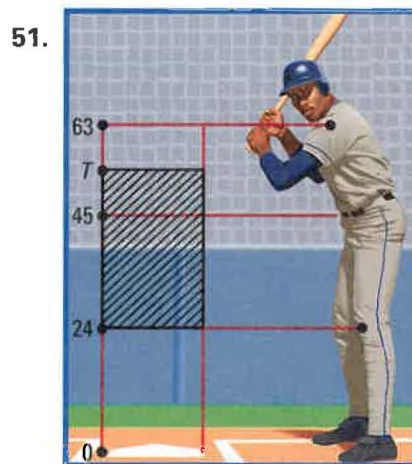
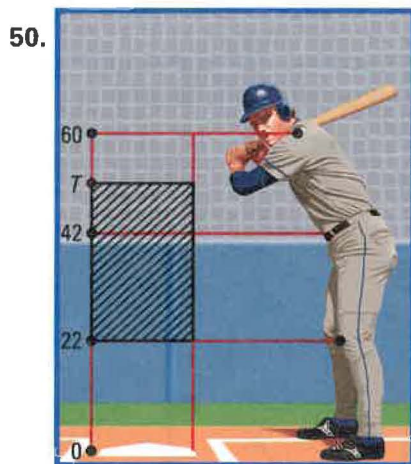
Visit our Web site
www.mcdougallittell.com
for help with Ex. 44–49.

XY USING ALGEBRA \overrightarrow{BD} bisects $\angle ABC$. Find the value of x .

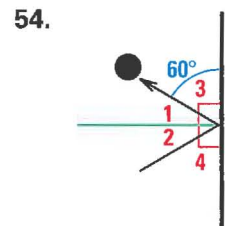
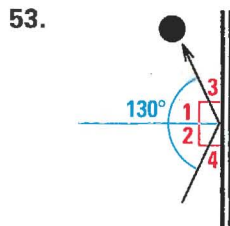
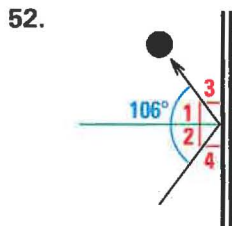


STRIKE ZONE In Exercises 50 and 51, use the information below. For each player, find the coordinate of T , a point on the top of the strike zone. In baseball, the “strike zone” is the region a baseball needs to pass through in order for an umpire to declare it a strike if it is not hit. The *top of the strike zone* is a horizontal plane passing through the midpoint between the top of the hitter’s shoulders and the top of the uniform pants when the player is in a batting stance.

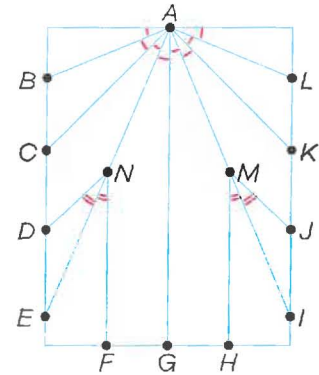
► Source: Major League Baseball



AIR HOCKEY When an air hockey puck is hit into the sideboards, it bounces off so that $\angle 1$ and $\angle 2$ are congruent. Find $m\angle 1$, $m\angle 2$, $m\angle 3$, and $m\angle 4$.



55. **PAPER AIRPLANES** The diagram represents an unfolded piece of paper used to make a paper airplane. The segments represent where the paper was folded to make the airplane.



Using the diagram, name as many pairs of congruent segments and as many congruent angles as you can.

56. **Writing** Explain, in your own words, how you would divide a line segment into four congruent segments using a compass and straightedge. Then explain how you could do it using the Midpoint Formula.
57. **MIDPOINT FORMULA REVISITED** Another version of the Midpoint Formula, for $A(x_1, y_1)$ and $B(x_2, y_2)$, is

$$M\left[x_1 + \frac{1}{2}(x_2 - x_1), y_1 + \frac{1}{2}(y_2 - y_1)\right].$$

Redo Exercises 17–24 using this version of the Midpoint Formula. Do you get the same answers as before? Use algebra to explain why the formula above is equivalent to the one in the lesson.

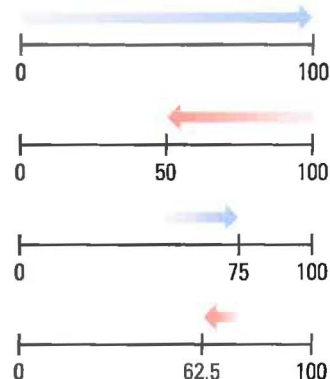


58. **MULTI-STEP PROBLEM** Sketch a triangle with three sides of different lengths.
- Using construction tools, find the midpoints of all three sides and the angle bisectors of all three angles of your triangle.
 - Determine whether or not the angle bisectors pass through the midpoints.
 - Writing** Write a brief paragraph explaining your results. Determine if your results would be different if you used a different kind of triangle.

★ Challenge

INFINITE SERIES A football team practices running back and forth on the field in a special way. First they run from one end of the 100 yd field to the other. Then they turn around and run half the previous distance. Then they turn around again and run half the previous distance, and so on.

59. Suppose the athletes continue the running drill with smaller and smaller distances. What is the coordinate of the point that they approach?
60. What is the total distance that the athletes cover?

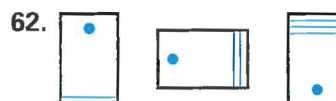


EXTRA CHALLENGE

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MIXED REVIEW

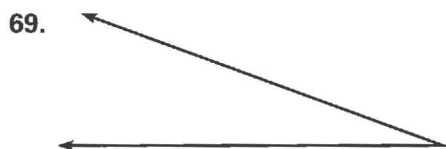
SKETCHING VISUAL PATTERNS Sketch the next figure in the pattern. (Review 1.1)



DISTANCE FORMULA Find the distance between the two points. (Review 1.3)

63. $A(3, 12), B(-5, -1)$ 64. $C(-6, 9), D(-2, -7)$ 65. $E(8, -8), F(2, 14)$
 66. $G(3, -8), H(0, -2)$ 67. $J(-4, -5), K(5, -1)$ 68. $L(-10, 1), M(-4, 9)$

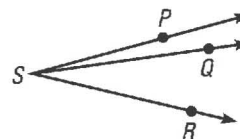
MEASURING ANGLES Use a protractor to find the measure of the angle. (Review 1.4 for 1.6)



QUIZ 2

Self-Test for Lessons 1.4 and 1.5

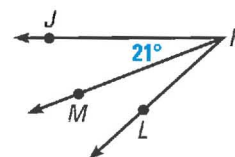
1. State the Angle Addition Postulate for the three angles shown at the right. (Lesson 1.4)



In a coordinate plane, plot the points and sketch $\angle DEF$. Classify the angle. Write the coordinates of a point that lies in the interior of the angle and the coordinates of a point that lies in the exterior of the angle. (Lesson 1.4)

- | | | | |
|---------------|----------------|---------------|---------------|
| 2. $D(-2, 3)$ | 3. $D(-6, -3)$ | 4. $D(-1, 8)$ | 5. $D(1, 10)$ |
| $E(4, -3)$ | $E(0, -5)$ | $E(-4, 0)$ | $E(1, 1)$ |
| $F(2, 6)$ | $F(8, -5)$ | $F(4, 0)$ | $F(8, 1)$ |

6. In the diagram, \overrightarrow{KM} is the angle bisector of $\angle JKL$. Find $m\angle MKL$ and $m\angle JKL$. (Lesson 1.5)



Angles and Intersecting Lines

You can use geometry software to construct intersecting lines and measure the angles formed by the lines.

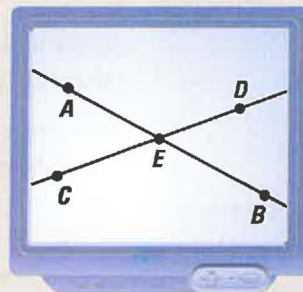
STUDENT HELP

INTERNET SOFTWARE HELP

Visit our Web site www.mcdougallittell.com to see instructions for several software applications.

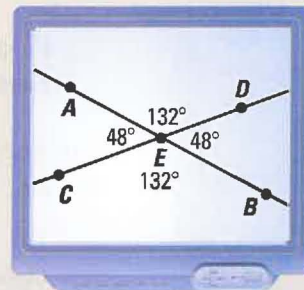
▶ **CONSTRUCT** Construct intersecting lines.

- 1 Draw a line. Label two points A and B on the line.
- 2 Draw a second line that intersects the first line. Label two points C and D on the line.
- 3 Select the two lines and construct their point of intersection. Label the point E .



▶ **INVESTIGATE**

1. Measure the four angles formed by the intersecting lines: $\angle AEC$, $\angle AED$, $\angle BEC$, and $\angle BED$. When measuring angles, select the points *in order*, with the vertex as the second point. Record the measures.
2. Move the lines into different positions by dragging the points. Record the measures of the four angles again.



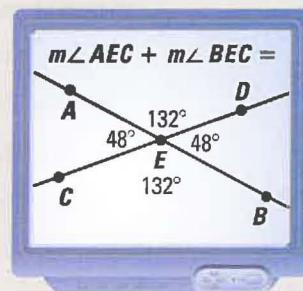
▶ **MAKE A CONJECTURE**

3. What do you notice about the angle measures?

▶ **INVESTIGATE**

You can use geometry software to perform calculations with measures.

4. Select all four angle measures you have made. Calculate the sum of the measures of any two adjacent angles.
5. Move the lines into different positions by dragging the points. Then find the sum of the two angle measures again.



▶ **MAKE A CONJECTURE**

6. What do you notice about the sum of the measures of adjacent angles formed by intersecting lines?

1.6

Angle Pair Relationships

What you should learn

GOAL 1 Identify vertical angles and linear pairs.

GOAL 2 Identify complementary and supplementary angles.

Why you should learn it

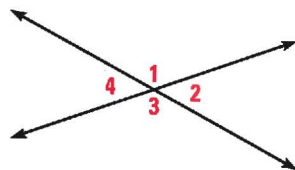
▼ To solve **real-life** problems, such as finding the measures of angles formed by the cables of a bridge in **Ex. 53**.



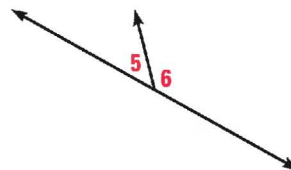
GOAL 1 VERTICAL ANGLES AND LINEAR PAIRS

In Lesson 1.4, you learned that two angles are *adjacent* if they share a common vertex and side but have no common interior points. In this lesson, you will study other relationships between pairs of angles.

Two angles are **vertical angles** if their sides form two pairs of opposite rays. Two adjacent angles are a **linear pair** if their noncommon sides are opposite rays.



$\angle 1$ and $\angle 3$ are vertical angles.
 $\angle 2$ and $\angle 4$ are vertical angles.

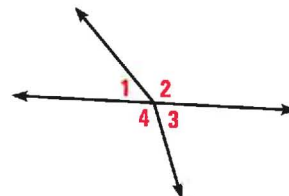


$\angle 5$ and $\angle 6$ are a linear pair.

In this book, you can assume from a diagram that two adjacent angles form a linear pair if the noncommon sides appear to lie on the same line.

EXAMPLE 1 Identifying Vertical Angles and Linear Pairs

- Are $\angle 2$ and $\angle 3$ a linear pair?
- Are $\angle 3$ and $\angle 4$ a linear pair?
- Are $\angle 1$ and $\angle 3$ vertical angles?
- Are $\angle 2$ and $\angle 4$ vertical angles?



SOLUTION

- No. The angles are adjacent but their noncommon sides are not opposite rays.
- Yes. The angles are adjacent and their noncommon sides are opposite rays.
- No. The sides of the angles do not form two pairs of opposite rays.
- No. The sides of the angles do not form two pairs of opposite rays.

.....

In Activity 1.6 on page 43, you may have discovered two results:

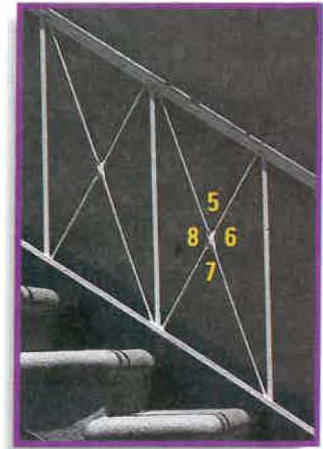
- *Vertical angles are congruent.*
- *The sum of the measures of angles that form a linear pair is 180° .*

Both of these results will be stated formally in Chapter 2.



EXAMPLE 2 Finding Angle Measures

In the stair railing shown at the right, $\angle 6$ has a measure of 130° . Find the measures of the other three angles.



SOLUTION

$\angle 6$ and $\angle 7$ are a linear pair. So, the sum of their measures is 180° .

$$m\angle 6 + m\angle 7 = 180^\circ$$

$$130^\circ + m\angle 7 = 180^\circ$$

$$m\angle 7 = 50^\circ$$

$\angle 6$ and $\angle 5$ are also a linear pair. So, it follows that $m\angle 5 = 50^\circ$.

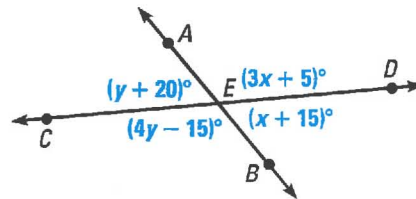
$\angle 6$ and $\angle 8$ are vertical angles. So, they are congruent and have the same measure.

$$m\angle 8 = m\angle 6 = 130^\circ$$



EXAMPLE 3 Finding Angle Measures

Solve for x and y .
Then find the angle measures.



SOLUTION

Use the fact that the sum of the measures of angles that form a linear pair is 180° .

$$m\angle AED + m\angle DEB = 180^\circ$$

$$m\angle AEC + m\angle CEB = 180^\circ$$

$$(3x + 5)^\circ + (x + 15)^\circ = 180^\circ$$

$$(y + 20)^\circ + (4y - 15)^\circ = 180^\circ$$

$$4x + 20 = 180$$

$$5y + 5 = 180$$

$$4x = 160$$

$$5y = 175$$

$$x = 40$$

$$y = 35$$

Use substitution to find the angle measures.

$$m\angle AED = (3x + 5)^\circ = (3 \cdot 40 + 5)^\circ = 125^\circ$$

$$m\angle DEB = (x + 15)^\circ = (40 + 15)^\circ = 55^\circ$$

$$m\angle AEC = (y + 20)^\circ = (35 + 20)^\circ = 55^\circ$$

$$m\angle CEB = (4y - 15)^\circ = (4 \cdot 35 - 15)^\circ = 125^\circ$$

- So, the angle measures are 125° , 55° , 55° , and 125° . Because the vertical angles are congruent, the result is reasonable.



GOAL 2 COMPLEMENTARY AND SUPPLEMENTARY ANGLES

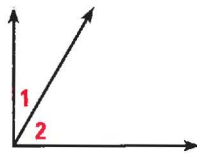
STUDENT HELP

Study Tip

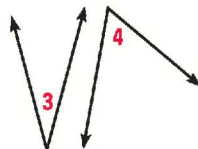
In mathematics, the word *complement* is related to the phrase *to complete*. When you draw the complement of an angle, you are “completing” a right angle. (The word *compliment* is different. It means something said in praise.)

Two angles are **complementary angles** if the sum of their measures is 90° . Each angle is the **complement** of the other. Complementary angles can be adjacent or nonadjacent.

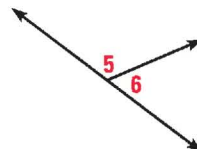
Two angles are **supplementary angles** if the sum of their measures is 180° . Each angle is the **supplement** of the other. Supplementary angles can be adjacent or nonadjacent.



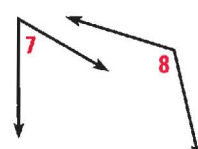
complementary adjacent



complementary nonadjacent



supplementary adjacent



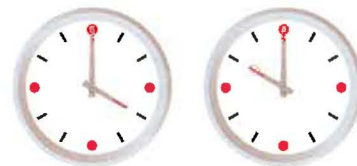
supplementary nonadjacent

EXAMPLE 4 Identifying Angles

State whether the two angles are complementary, supplementary, or neither.

SOLUTION

The angle showing 4:00 has a measure of 120° and the angle showing 10:00 has a measure of 60° . Because the sum of these two measures is 180° , the angles are supplementary.



EXAMPLE 5 Finding Measures of Complements and Supplements

- Given that $\angle A$ is a complement of $\angle C$ and $m\angle A = 47^\circ$, find $m\angle C$.
- Given that $\angle P$ is a supplement of $\angle R$ and $m\angle R = 36^\circ$, find $m\angle P$.

SOLUTION

- $m\angle C = 90^\circ - m\angle A = 90^\circ - 47^\circ = 43^\circ$
- $m\angle P = 180^\circ - m\angle R = 180^\circ - 36^\circ = 144^\circ$



EXAMPLE 6 Finding the Measure of a Complement

$\angle W$ and $\angle Z$ are complementary. The measure of $\angle Z$ is five times the measure of $\angle W$. Find $m\angle W$.

SOLUTION

Because the angles are complementary, $m\angle W + m\angle Z = 90^\circ$. But $m\angle Z = 5(m\angle W)$, so $m\angle W + 5(m\angle W) = 90^\circ$. Because $6(m\angle W) = 90^\circ$, you know that $m\angle W = 15^\circ$.

GUIDED PRACTICE

Vocabulary Check ✓

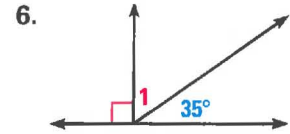
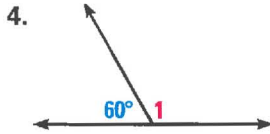
1. Explain the difference between *complementary angles* and *supplementary angles*.

Concept Check ✓

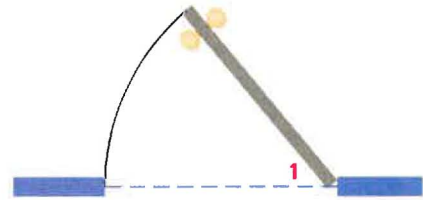
2. Sketch examples of acute vertical angles and obtuse vertical angles.
3. Sketch examples of adjacent congruent complementary angles and adjacent congruent supplementary angles.

Skill Check ✓

FINDING ANGLE MEASURES Find the measure of $\angle 1$.



7. **OPENING A DOOR** The figure shows a doorway viewed from above. If you open the door so that the measure of $\angle 1$ is 50° , how many more degrees would you have to open the door so that the angle between the wall and the door is 90° ?



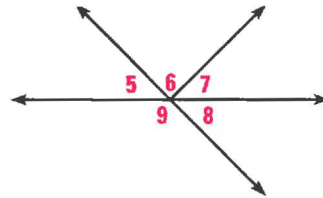
PRACTICE AND APPLICATIONS

STUDENT HELP

➔ **Extra Practice** to help you master skills is on p. 804.

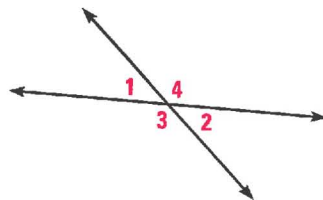
IDENTIFYING ANGLE PAIRS Use the figure at the right.

8. Are $\angle 5$ and $\angle 6$ a linear pair?
9. Are $\angle 5$ and $\angle 9$ a linear pair?
10. Are $\angle 5$ and $\angle 8$ a linear pair?
11. Are $\angle 5$ and $\angle 8$ vertical angles?
12. Are $\angle 5$ and $\angle 7$ vertical angles?
13. Are $\angle 9$ and $\angle 6$ vertical angles?



EVALUATING STATEMENTS Decide whether the statement is *always*, *sometimes*, or *never* true.

14. If $m\angle 1 = 40^\circ$, then $m\angle 2 = 140^\circ$.
15. If $m\angle 4 = 130^\circ$, then $m\angle 2 = 50^\circ$.
16. $\angle 1$ and $\angle 4$ are congruent.
17. $m\angle 2 + m\angle 3 = m\angle 1 + m\angle 4$
18. $\angle 2 \cong \angle 1$
19. $m\angle 2 = 90^\circ - m\angle 3$



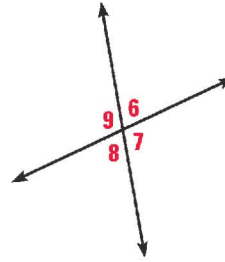
STUDENT HELP

➔ HOMEWORK HELP

- Example 1:** Exs. 8–13
Example 2: Exs. 14–27
Example 3: Exs. 28–36
Example 4: Exs. 37–40
Example 5: Exs. 41, 42
Example 6: Exs. 43, 44

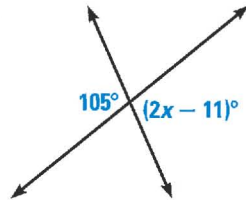
FINDING ANGLE MEASURES Use the figure at the right.

20. If $m\angle 6 = 72^\circ$, then $m\angle 7 = \underline{\quad? \quad}$.
21. If $m\angle 8 = 80^\circ$, then $m\angle 6 = \underline{\quad? \quad}$.
22. If $m\angle 9 = 110^\circ$, then $m\angle 8 = \underline{\quad? \quad}$.
23. If $m\angle 9 = 123^\circ$, then $m\angle 7 = \underline{\quad? \quad}$.
24. If $m\angle 7 = 142^\circ$, then $m\angle 8 = \underline{\quad? \quad}$.
25. If $m\angle 6 = 13^\circ$, then $m\angle 9 = \underline{\quad? \quad}$.
26. If $m\angle 9 = 170^\circ$, then $m\angle 6 = \underline{\quad? \quad}$.
27. If $m\angle 8 = 26^\circ$, then $m\angle 7 = \underline{\quad? \quad}$.

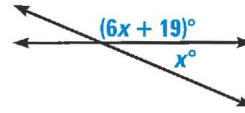


USING ALGEBRA Find the value(s) of the variable(s).

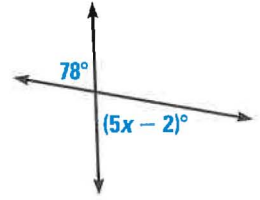
28.



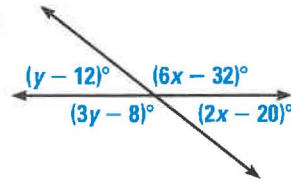
29.



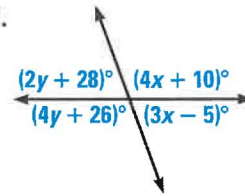
30.



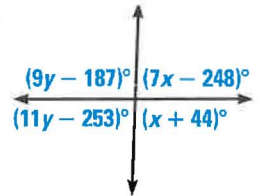
31.



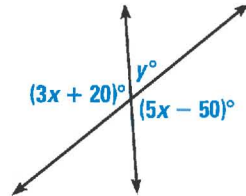
32.



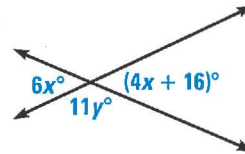
33.



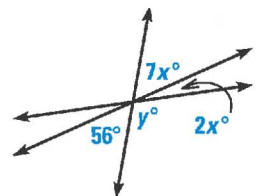
34.



35.

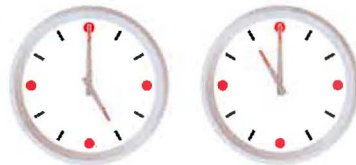


36.



IDENTIFYING ANGLES State whether the two angles shown are *complementary*, *supplementary*, or *neither*.

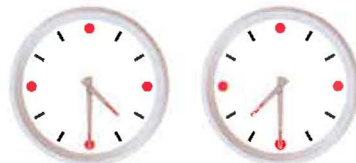
37.



38.



39.



40.



41. **FINDING COMPLEMENTS** In the table, assume that $\angle 1$ and $\angle 2$ are complementary. Copy and complete the table.

$m\angle 1$	2°	10°	25°	33°	40°	49°	55°	62°	76°	86°
$m\angle 2$?	?	?	?	?	?	?	?	?	?

42. **FINDING SUPPLEMENTS** In the table, assume that $\angle 1$ and $\angle 2$ are supplementary. Copy and complete the table.

$m\angle 1$	4°	16°	48°	72°	90°	99°	120°	152°	169°	178°
$m\angle 2$?	?	?	?	?	?	?	?	?	?

43. **xy USING ALGEBRA** $\angle A$ and $\angle B$ are complementary. The measure of $\angle B$ is three times the measure of $\angle A$. Find $m\angle A$ and $m\angle B$.
44. **xy USING ALGEBRA** $\angle C$ and $\angle D$ are supplementary. The measure of $\angle D$ is eight times the measure of $\angle C$. Find $m\angle C$ and $m\angle D$.

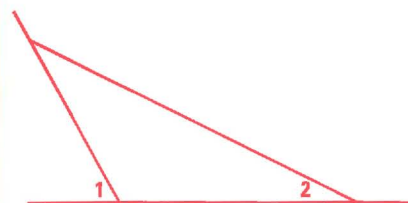
FINDING ANGLES $\angle A$ and $\angle B$ are complementary. Find $m\angle A$ and $m\angle B$.

45. $m\angle A = 5x + 8$
 $m\angle B = x + 4$
46. $m\angle A = 3x - 7$
 $m\angle B = 11x - 1$
47. $m\angle A = 8x - 7$
 $m\angle B = x - 11$
48. $m\angle A = \frac{3}{4}x - 13$
 $m\angle B = 3x - 17$

FINDING ANGLES $\angle A$ and $\angle B$ are supplementary. Find $m\angle A$ and $m\angle B$.

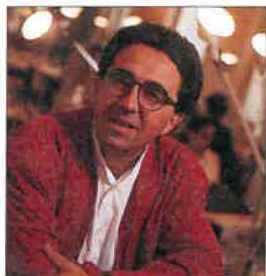
49. $m\angle A = 3x$
 $m\angle B = x + 8$
50. $m\angle A = 6x - 1$
 $m\angle B = 5x - 17$
51. $m\angle A = 12x + 1$
 $m\angle B = x + 10$
52. $m\angle A = \frac{3}{8}x + 50$
 $m\angle B = x + 31$

53. **BRIDGES** The Alamillo Bridge in Seville, Spain, was designed by Santiago Calatrava. In the bridge, $m\angle 1 = 58^\circ$ and $m\angle 2 = 24^\circ$. Find the supplements of both $\angle 1$ and $\angle 2$.



54. **BASEBALL** The foul lines of a baseball field intersect at home plate to form a right angle. Suppose you hit a baseball whose path forms an angle of 34° with the third base foul line. What is the angle between the first base foul line and the path of the baseball?

FOCUS ON PEOPLE



REAL LIFE **SANTIAGO CALATRAVA**, a Spanish born architect, has developed designs for bridges, train stations, stadiums, and art museums.

APPLICATION LINK
www.mcdougallittell.com

Test Preparation

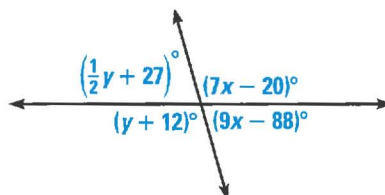


55. **PLANTING TREES** To support a young tree, you attach wires from the trunk to the ground. The obtuse angle the wire makes with the ground is supplementary to the acute angle the wire makes, and it is three times as large. Find the measures of the angles.

56. **Writing** Give an example of an angle that *does not* have a complement. In general, what is true about an angle that has a complement?

57. **MULTIPLE CHOICE** In the diagram shown at the right, what are the values of x and y ?

- (A) $x = 74, y = 106$
- (B) $x = 16, y = 88$
- (C) $x = 74, y = 16$
- (D) $x = 18, y = 118$
- (E) $x = 18, y = 94$

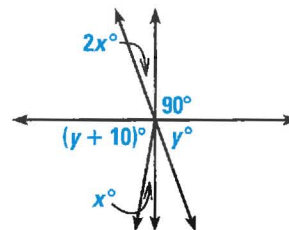


58. **MULTIPLE CHOICE** $\angle F$ and $\angle G$ are supplementary. The measure of $\angle G$ is six and one half times the measure of $\angle F$. What is $m\angle F$?

- (A) 20°
- (B) 24°
- (C) 24.5°
- (D) 26.5°
- (E) 156°

★ Challenge

59. **xy USING ALGEBRA** Find the values of x and y in the diagram shown at the right.



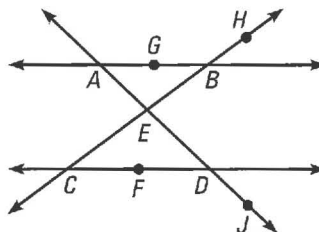
MIXED REVIEW

SOLVING EQUATIONS Solve the equation. (Skills Review, p. 802, for 1.7)

- 60. $3x = 96$
- 61. $\frac{1}{2} \cdot 5 \cdot h = 20$
- 62. $\frac{1}{2} \cdot b \cdot 6 = 15$
- 63. $s^2 = 200$
- 64. $2 \cdot 3.14 \cdot r = 40$
- 65. $3.14 \cdot r^2 = 314$

FINDING COLLINEAR POINTS Use the diagram to find a third point that is collinear with the given points. (Review 1.2)

- 66. A and J
- 67. D and F
- 68. H and E
- 69. B and G



FINDING THE MIDPOINT Find the coordinates of the midpoint of a segment with the given endpoints. (Review 1.5)

- 70. $A(0, 0), B(-6, -4)$
- 71. $F(2, 5), G(-10, 7)$
- 72. $K(8, -6), L(-2, -2)$
- 73. $M(-14, -9), N(0, 11)$
- 74. $P(-1.5, 4), Q(5, -9)$
- 75. $S(-2.4, 5), T(7.6, 9)$

1.7

Introduction to Perimeter, Circumference, and Area

What you should learn

GOAL 1 Find the perimeter and area of common plane figures.

GOAL 2 Use a general problem-solving plan.

Why you should learn it

▼ To solve **real-life** problems about perimeter and area, such as finding the number of bags of seed you need for a field in **Example 4**.

GOAL 1 REVIEWING PERIMETER, CIRCUMFERENCE, AND AREA

In this lesson, you will review some common formulas for perimeter, circumference, and area. You will learn more about area in Chapters 6, 11, and 12.

PERIMETER, CIRCUMFERENCE, AND AREA FORMULAS

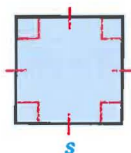
Formulas for the perimeter P , area A , and circumference C of some common plane figures are given below.

SQUARE

side length s

$$P = 4s$$

$$A = s^2$$

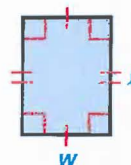


RECTANGLE

length ℓ and width w

$$P = 2\ell + 2w$$

$$A = \ell w$$

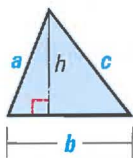


TRIANGLE

side lengths a , b , and c , base b , and height h

$$P = a + b + c$$

$$A = \frac{1}{2}bh$$

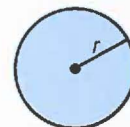


CIRCLE

radius r

$$C = 2\pi r$$

$$A = \pi r^2$$



π (π) is the ratio of the circle's circumference to its diameter.

The measurements of perimeter and circumference use units such as centimeters, meters, kilometers, inches, feet, yards, and miles. The measurements of area use units such as square centimeters (cm^2), square meters (m^2), and so on.

EXAMPLE 1 Finding the Perimeter and Area of a Rectangle

Find the perimeter and area of a rectangle of length 12 inches and width 5 inches.

SOLUTION

Begin by drawing a diagram and labeling the length and width. Then, use the formulas for perimeter and area of a rectangle.

$$P = 2\ell + 2w$$

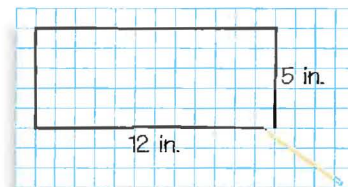
$$= 2(12) + 2(5)$$

$$= 34$$

$$A = \ell w$$

$$= (12)(5)$$

$$= 60$$



► So, the perimeter is 34 inches and the area is 60 square inches.

EXAMPLE 2 Finding the Area and Circumference of a Circle

Find the diameter, radius, circumference, and area of the circle shown at the right. Use 3.14 as an approximation for π .

STUDENT HELP**Study Tip**

Some approximations for $\pi = 3.141592654\dots$ are 3.14 and $\frac{22}{7}$.

SOLUTION

From the diagram, you can see that the diameter of the circle is

$$d = 13 - 5 = 8 \text{ cm.}$$

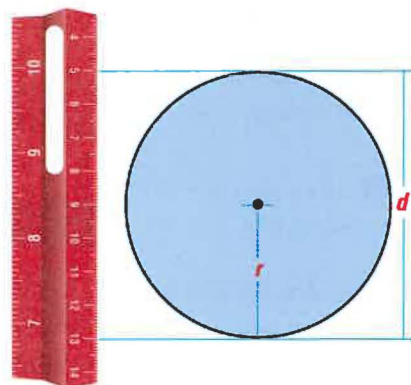
The radius is one half the diameter.

$$r = \frac{1}{2}(8) = 4 \text{ cm}$$

Using the formulas for circumference and area, you have

$$C = 2\pi r \approx 2(3.14)(4) \approx 25.1 \text{ cm}$$

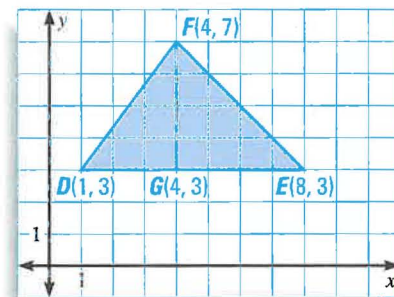
$$A = \pi r^2 \approx 3.14(4^2) \approx 50.2 \text{ cm}^2.$$

**EXAMPLE 3** Finding Measurements of a Triangle in a Coordinate Plane

Find the area and perimeter of the triangle defined by $D(1, 3)$, $E(8, 3)$, and $F(4, 7)$.

SOLUTION

Plot the points in a coordinate plane. Draw the height from F to side \overline{DE} . Label the point where the height meets \overline{DE} as G . Point G has coordinates $(4, 3)$.



$$\text{base: } DE = 8 - 1 = 7$$

$$\text{height: } FG = 7 - 3 = 4$$

$$A = \frac{1}{2}(\text{base})(\text{height})$$

$$= \frac{1}{2}(7)(4)$$

$$= 14 \text{ square units}$$

To find the perimeter, use the Distance Formula.

$$EF = \sqrt{(4 - 8)^2 + (7 - 3)^2}$$

$$= \sqrt{(-4)^2 + 4^2}$$

$$= \sqrt{32}$$

$$= 4\sqrt{2} \text{ units}$$

$$DF = \sqrt{(4 - 1)^2 + (7 - 3)^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{25}$$

$$= 5 \text{ units}$$

► So, the perimeter is $DE + EF + DF = (7 + 4\sqrt{2} + 5)$, or $12 + 4\sqrt{2}$, units.

STUDENT HELP**Skills Review**

For help with simplifying radicals, see page 799.


GOAL 2 USING A PROBLEM-SOLVING PLAN

A problem-solving plan can help you organize solutions to geometry problems.

A PROBLEM-SOLVING PLAN

1. Ask yourself what you need to solve the problem. Write a **verbal model** or **draw a sketch** that will help you find what you need to know.
2. **Label known and unknown facts** on or near your sketch.
3. Use labels and facts to **choose related definitions, theorems, formulas**, or other results you may need.
4. **Reason logically** to link the facts, using a proof or other written argument.
5. Write a **conclusion** that answers the original problem. **Check** that your reasoning is correct.

EXAMPLE 4 Using the Area of a Rectangle

 **SOCCER FIELD** You have a part-time job at a school. You need to buy enough grass seed to cover the school's soccer field. The field is 50 yards wide and 100 yards long. The instructions on the seed bags say that one bag will cover 5000 square feet. How many bags do you need?

SOLUTION

Begin by rewriting the dimensions of the field in feet. Multiplying each of the dimensions by 3, you find that the field is 150 feet wide and 300 feet long.

PROBLEM
SOLVING
STRATEGY

VERBAL MODEL	$\boxed{\text{Area of field}} = \boxed{\text{Bags of seed}} \cdot \boxed{\text{Coverage per bag}}$
LABELS	Area of field = $150 \cdot 300$ (square feet) Bags of seed = n (bags) Coverage per bag = 5000 (square feet per bag)
REASONING	$150 \cdot 300 = n \cdot 5000$ Write model for area of field. $\frac{150 \cdot 300}{5000} = n$ Divide each side by 5000. $9 = n$ Simplify.

▶ You need 9 bags of seed.

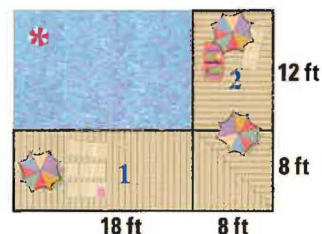
✓ **UNIT ANALYSIS** You can use *unit analysis* to verify the units of measure.

$$\text{ft}^2 = \text{bags} \cdot \frac{\text{ft}^2}{\text{bag}}$$

EXAMPLE 5 Using the Area of a Square



SWIMMING POOL You are planning a deck along two sides of a pool. The pool measures 18 feet by 12 feet. The deck is to be 8 feet wide. What is the area of the deck?



SOLUTION

PROBLEM SOLVING STRATEGY

DRAW A SKETCH

VERBAL MODEL

LABELS

REASONING

From your diagram, you can see that the area of the deck can be represented as the sum of the areas of two rectangles and a square.

$$\boxed{\text{Area of deck}} = \boxed{\text{Area of rectangle 1}} + \boxed{\text{Area of rectangle 2}} + \boxed{\text{Area of square}}$$

$$\text{Area of deck} = A \quad (\text{square feet})$$

$$\text{Area of rectangle 1} = 8 \cdot 18 \quad (\text{square feet})$$

$$\text{Area of rectangle 2} = 8 \cdot 12 \quad (\text{square feet})$$

$$\text{Area of square} = 8 \cdot 8 \quad (\text{square feet})$$

$$A = 8 \cdot 18 + 8 \cdot 12 + 8 \cdot 8 \quad \text{Write model for deck area.}$$

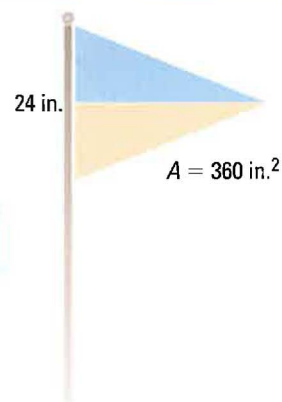
$$= 304 \quad \text{Simplify.}$$

▶ The area of the deck is 304 square feet.

EXAMPLE 6 Using the Area of a Triangle



FLAG DESIGN You are making a triangular flag with a base of 24 inches and an area of 360 square inches. How long should it be?



SOLUTION

PROBLEM SOLVING STRATEGY

VERBAL MODEL

LABELS

REASONING

$$\boxed{\text{Area of flag}} = \frac{1}{2} \cdot \boxed{\text{Base of flag}} \cdot \boxed{\text{Length of flag}}$$

$$\text{Area of flag} = 360 \quad (\text{square inches})$$

$$\text{Base of flag} = 24 \quad (\text{inches})$$

$$\text{Length of flag} = L \quad (\text{inches})$$

$$360 = \frac{1}{2}(24)L \quad \text{Write model for flag area.}$$

$$360 = 12L \quad \text{Simplify.}$$

$$30 = L \quad \text{Divide each side by 12.}$$

▶ The flag should be 30 inches long.

GUIDED PRACTICE

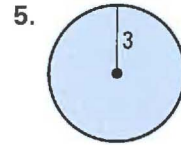
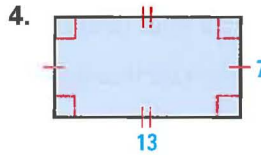
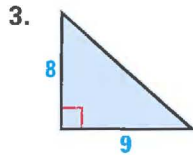
Vocabulary Check ✓

Concept Check ✓

Skill Check ✓

1. The perimeter of a circle is called its ____? ____.
2. Explain how to find the perimeter of a rectangle.

In Exercises 3–5, find the area of the figure. (Where necessary, use $\pi \approx 3.14$.)



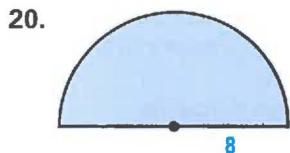
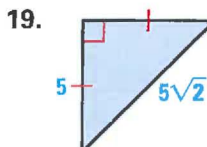
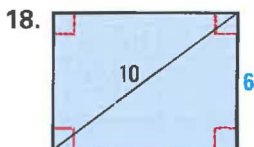
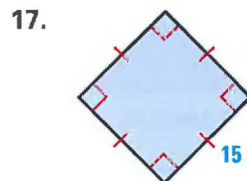
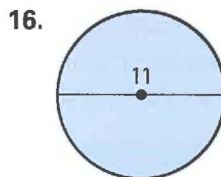
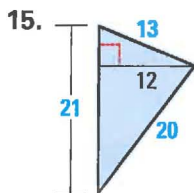
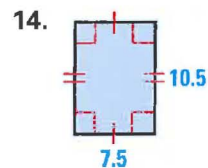
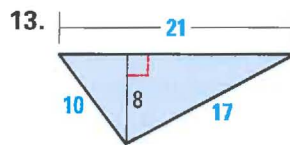
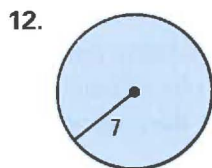
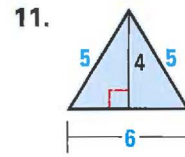
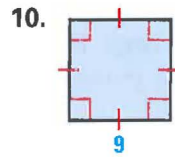
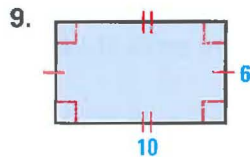
6. The perimeter of a square is 12 meters. What is the length of a side of the square?
7. The radius of a circle is 4 inches. What is the circumference of the circle? (Use $\pi \approx 3.14$.)
8. **FENCING** You are putting a fence around a rectangular garden with length 15 feet and width 8 feet. What is the length of the fence that you will need?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 804.

FINDING PERIMETER, CIRCUMFERENCE, AND AREA Find the perimeter (or circumference) and area of the figure. (Where necessary, use $\pi \approx 3.14$.)



STUDENT HELP

HOMEWORK HELP

- Example 1: Exs. 9–26
- Example 2: Exs. 9–26
- Example 3: Exs. 27–33
- Example 4: Exs. 34–40
- Example 5: Exs. 34–40
- Example 6: Exs. 41–48

STUDENT HELP



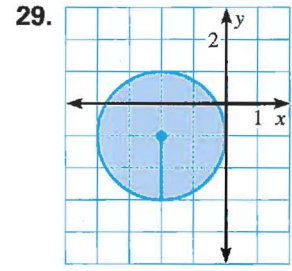
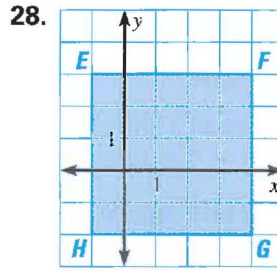
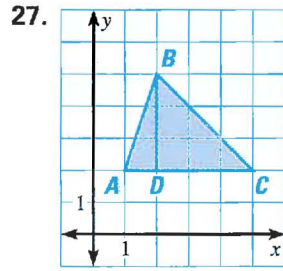
HOMEWORK HELP

Visit our Web site
www.mcdougallittell.com
for help with problem
solving in Exs. 21–26.

FINDING AREA Find the area of the figure described.

21. Triangle with height 6 cm and base 5 cm
22. Rectangle with length 12 yd and width 9 yd
23. Square with side length 8 ft
24. Circle with radius 10 m (Use $\pi \approx 3.14$.)
25. Square with perimeter 24 m
26. Circle with diameter 100 ft (Use $\pi \approx 3.14$.)

FINDING AREA Find the area of the figure.



FINDING AREA Draw the figure in a coordinate plane and find its area.

30. Triangle defined by $A(3, 4)$, $B(7, 4)$, and $C(5, 7)$
31. Triangle defined by $R(-2, -3)$, $S(6, -3)$, and $T(5, 4)$
32. Rectangle defined by $L(-2, -4)$, $M(-2, 1)$, $N(7, 1)$, and $P(7, -4)$
33. Square defined by $W(5, 0)$, $X(0, 5)$, $Y(-5, 0)$, and $Z(0, -5)$

34. **CARPETING** How many square yards of carpet are needed to carpet a room that is 15 feet by 25 feet?

35. **WINDOWS** A rectangular pane of glass measuring 12 inches by 18 inches is surrounded by a wooden frame that is 2 inches wide. What is the area of the window, including the frame?

36. **MILLENNIUM DOME** The largest fabric dome in the world, the Millennium Dome covers a circular plot of land with a diameter of 320 meters. What is the circumference of the covered land? What is its area? (Use $\pi \approx 3.14$.)

37. **SPREADSHEET** Use a spreadsheet to show many different possible values of length and width for a rectangle with an area of 100 m^2 . For each possible rectangle, calculate the perimeter. What are the dimensions of the rectangle with the smallest perimeter?

Perimeter of Rectangle								
	A	B	C	D	E	F	G	H
1	Length	1.00	2.00	3.00	4.00	5.00	6.00	...
2	Width	100.00	50.00	33.33	25.00	20.00	16.67	...
3	Area	100.00	100.00	100.00	100.00	100.00	100.00	...
4	Perimeter	202.00	104.00	72.67	58.00	50.00	45.33	...
5								

FOCUS ON APPLICATIONS



MILLENNIUM DOME

Built for the year 2000, this dome in Greenwich, England, is over 50 m tall and is covered by more than 100,000 square meters of fabric.

APPLICATION LINK

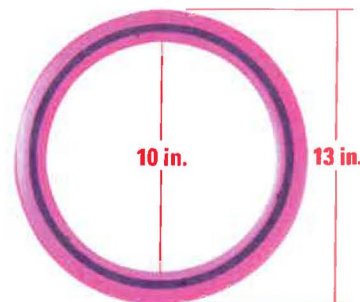
www.mcdougallittell.com

FOCUS ON APPLICATIONS



REAL LIFE
CRANBERRIES
 Cranberries were once called “bounceberries” because they bounce when they are ripe.

38. **CRANBERRY HARVEST** To harvest cranberries, the field is flooded so that the berries float. The berries are gathered with an inflatable boom. What area of cranberries can be gathered into a circular region with a radius of 5.5 meters? (Use $\pi \approx 3.14$.)
39. **BICYCLES** How many times does a bicycle tire that has a radius of 21 inches rotate when it travels 420 inches? (Use $\pi \approx 3.14$.)
40. **FLYING DISC** A plastic flying disc is circular and has a circular hole in the middle. If the diameter of the outer edge of the ring is 13 inches and the diameter of the inner edge of the ring is 10 inches, what is the area of plastic in the ring? (Use $\pi \approx 3.14$.)



LOGICAL REASONING Use the given measurements to find the unknown measurement. (Where necessary, use $\pi \approx 3.14$.)

41. A rectangle has an area of 36 in.^2 and a length of 9 in. Find its perimeter.
42. A square has an area of $10,000 \text{ m}^2$. Find its perimeter.
43. A triangle has an area of 48 ft^2 and a base of 16 ft. Find its height.
44. A triangle has an area of 52 yd^2 and a height of 13 yd. Find its base.
45. A circle has an area of $200\pi \text{ cm}^2$. Find its radius.
46. A circle has an area of 1 m^2 . Find its diameter.
47. A circle has a circumference of 100 yd. Find its area.
48. A right triangle has sides of length 4.5 cm, 6 cm, and 7.5 cm. Find its area.

Test Preparation



49. **MULTI-STEP PROBLEM** Use the following information.
 Earth has a radius of about 3960 miles at the equator. Because there are 5280 feet in one mile, the radius of Earth is about 20,908,800 feet.
- Suppose you could wrap a cable around Earth to form a circle that is snug against the ground. Find the length of the cable in feet by finding the circumference of Earth. (Assume that Earth is perfectly round. Use $\pi \approx 3.14$.)
 - Suppose you add 6 feet to the cable length in part (a). Use this length as the circumference of a new circle. Find the radius of the larger circle.
 - Use your results from parts (a) and (b) to find how high off of the ground the longer cable would be if it was evenly spaced around Earth.
 - Would the answer to part (c) be different on a planet with a different radius? Explain.


★ Challenge

50. **DOUBLING A RECTANGLE'S SIDES** The length and width of a rectangle are doubled. How do the perimeter and area of the new rectangle compare with the perimeter and area of the original rectangle? Illustrate your answer.

MIXED REVIEW

SKETCHING FIGURES Sketch the points, lines, segments, and rays.
(Review 1.2 for 2.1)

51. Draw opposite rays using the points A , B , and C , with B as the initial point for both rays.
52. Draw four noncollinear points, W , X , Y , and Z , no three of which are collinear. Then sketch \overleftrightarrow{XY} , \overrightarrow{YW} , \overline{XZ} and \overleftrightarrow{ZY} .

 **USING ALGEBRA** Plot the points in a coordinate plane and sketch $\angle DEF$. Classify the angle. Write the coordinates of one point in the interior of the angle and one point in the exterior of the angle. (Review 1.4)

- | | | | |
|----------------|---------------|---------------|-----------------|
| 53. $D(2, -2)$ | 54. $D(0, 0)$ | 55. $D(0, 1)$ | 56. $D(-3, -2)$ |
| $E(4, -3)$ | $E(-3, 0)$ | $E(2, 3)$ | $E(3, -4)$ |
| $F(6, -2)$ | $F(0, -2)$ | $F(4, 1)$ | $F(1, 3)$ |

FINDING THE MIDPOINT Find the coordinates of the midpoint of a segment with the given endpoints. (Review 1.5)

- | | |
|---------------------------|---------------------------|
| 57. $A(0, 0), B(5, 3)$ | 58. $C(2, -3), D(4, 4)$ |
| 59. $E(-3, 4), F(-2, -1)$ | 60. $G(-2, 0), H(-7, -6)$ |
| 61. $J(0, 5), K(14, 1)$ | 62. $M(-44, 9), N(6, -7)$ |


QUIZ 3

Self-Test for Lessons 1.6 and 1.7

In Exercises 1–4, find the measure of the angle. (Lesson 1.6)

- | | |
|--|---|
| 1. Complement of $\angle A$; $m\angle A = 41^\circ$ | 2. Supplement of $\angle B$; $m\angle B = 127^\circ$ |
| 3. Supplement of $\angle C$; $m\angle C = 22^\circ$ | 4. Complement of $\angle D$; $m\angle D = 35^\circ$ |
5. $\angle A$ and $\angle B$ are complementary. The measure of $\angle A$ is five times the measure of $\angle B$. Find $m\angle A$ and $m\angle B$. (Lesson 1.6)

In Exercises 6–9, use the given information to find the unknown measurement. (Lesson 1.7)

6. Find the area and circumference of a circle with a radius of 18 meters. (Use $\pi \approx 3.14$.)
7. Find the area of a triangle with a base of 13 inches and a height of 11 inches.
8. Find the area and perimeter of a rectangle with a length of 10 centimeters and a width of 4.6 centimeters.
9. Find the area of a triangle defined by $P(-3, 4)$, $Q(7, 4)$, and $R(-1, 12)$.
10.  **WALLPAPER** You are buying rolls of wallpaper to paper the walls of a rectangular room. The room measures 12 feet by 24 feet and the walls are 8 feet high. A roll of wallpaper contains 28 ft^2 . About how many rolls of wallpaper will you need? (Lesson 1.7)

WHAT did you learn?

- Find and describe patterns. (1.1)
- Use inductive reasoning. (1.1)
- Use defined and undefined terms. (1.2)
- Sketch intersections of lines and planes. (1.2)
- Use segment postulates and the Distance Formula. (1.3)
- Use angle postulates and classify angles. (1.4)
- Bisect a segment and bisect an angle. (1.5)
- Identify vertical angles, linear pairs, complementary angles, and supplementary angles. (1.6)
- Find the perimeter, circumference, and area of common plane figures. (1.7)
- Use a general problem-solving plan. (1.7)

WHY did you learn it?

- Use a pattern to predict a figure or number in a sequence. (p. 3)
- Make and verify conjectures such as a conjecture about the frequency of full moons. (p. 5)
- Understand the basic elements of geometry.
- Visualize the basic elements of geometry and the ways they can intersect.
- Solve real-life problems, such as finding the distance between two points on a map. (p. 20)
- Solve problems in geometry and in real life, such as finding the measure of the angle of vision for a horse wearing blinkers. (p. 27)
- Solve problems in geometry and in real life, such as finding an angle measure of a kite. (p. 37)
- Find the angle measures of geometric figures and real-life structures, such as intersecting metal supports of a stair railing. (p. 45)
- To solve problems related to measurement, such as finding the area of a deck for a pool. (p. 54)
- To solve problems related to mathematics and real life, such as finding the number of bags of grass seed you need for a soccer field. (p. 53)

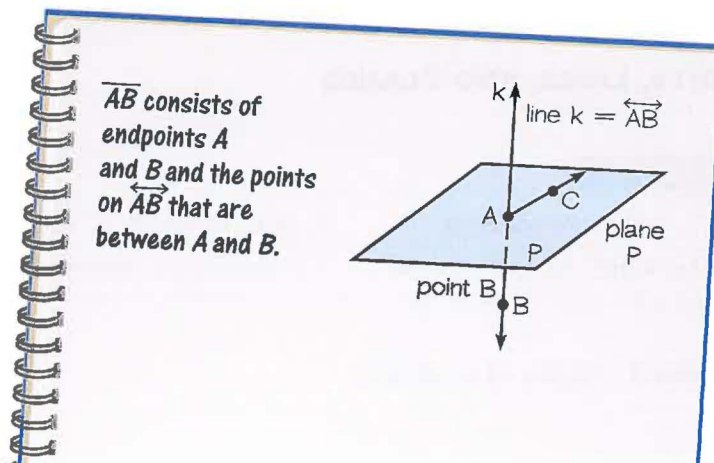
How does Chapter 1 fit into the BIGGER PICTURE of geometry?

In this chapter, you learned a basic reasoning skill—inductive reasoning. You also learned many fundamental terms—*point*, *line*, *plane*, *segment*, and *angle*, to name a few. Added to this were four basic postulates. These building blocks will be used throughout the remainder of this book to develop new terms, postulates, and theorems to explain the geometry of the world around you.

STUDY STRATEGY

How did you use your vocabulary pages?

The definitions of vocabulary terms you made, using the Study Strategy on page 2, may resemble this one.



VOCABULARY

- conjecture, p. 4
- inductive reasoning, p. 4
- counterexample, p. 4
- definition, undefined, p. 10
- point, line, plane, p. 10
- collinear, coplanar, p. 10
- line segment, p. 11
- endpoints, p. 11
- ray, p. 11
- initial point, p. 11
- opposite rays, p. 11
- intersect, intersection, p. 12
- postulates, or axioms, p. 17
- coordinate, p. 17
- distance, length, p. 17
- between, p. 18
- Distance Formula, p. 19
- congruent segments, p. 19
- angle, p. 26
- sides, vertex of an angle, p. 26
- congruent angles, p. 26
- measure of an angle, p. 27
- interior of an angle, p. 27
- exterior of an angle, p. 27
- acute, obtuse angles, p. 28
- right, straight angles, p. 28
- adjacent angles, p. 28
- midpoint, p. 34
- bisect, p. 34
- segment bisector, p. 34
- compass, straightedge, p. 34
- construct, construction, p. 34
- Midpoint Formula, p. 35
- angle bisector, p. 36
- vertical angles, p. 44
- linear pair, p. 44
- complementary angles, p. 46
- complement of an angle, p. 46
- supplementary angles, p. 46
- supplement of an angle, p. 46

1.1

PATTERNS AND INDUCTIVE REASONING

Examples on
pp. 3–5

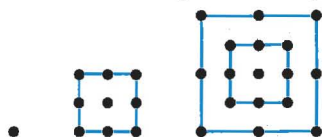
EXAMPLE Make a conjecture based on the results shown.

<p>Conjecture: Given a 3-digit number, form a 6-digit number by repeating the digits. Divide the number by 7, then 11, then 13. The result is the original number.</p>	$456,456 \div 7 \div 11 \div 13 = 456$ $562,562 \div 7 \div 11 \div 13 = 562$ $109,109 \div 7 \div 11 \div 13 = 109$
---	--

In Exercises 1–3, describe a pattern in the sequence of numbers.

1. 5, 12, 19, 26, 33, ... 2. 0, 2, 6, 14, 30, ... 3. 4, 12, 36, 108, 324, ...

4. Sketch the next figure in the pattern. 5. Make a conjecture based on the results.



$$4 \cdot 5 \cdot 6 \cdot 7 + 1 = 29 \cdot 29$$

$$5 \cdot 6 \cdot 7 \cdot 8 + 1 = 41 \cdot 41$$

$$6 \cdot 7 \cdot 8 \cdot 9 + 1 = 55 \cdot 55$$

6. Show the conjecture is false by finding a counterexample:

Conjecture: The cube of a number is always greater than the number.

1.2

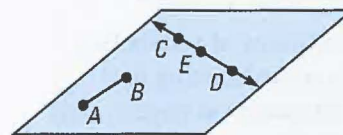
POINTS, LINES, AND PLANES

Examples on
pp. 10–12

EXAMPLE

C , E , and D are collinear.
 \overleftrightarrow{CD} is a line. \overline{AB} is a segment.

A , B , C , D , and E are coplanar.
 \overrightarrow{EC} and \overrightarrow{ED} are opposite rays.



- Draw five coplanar points, $A, B, C, D,$ and E so that \overrightarrow{BA} and \overrightarrow{BC} are opposite rays, and \overrightarrow{DE} intersects \overrightarrow{AC} at B .
- Sketch three planes that do not intersect.
- Sketch two lines that are not coplanar and do not intersect.

1.3

SEGMENTS AND THEIR MEASURES

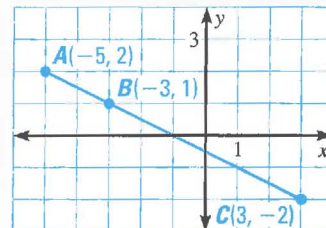
Examples on pp. 17–20

EXAMPLE B is between A and C , so $AB + BC = AC$. Use the Distance Formula to find AB and BC .

$$AB = \sqrt{[-3 - (-5)]^2 + (1 - 2)^2} = \sqrt{2^2 + (-1)^2} = \sqrt{5}$$

$$BC = \sqrt{[3 - (-3)]^2 + (-2 - 1)^2} = \sqrt{6^2 + (-3)^2} = \sqrt{45}$$

Because $AB \neq BC$, \overline{AB} and \overline{BC} are *not* congruent segments.



- Q is between P and S . R is between Q and S . S is between Q and T . $PT = 30$, $QS = 16$, and $PQ = QR = RS$. Find PQ , ST , and RP .

Use the Distance Formula to decide whether $\overline{PQ} \cong \overline{QR}$.

11. $P(-4, 3)$
 $Q(-2, 1)$
 $R(0, -1)$

12. $P(-3, 5)$
 $Q(1, 3)$
 $R(4, 1)$

13. $P(-2, -2)$
 $Q(0, 1)$
 $R(1, 4)$

1.4

ANGLES AND THEIR MEASURES

Examples on pp. 26–28

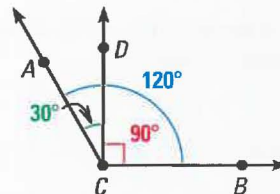
EXAMPLE

$$m\angle ACD + m\angle DCB = m\angle ACB$$

$\angle ACD$ is an acute angle: $m\angle ACD < 90^\circ$.

$\angle DCB$ is a right angle: $m\angle DCB = 90^\circ$.

$\angle ACB$ is an obtuse angle: $m\angle ACB > 90^\circ$.



Classify the angle as *acute*, *right*, *obtuse*, or *straight*. Sketch the angle. Then use a protractor to check your results.

14. $m\angle KLM = 180^\circ$

15. $m\angle A = 150^\circ$

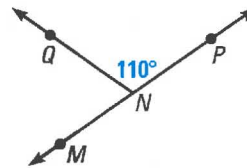
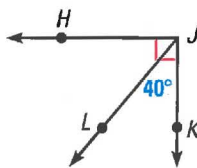
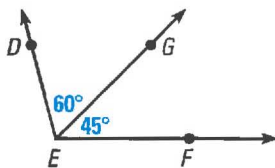
16. $m\angle Y = 45^\circ$

Use the Angle Addition Postulate to find the measure of the unknown angle.

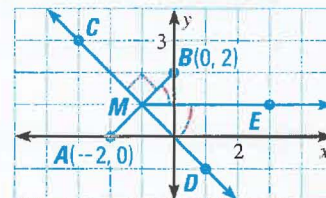
17. $m\angle DEF$

18. $m\angle HJL$

19. $m\angle QNM$



EXAMPLE If \overleftrightarrow{CD} is a bisector of \overline{AB} , then \overleftrightarrow{CD} intersects \overline{AB} at its midpoint M : $M = \left(\frac{-2+0}{2}, \frac{0+2}{2}\right) = (-1, 1)$.
 \overline{ME} bisects $\angle BMD$, so $m\angle BME = m\angle EMD = 45^\circ$.



Find the coordinates of the midpoint of a segment with the given endpoints.

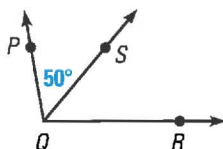
20. $A(0, 0), B(-8, 6)$

21. $J(-1, 7), K(3, -3)$

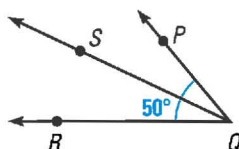
22. $P(-12, -9), Q(2, 10)$

\overleftrightarrow{QS} is the bisector of $\angle PQR$. Find any angle measures not given in the diagram.

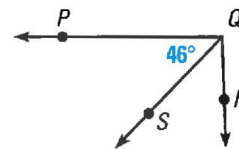
23.



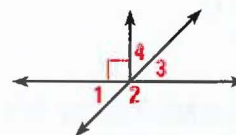
24.



25.



EXAMPLE $\angle 1$ and $\angle 3$ are vertical angles.
 $\angle 1$ and $\angle 2$ are a linear pair and are supplementary angles.
 $\angle 3$ and $\angle 4$ are complementary angles.



Use the diagram above to decide whether the statement is *always*, *sometimes*, or *never* true.

26. If $m\angle 2 = 115^\circ$, then $m\angle 3 = 65^\circ$.

27. $\angle 3$ and $\angle 4$ are congruent.

28. If $m\angle 1 = 40^\circ$, then $m\angle 3 = 50^\circ$.

29. $\angle 1$ and $\angle 4$ are complements.

EXAMPLES A circle has diameter 24 ft.
 Its circumference is $C = 2\pi r \approx 2(3.14)(12) = 75.36$ feet.
 Its area is $A = \pi r^2 \approx 3.14(12^2) = 452.16$ square feet.

Find the perimeter (or circumference) and area of the figure described.

30. Rectangle with length 10 cm and width 4.5 cm

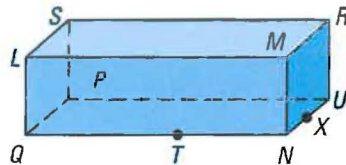
31. Circle with radius 9 in. (Use $\pi \approx 3.14$.)

32. Triangle defined by $A(-6, 0)$, $B(2, 0)$, and $C(-2, -3)$

33. A square garden has sides of length 14 ft. What is its perimeter?

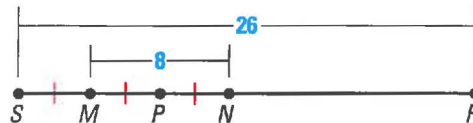
Use the diagram to name the figures.

- Three collinear points
- Four noncoplanar points
- Two opposite rays
- Two intersecting lines
- The intersection of plane LMN and plane QLS



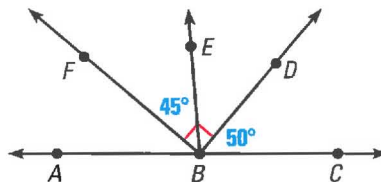
Find the length of the segment.

- \overline{MP}
- \overline{SM}
- \overline{NR}
- \overline{MR}

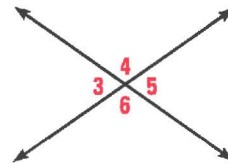


Find the measure of the angle.

- $\angle DBE$
- $\angle FBC$
- $\angle ABF$
- $\angle DBA$

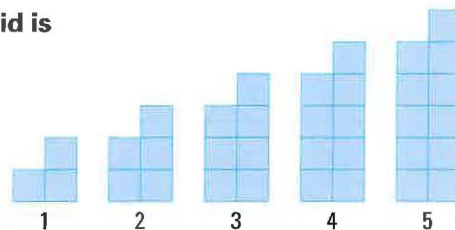


- Refer to the diagram for Exercises 10–13. Name an obtuse angle, an acute angle, a right angle, and two complementary angles.
- Q is between P and R . $PQ = 2w - 3$, $QR = 4 + w$, and $PR = 34$. Find the value of w . Then find the lengths of \overline{PQ} and \overline{QR} .
- \overline{RT} has endpoints $R(-3, 8)$ and $T(3, 6)$. Find the coordinates of the midpoint, S , of \overline{RT} . Then use the Distance Formula to verify that $RS = ST$.
- Use the diagram. If $m\angle 3 = 68^\circ$, find the measures of $\angle 5$ and $\angle 4$.
- Suppose $m\angle PQR = 130^\circ$. If \overrightarrow{QT} bisects $\angle PQR$, what is the measure of $\angle PQT$?



The first five figures in a pattern are shown. Each square in the grid is 1 unit \times 1 unit.

- Make a table that shows the distance around each figure at each stage.
- Describe the pattern of the distances and use it to predict the distance around the figure at stage 20.



A center pivot irrigation system uses a fixed water supply to water a circular region of a field. The radius of the watering system is 560 feet long. (Use $\pi \approx 3.14$.)

- If some workers walked around the circumference of the watered region, how far would they have to walk? Round to the nearest foot.
- Find the area of the region watered. Round to the nearest square foot.

Chapter Standardized Test

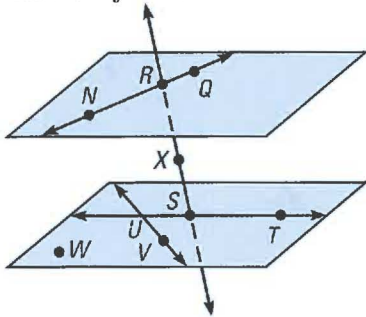
TEST-TAKING STRATEGY Work as quickly as you can through the easier sections, but avoid making careless errors on easy questions.

1. **MULTIPLE CHOICE** What is the next number in the sequence?

4488; 44,088; 440,088; 4,400,088; ...

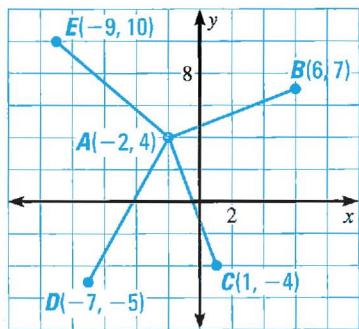
- (A) 400,008 (B) 40,000,088
(C) 44,000,088 (D) 440,000,088
(E) 44,000,008

2. **MULTIPLE CHOICE** Which of the following statements is *false*?



- (A) $S, T, V,$ and W are coplanar.
(B) $X, T, S,$ and U are coplanar.
(C) $Q, N,$ and R are collinear.
(D) $S, R,$ and X are collinear.
(E) \overrightarrow{TS} and \overrightarrow{TU} are opposite rays.

3. **MULTIPLE CHOICE** Which of the line segments shown in the coordinate plane are congruent?



- (A) \overline{AC} and \overline{AE} (B) \overline{AB} and \overline{AE}
(C) \overline{AD} and \overline{AC} (D) \overline{AD} and \overline{AB}
(E) \overline{AB} and \overline{AC}

4. **MULTIPLE CHOICE** B is between A and C , D is between B and C , and C is between B and E . $AE = 28$, $BC = 10$, and $AB = DB = DC$. What is the length of \overline{CE} ?

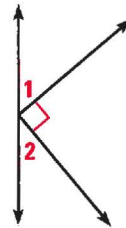
- (A) 5 (B) 10 (C) 12
(D) 13 (E) 15

5. **MULTIPLE CHOICE** If $\angle 4$ and $\angle 5$ are complementary and $m\angle 4 = 19^\circ$, find $m\angle 5$.

- (A) 19° (B) 71° (C) 109°
(D) 161° (E) cannot be determined

6. **MULTIPLE CHOICE** $\angle 1$ and $\angle 2$ in the diagram are _____.

- (A) complementary
(B) supplementary
(C) congruent
(D) vertical angles
(E) a linear pair

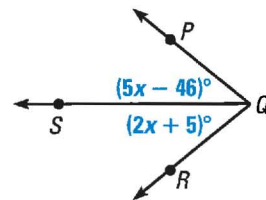


7. **MULTIPLE CHOICE** The midpoint of \overline{BC} is $M(-10, -16)$. One endpoint is $B(-1, 8)$. What are the coordinates of C ?

- (A) $(-21, -40)$ (B) $(-20, -40)$
(C) $(-19, -40)$ (D) $(-21, -24)$
(E) $(8, 32)$

8. **MULTIPLE CHOICE** If \overrightarrow{QS} bisects $\angle PQR$, find the measure of $\angle PQR$.

- (A) 17°
(B) 56°
(C) 21°
(D) 39°
(E) 78°



9. **MULTIPLE CHOICE** Two angles are complementary and one angle has a measure that is 9 times the measure of the other angle. What is the angle measure of the larger angle?

- (A) 9°
(B) 18°
(C) 81°
(D) 90°
(E) 162°

10. **QUANTITATIVE COMPARISON** Consider the areas of the two triangles that are described below.

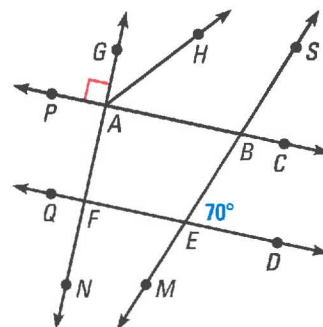
COLUMN A	COLUMN B
The area of a triangle defined by $A(-6, 7)$, $B(-6, -1)$, and $C(-3, 2)$	The area of a triangle defined by $D(0, 4)$, $E(6, 4)$, and $F(6, 0)$

Choose the statement that is true.

- (A) The quantity in column A is greater.
 (B) The quantity in column B is greater.
 (C) The two quantities are equal.
 (D) The relationship cannot be determined from the information given.

MULTI-STEP PROBLEM In Exercises 11–14, use the figure at the right.

11. Name an angle that is (a) acute, (b) obtuse, (c) straight, and (d) right.
 12. Classify each pair of angles as *complementary*, *supplementary*, or *vertical angles*.
 a. $\angle ABS$ and $\angle SBC$
 b. $\angle BAH$ and $\angle GAH$
 c. $\angle BEF$ and $\angle FEM$
 d. $\angle ABS$ and $\angle EBC$
 13. If \overrightarrow{AH} bisects $\angle GAB$, find the measures of $\angle GAH$ and $\angle BAH$.
 14. If $m\angle QFN = x^\circ$, express the measures of $\angle QFA$, $\angle AFE$, and $\angle EFN$ in terms of x .



MULTI-STEP PROBLEM Consider some rectangles with a perimeter of 24 inches.

15. Copy and complete the table below.

Width (in.)	Perimeter (in.)	Length (in.)	Area (in. ²)
1	?	?	?
2	?	?	?
3	?	?	?
4	?	?	?
5	?	?	?
6	?	?	?
7	?	?	?

16. Which rectangle in the table has the greatest area?
 17. Look at the entries in the table. Describe a pattern in the widths and lengths. Use the pattern to predict the length of a rectangle with a width of 3.5 inches.
 18. Make a conjecture about the dimensions of a rectangle with greatest area if the perimeter of the rectangle is known. Describe a way to test your conjecture.

Taxicab Geometry

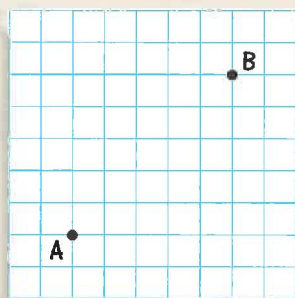
OBJECTIVE Compare distances in taxicab geometry to distances in Euclidean geometry.

Materials: ruler, graph paper, colored pencils, poster paper

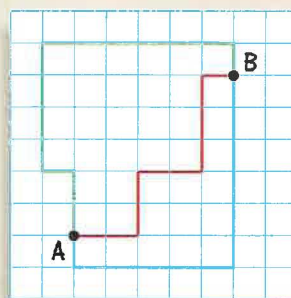
In *taxicab geometry*, distances are measured along paths that are made of horizontal and vertical segments. Diagonal paths are not allowed. This simulates the movement of taxicabs in a city, which can travel only on streets, never through buildings.

FINDING TAXICAB DISTANCES

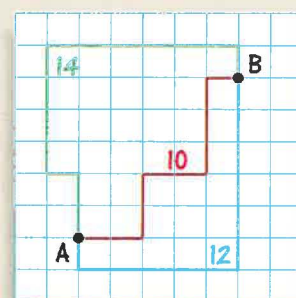
Follow these steps to learn more about distance in taxicab geometry.



1 Copy points *A* and *B* on a piece of graph paper.



2 Trace paths from *A* to *B* using the grid lines. You may move horizontally and vertically but not diagonally.



3 Calculate the distances covered by your paths.

INVESTIGATION

Repeat Steps 2 and 3 above several times, then answer the exercises.

1. Compare your paths and those of your classmates. Are the distances always the same? What is the length of the shortest possible path from *A* to *B*?
2. The length of the shortest path from *A* to *B* is called the *taxicab distance* from *A* to *B*. Can you find other paths from *A* to *B* that have the same distance?
3. Use the Distance Formula to find the Euclidean distance from *A* to *B*. Which is greater, the taxicab distance from *A* to *B* or the Euclidean distance?
4. On another piece of graph paper, plot a new pair of points. Find the taxicab distance and Euclidean distance for the points. Repeat this for several pairs of points. Write a general statement that compares taxicab distance and Euclidean distance.
5. Write a general formula for the taxicab distance from $A(x_1, y_1)$ to $B(x_2, y_2)$.

TAXICAB CIRCLES

Suppose you are in a city with a square grid and want to find all the places you can reach if you walk 4 blocks along the streets. This forms a figure in taxicab geometry in the same way that a circle is formed in Euclidean geometry.

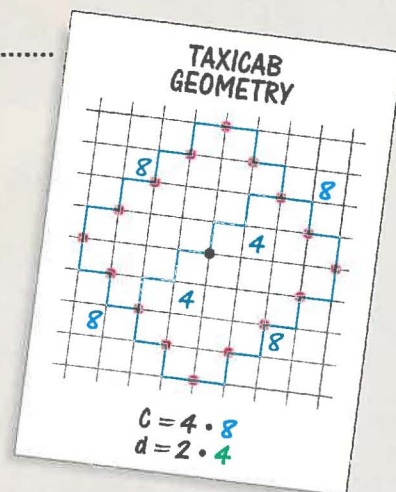
INVESTIGATION

- On a piece of graph paper, plot a point $O(0, 0)$. Plot all the points that have a taxicab distance of 4 units from point O . How are the points arranged?
- To find the circumference of your taxicab circle, imagine that you are walking a shortest taxicab path that passes through all the points you drew in Investigation Exercise 6. Draw one such path. What is its total length? (Remember that all distances in taxicab geometry are horizontal or vertical.)
- To find the diameter of your taxicab circle, draw a shortest taxicab path that joins two points on the circumference and also passes through the center O . What is its length?
- In Euclidean geometry, the constant π is defined as the ratio of a circle's circumference to its diameter. If π is defined in the same way in taxicab geometry, does it have a constant value? If so, what is the value?

PRESENT YOUR RESULTS

Gather your work and present it as a poster.

- Include your answers to the Investigation Exercises.
- Include your calculations and drawings you used to find the value of π in taxicab geometry.
- Summarize how taxicab geometry and Euclidean geometry are different.



EXTENSIONS

In Euclidean geometry, the points that lie between two points A and B form a segment. In taxicab geometry, what kind of region do the points that lie *between* two points A and B form? (*Hint*: A point C lies *between* A and B in taxicab geometry if this equation is true for the taxicab distances: $AC + CB = AB$.)

In Euclidean geometry, the midpoint of \overline{AB} *bisects* the segment into two parts. All the points in one part are closer to A than to B , and all the points in the other part are closer to B than to A . Is there a set of points that *bisects* the set of points between A and B in taxicab geometry? What does it look like?