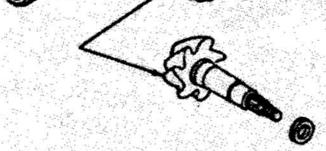
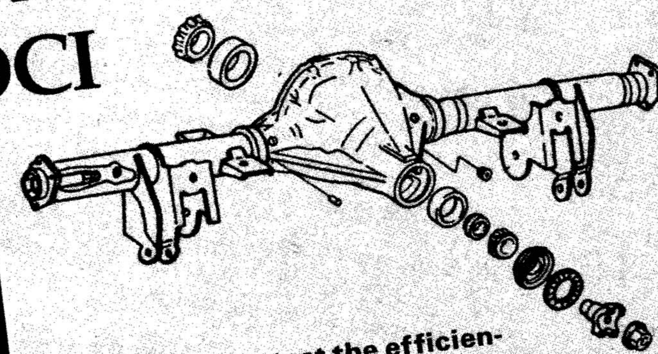
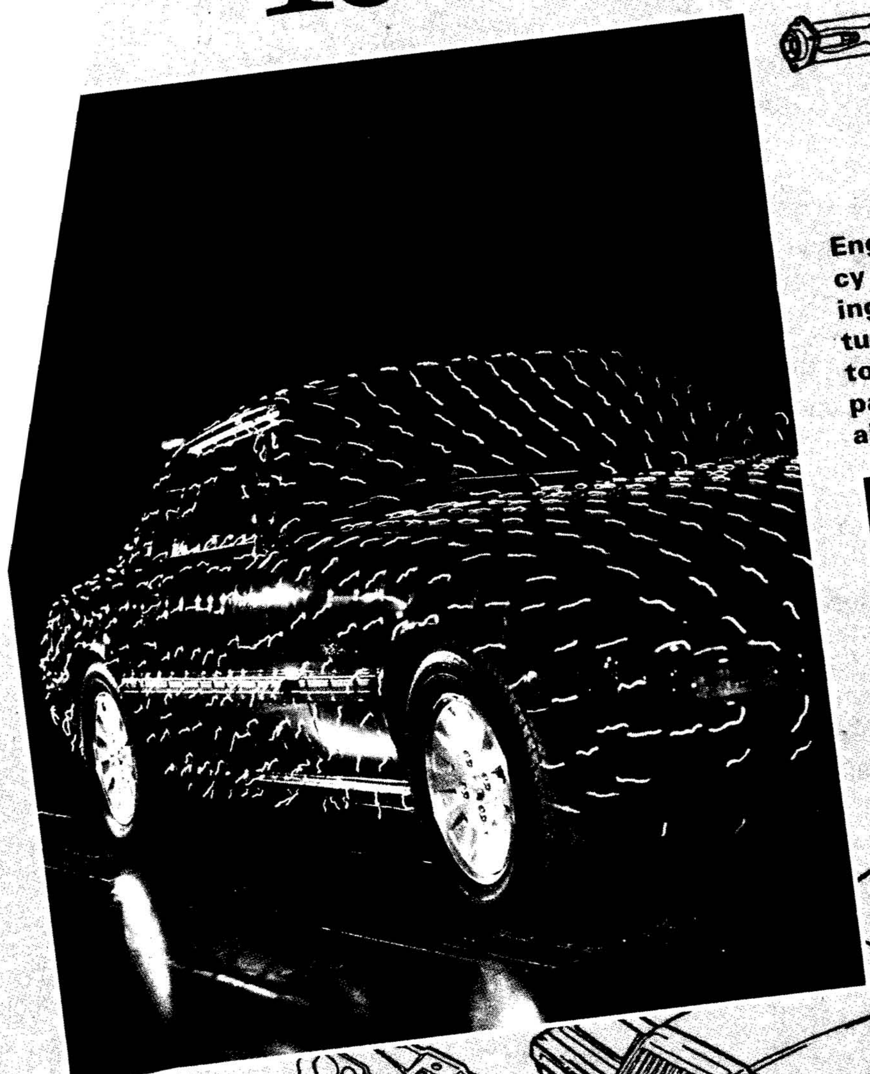
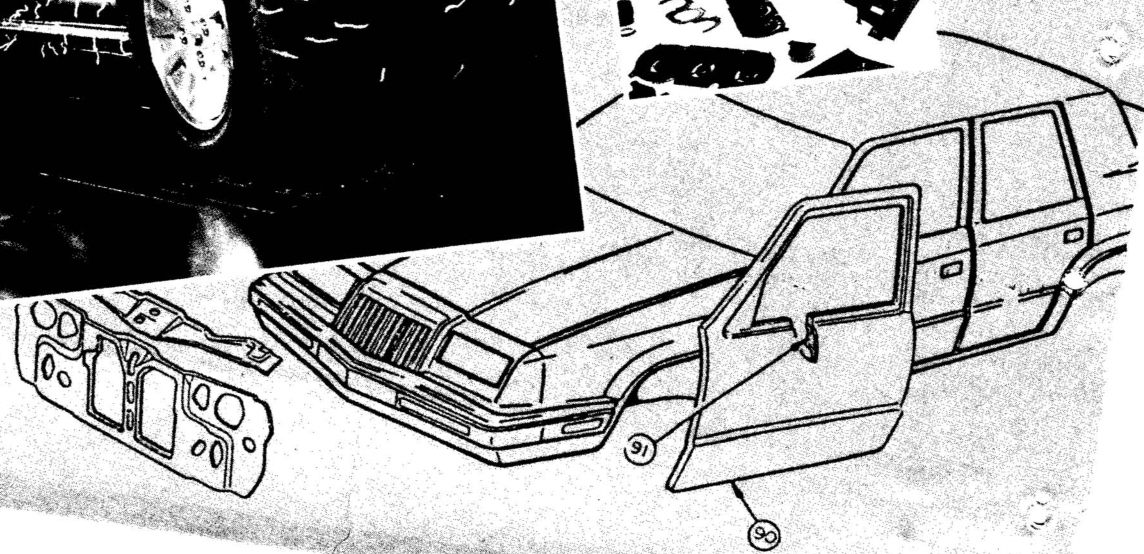


10 CONSTRUCTIONS AND LOCI





Engineers can test the efficiency of a car's design by observing the air currents in a wind tunnel. Strips of cloth attached to the body, as well as smoke patterns, indicate the flow of air around the car.



Basic Constructions

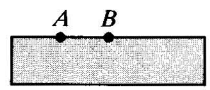
Objectives

1. Perform seven basic constructions.
2. Use these basic constructions in original construction exercises.
3. State and apply theorems involving concurrent lines.

10-1 *What Construction Means*

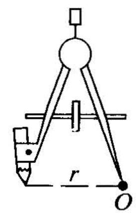
In Chapters 1–9 we have used rulers and protractors to draw segments with certain lengths and angles with certain measures. In this chapter we will *construct* geometric figures using only two instruments, a *straightedge* and a *compass*. (You may use a ruler as a straightedge as long as you do not use the marks on the ruler.)

Using a Straightedge in Constructions



Given two points A and B, we know from Postulate 6 that there is exactly one line through A and B. We agree that we can use a straightedge to draw $\overleftrightarrow{AB}$ or parts of the line, such as $\overline{AB}$ and $\overrightarrow{AB}$.

Using a Compass in Constructions



Given a point O and a length r, we know from the definition of a circle that there is exactly one circle with center O and radius r. We agree that we can use a compass to draw this circle or arcs of the circle.

Construction 1

Given a segment, construct a segment congruent to the given segment.

Given: $\overline{AB}$

Construct: A segment congruent to $\overline{AB}$

Procedure:

1. Use a straightedge to draw a line. Call it l.
2. Choose any point on l and label it X.
3. Set your compass for radius AB. Using X as center, draw an arc intersecting line l. Label the point of intersection Y.

$\overline{XY}$ is congruent to $\overline{AB}$.

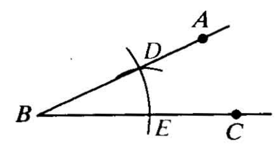
Justification: Since you used AB for the radius of $\odot X$, $\overline{XY} \cong \overline{AB}$.

Construction 2

Given an angle, construct an angle congruent to the given angle.

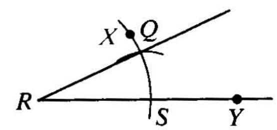
Given: $\angle ABC$

Construct: An angle congruent to $\angle ABC$



Procedure:

1. Draw a ray. Label it $\overrightarrow{RY}$.
2. Using B as center and any radius, draw an arc intersecting $\overrightarrow{BA}$ and $\overrightarrow{BC}$. Label the points of intersection D and E, respectively.
3. Using R as center and the same radius as in Step 2, draw an arc intersecting $\overrightarrow{RY}$. Label the arc $\overset{\frown}{XS}$, with S the point where the arc intersects $\overrightarrow{RY}$.
4. Using S as center and a radius equal to DE, draw an arc that intersects $\overset{\frown}{XS}$ at a point Q.
5. Draw $\overrightarrow{RQ}$.

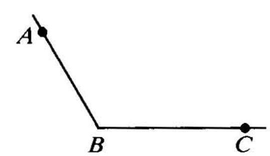


$\angle QRS$ is congruent to $\angle ABC$.

Justification: If you draw $\overline{DE}$ and $\overline{QS}$, $\triangle DBE \cong \triangle QRS$ (SSS Postulate). Then $\angle QRS \cong \angle ABC$.

Construction 3

Given an angle, construct the bisector of the angle.

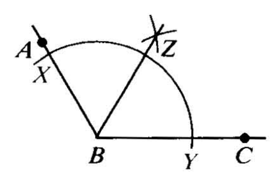


Given: $\angle ABC$

Construct: The bisector of $\angle ABC$

Procedure:

1. Using B as center and any radius, draw an arc that intersects $\overrightarrow{BA}$ at X and $\overrightarrow{BC}$ at Y.
2. Using X as center and a suitable radius, draw an arc. Using Y as center and the same radius, draw an arc that intersects the arc with center X at a point Z.
3. Draw $\overrightarrow{BZ}$.

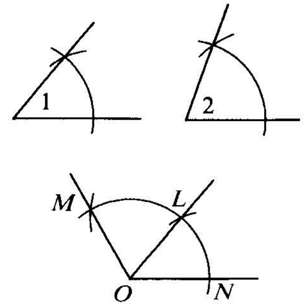


$\overrightarrow{BZ}$ bisects $\angle ABC$.

Justification: If you draw $\overline{XZ}$ and $\overline{YZ}$, $\triangle XBZ \cong \triangle YBZ$ (SSS Postulate). Then $\angle XBZ \cong \angle YBZ$ and $\overrightarrow{BZ}$ bisects $\angle ABC$.

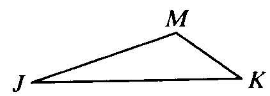
Example Given $\angle 1$ and $\angle 2$, construct an angle whose measure is equal to $m\angle 1 + m\angle 2$.

Solution First use Construction 2 to construct $\angle LON$ congruent to $\angle 1$. Then use the same method to construct $\angle MOL$ congruent to $\angle 2$ (as shown) so that $m\angle MON = m\angle 1 + m\angle 2$.

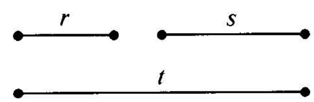


In construction exercises, you won't ordinarily have to write out the procedure and the justification. However, you should be able to supply them when asked to do so.

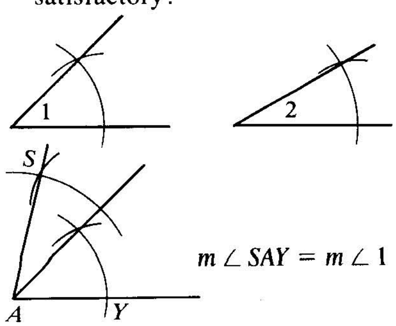
Classroom Exercises



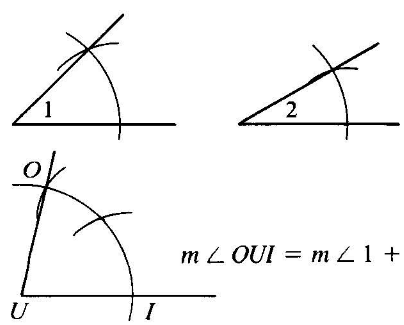
1. Given: $\triangle JKM$
   Explain how to construct a triangle that is congruent to $\triangle JKM$.
2. Draw any $\overline{AB}$.
   a. Construct $\overline{XY}$ so that $XY = AB$.
   b. Using X and Y as centers, and a radius equal to AB, draw arcs that intersect. Label the point of intersection Z.
   c. Draw $\overline{XZ}$ and $\overline{YZ}$.
   d. What kind of triangle is $\triangle XYZ$?
3. Explain how you could construct a 30° angle.
4. Exercise 3 suggests that you could construct other angles with certain measures. Name some.
5. Suppose you are given the three lengths shown and are asked to construct a triangle whose sides have lengths r, s, and t. Can you do so? State the theorem from Chapter 6 that applies.



6. $\angle 1$ and $\angle 2$ are given. You see two attempts at constructing an angle whose measure is equal to $m\angle 1 + m\angle 2$. Are both constructions satisfactory?



$m\angle SAY = m\angle 1 + m\angle 2$



$m\angle OUI = m\angle 1 + m\angle 2$

Written Exercises

On your paper, draw two segments roughly like those shown. Use these segments in Exercises 1–4 to construct a segment having the indicated length.

A **1.** $a + b$ **2.** $b - a$ **3.** $3a - b$ **4.** $a + 2b$

5. Using any convenient length for a side, construct an equilateral triangle.

6. **a.** Construct a 30° angle. **b.** Construct a 15° angle.

7. Draw any acute $\triangle ACU$. Use a method based on the SSS Postulate to construct a triangle congruent to $\triangle ACU$.

8. Draw any obtuse $\triangle OBT$. Use the SSS method to construct a triangle congruent to $\triangle OBT$.

9. Repeat Exercise 7, but use the SAS method.

10. Repeat Exercise 8, but use the ASA method.

On your paper, draw two angles roughly like those shown. Then for Exercises 11–14 construct an angle having the indicated measure.

11. $x + y$ **12.** $x - y$ **13.** $\frac{3}{4}x$ **14.** $180 - 2y$

B **15.** **a.** Draw any acute triangle. Bisect each of the three angles.
b. Draw any obtuse triangle. Bisect each of the three angles.
c. What do you notice about the points of intersection of the bisectors in parts (a) and (b)?

16. Construct a six-pointed star using the following procedure.
1. Draw a ray, $\overrightarrow{AB}$. On $\overrightarrow{AB}$ mark off, in order, points C and D such that $AB = BC = CD$.
2. Construct equilateral $\triangle ADG$.
3. On $\overline{AG}$ mark off points E and F so that both AE and EF equal AB.
4. On $\overline{GD}$ mark off points H and I so that both GH and HI equal AB.
5. To complete the star, draw the three lines $\overleftrightarrow{FH}$, $\overleftrightarrow{EB}$, and $\overleftrightarrow{CI}$.

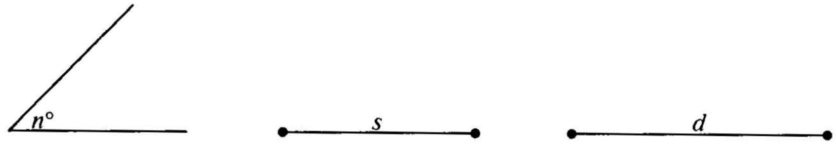
Construct an angle having the indicated measure.

17. 120 **18.** 150 **19.** 165 **20.** 45

21. Draw any $\triangle ABC$. Construct $\triangle DEF$ so that $\triangle DEF \sim \triangle ABC$ and $DE = 2AB$.

22. Construct a $\triangle RST$ such that $RS:ST:TR = 4:6:7$.

On your paper draw figures roughly like those shown. Use them in constructing the figures described in Exercises 23–25.

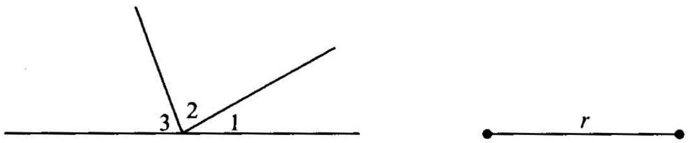


23. An isosceles triangle with a vertex angle of $n°$ and legs of length d

24. An isosceles triangle with a vertex angle of $n°$ and base of length s

C 25. A parallelogram with an $n°$ angle, longer side of length s, and longer diagonal of length d

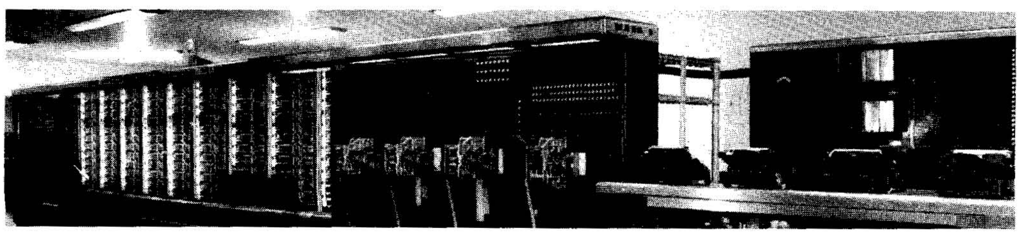
★ **26.** On your paper draw figures roughly like the ones shown. Then construct a triangle whose three angles are congruent to $\angle 1$, $\angle 2$, and $\angle 3$, and whose circumscribed circle has radius r.



Biographical Note — *Grace Hopper*

In 1944 the Mark I, the first working computing machine, started operations at Harvard. It could do three additions per second; calculations that took six months by hand could now be done in a day. Today, computers are one *billion* times as fast, partly because software (programming) has become more efficient, but mostly because of advances in hardware (electronics) such as the development of integrated circuits and silicon chips.

Rear Adm. Grace Hopper, U.S. Navy (Ret.) worked on that first computing machine and many others since. After getting her Ph.D. in mathematics in 1934 from Yale and teaching for several years, Hopper joined the Navy in 1943 and was assigned to Harvard as a programmer of the Mark I. In 1957, her work on making programming faster and easier resulted in her language called Flowmatic, based on the novel idea of using English words in a computer language. The first machine-independent language, COBOL, was announced in 1960 and was based on her language. She continues today to promote computers and learning, saying computers are the "first tool to assist man's brain instead of his arm."



Mixed Review Exercises

Complete.

1. A median of a triangle is a segment from a vertex to the __?__ of the opposite side.
2. A quadrilateral with both pairs of opposite angles congruent is a __?__.
3. A parallelogram with congruent diagonals is a __?__.
4. A parallelogram with perpendicular diagonals is a __?__.
5. If a side of a square has length 5 cm, then a diagonal of the square has length __?__ cm.
6. The measure of each interior angle of a regular pentagon is __?__.

10-2 *Perpendiculars and Parallels*

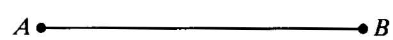
The next three constructions are based on a theorem and postulate from earlier chapters. The theorem and postulate are repeated here for your use.

(1) If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of the segment.

(2) Through any two points there is exactly one line.

Construction 4

Given a segment, construct the perpendicular bisector of the segment.

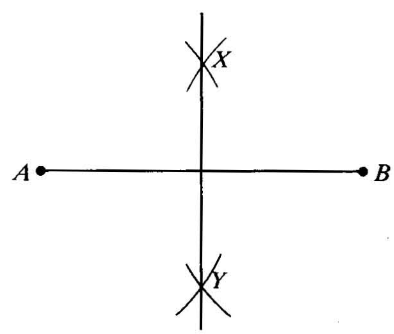


Given: $\overline{AB}$

Construct: The perpendicular bisector of $\overline{AB}$

Procedure:

1. Using any radius greater than $\frac{1}{2}AB$, draw four arcs of equal radii, two with center A and two with center B. Label the points of intersections of these arcs X and Y.
2. Draw $\overleftrightarrow{XY}$.



$\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$.

Justification: Points X and Y are equidistant from A and B. Thus $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$.

Note that you can use Construction 4 to find the midpoint of a segment.

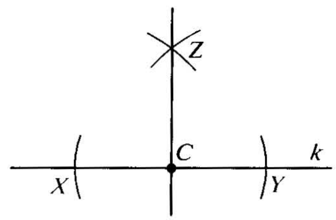
Construction 5

Given a point on a line, construct the perpendicular to the line at the given point.

Given: Point C on line k

Construct: The perpendicular to k at C

Procedure:



1. Using C as center and any radius, draw arcs intersecting k at X and Y.
2. Using X as center and a radius greater than CX, draw an arc. Using Y as center and the same radius, draw an arc intersecting the arc with center X at a point Z.
3. Draw $\overleftrightarrow{CZ}$.

$\overleftrightarrow{CZ}$ is perpendicular to k at C.

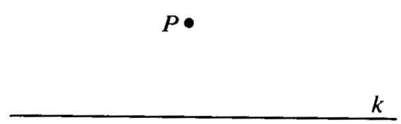
Justification: You constructed points X and Y so that C is equidistant from X and Y. Then you constructed point Z so that Z is equidistant from X and Y. Thus $\overleftrightarrow{CZ}$ is the perpendicular bisector of $\overline{XY}$, and $\overleftrightarrow{CZ} \perp k$ at C.

Construction 6

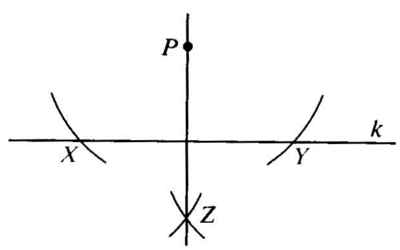
Given a point outside a line, construct the perpendicular to the line from the given point.

Given: Point P outside line k

Construct: The perpendicular to k from P



Procedure:



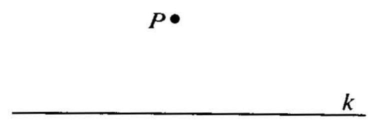
1. Using P as center, draw two arcs of equal radii that intersect k at points X and Y.
2. Using X and Y as centers and a suitable radius, draw arcs that intersect at a point Z.
3. Draw $\overleftrightarrow{PZ}$.

$\overleftrightarrow{PZ}$ is perpendicular to k.

Justification: Both P and Z are equidistant from X and Y. Thus $\overleftrightarrow{PZ}$ is the perpendicular bisector of $\overline{XY}$, and $\overleftrightarrow{PZ} \perp k$.

Construction 7

Given a point outside a line, construct the parallel to the given line through the given point.



Given: Point P outside line k

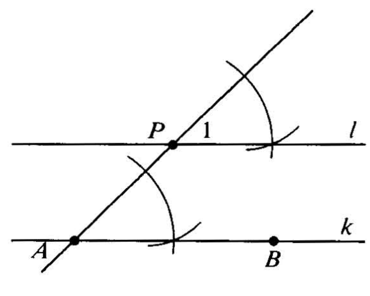
Construct: The line through P parallel to k

Procedure:

1. Let A and B be two points on line k. Draw $\overleftrightarrow{PA}$.
2. At P, construct $\angle 1$ so that $\angle 1$ and $\angle PAB$ are congruent corresponding angles. Let l be the line containing the ray you just constructed.

l is the line through P parallel to k.

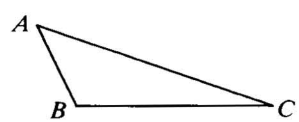
Justification: If two lines are cut by a transversal and corresponding angles are congruent, then the lines are parallel. (Postulate 11)



Classroom Exercises

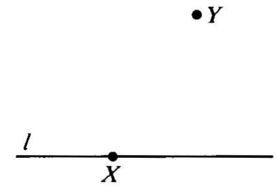
1. Suggest an alternative procedure for Construction 7 that uses Constructions 5 and 6.

Describe how you would construct each of the following.



2. The midpoint of $\overline{BC}$
3. The median of $\triangle ABC$ that contains vertex B
4. The altitude of $\triangle ABC$ that contains vertex B
5. The altitude of $\triangle ABC$ that contains vertex A
6. The perpendicular to $\overline{BC}$ at C
7. A square whose sides each have length AC
8. A square whose perimeter equals AC
9. A right triangle with hypotenuse and one leg equal to AC and BC, respectively
10. A triangle whose sides are in the ratio $1:2:\sqrt{5}$

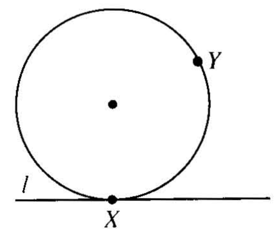
Exercises 11–13 will analyze the following problem.



Given: Line l; points X and Y

Construct: A circle through Y and tangent to l at X

If the problem had been solved, we would have a diagram something like the one shown.



11. Where does the center of the circle lie with respect to line l and point X?

12. Where does the center of the circle lie with respect to $\overline{XY}$?

13. Explain how to carry out the construction of the circle.

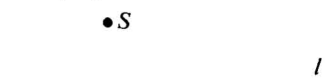
Written Exercises

Draw a figure roughly like the one shown, but larger. Do the indicated construction clearly enough so that your method can be understood easily.

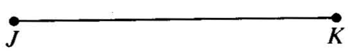
A

1. The perpendicular to l at P

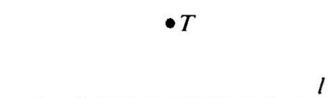
2. The perpendicular to l from S



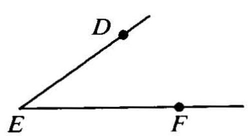
3. The perpendicular bisector of $\overline{JK}$



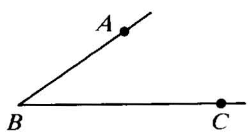
4. The parallel to l through T



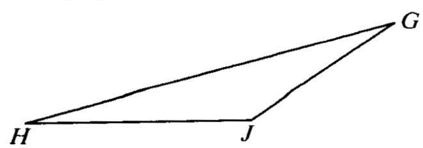
5. The parallel to $\overleftrightarrow{ED}$ through F



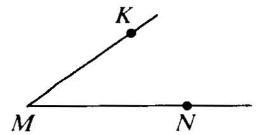
6. The perpendicular to $\overleftrightarrow{BA}$ at A



7. The perpendicular to $\overleftrightarrow{HJ}$ from G



8. A complement of $\angle KMN$



Construct an angle with the indicated measure.

9. 45 **10.** 135

11. $22\frac{1}{2}$ **12.** 105

13. Draw a segment $\overline{AB}$. Construct a segment $\overline{XY}$ whose length equals $\frac{3}{4}AB$.

B **14.** **a.** Draw an acute triangle. Construct the perpendicular bisector of each side.
b. Do the perpendicular bisectors intersect in one point?
c. Repeat parts (a) and (b) using an obtuse triangle.

15. **a.** Draw an acute triangle. Construct the three altitudes.
b. Do the lines that contain the altitudes intersect in one point?
c. Repeat parts (a) and (b) using an obtuse triangle.

16. **a.** Draw a very large acute triangle. Construct the three medians.
b. Do the lines that contain the medians intersect in one point?
c. Repeat parts (a) and (b) using an obtuse triangle.

On your paper draw figures roughly like those shown. Use them in constructing the figures described in Exercises 17–24.

17. A parallelogram with an $n°$ angle and sides of lengths a and b

18. A rectangle with sides of lengths a and b

19. A square with perimeter $2a$

20. A rhombus with diagonals of lengths a and b

21. A square with diagonals of length b

22. A segment of length $\sqrt{a^2 + b^2}$

23. A square with diagonals of length $b\sqrt{2}$

24. A right triangle with hypotenuse of length a and one leg of length b

C **25.** Draw a segment and let its length be s. Construct a segment whose length is $s\sqrt{3}$.

26. Draw a diagram roughly like the one shown. Without laying your straightedge across any part of the lake, construct more of $\overrightarrow{RS}$.

27. Draw three noncollinear points R, S, and T. Construct a triangle whose sides have R, S, and T as midpoints. (*Hint*: How is $\overline{RT}$ related to the side of the triangle that has S as its midpoint?)

28. Draw a segment and let its length be 1.

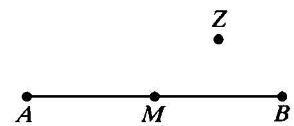
a. Construct a segment of length $\sqrt{5}$.

b. Construct a segment of length $\frac{1}{2} + \frac{\sqrt{5}}{2}$, or $\frac{1 + \sqrt{5}}{2}$.

c. Construct a *golden rectangle* (as discussed on page 253) whose sides are in the ratio $1 : \frac{1 + \sqrt{5}}{2}$.

Challenge

Given $\overline{AB}$, its midpoint M, and a point Z outside $\overline{AB}$, use only a straightedge (and *no* compass) to construct a line through Z parallel to $\overleftrightarrow{AB}$. (*Hint*: Use Ceva's Theorem, Exercise 33, page 273.)



Explorations

These exploratory exercises can be done using a computer with a program that draws and measures geometric figures.

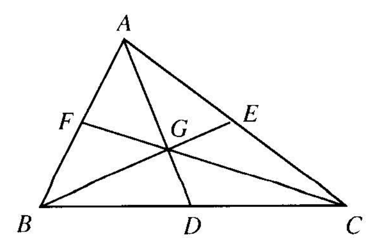
1. Draw any $\triangle ABC$. Draw the bisectors of the angles of the triangle. They should intersect in one point. Draw a perpendicular segment from this point to each of the sides. Measure the length of each perpendicular segment. What do you notice?

2. a. Draw any acute $\triangle ABC$. Draw the perpendicular bisector of each side of the triangle. They should intersect in one point. Measure the distance from this point of intersection to each of the vertices of the triangle. What do you notice?

b. Repeat using an obtuse triangle and a right triangle. Is the same result true for these triangles as well?

c. In a right triangle, the perpendicular bisectors of the sides intersect in what point?

3. Draw any $\triangle ABC$. Draw the three medians. They should intersect in one point, as shown in the diagram at the right. Find the ratios $\frac{AG}{AD}$, $\frac{BG}{BE}$, and $\frac{CG}{CF}$. What do you notice?



10-3 *Concurrent Lines*

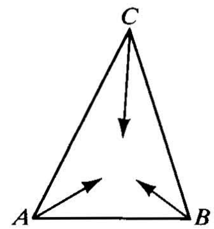
When two or more lines intersect in one point, the lines are said to be **concurrent.** For example, as you saw in Exercise 15, page 378, the bisectors of the angles of a triangle are concurrent.

Theorem 10-1

The bisectors of the angles of a triangle intersect in a point that is equidistant from the three sides of the triangle.

Given: $\triangle ABC$; the bisectors of $\angle A$, $\angle B$, and $\angle C$

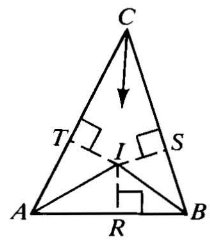
Prove: The angle bisectors intersect in a point; that point is equidistant from $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$.



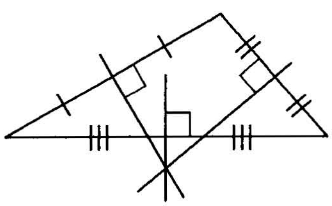
Proof:

The bisectors of $\angle A$ and $\angle B$ intersect at some point I. We will show that point I also lies on the bisector of $\angle C$ and that I is equidistant from $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$.

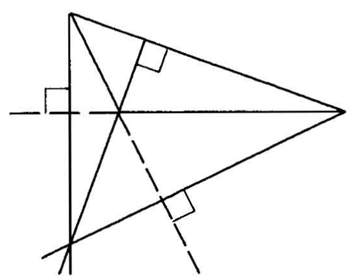
Draw perpendiculars from I intersecting $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ at R, S, and T, respectively. Since any point on the bisector of an angle is equidistant from the sides of the angle (Theorem 4-7, page 154), $IT = IR$ and $IR = IS$. Thus $IT = IS$. Since any point equidistant from the sides of an angle is on the bisector of the angle (Theorem 4-8, page 154), I is on the bisector of $\angle C$. Since $IR = IS = IT$, point I is equidistant from $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$.



In Exercises 14–16, page 384, you discovered three other sets of concurrent lines related to triangles: the perpendicular bisectors of the sides, the lines containing the altitudes, and the medians. As you can see in the diagrams below, concurrent lines may intersect in a point outside the triangle. The intersection point may also lie on the triangle (see Classroom Exercise 4, page 388).



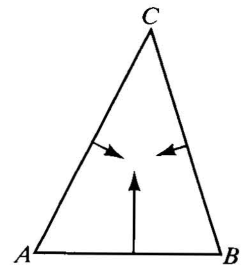
Perpendicular bisectors



Lines containing altitudes

Theorem 10-2

The perpendicular bisectors of the sides of a triangle intersect in a point that is equidistant from the three vertices of the triangle.

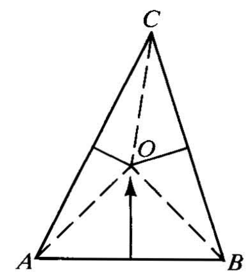


Given: $\triangle ABC$; the $\perp$ bisectors of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$

Prove: The $\perp$ bisectors intersect in a point; that point is equidistant from A, B, and C.

Proof:

The perpendicular bisectors of $\overline{AC}$ and $\overline{BC}$ intersect at some point O. We will show that point O lies on the perpendicular bisector of $\overline{AB}$ and is equidistant from A, B, and C.



Draw $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$. Since any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment (Theorem 4-5, page 153), $OA = OC$ and $OC = OB$. Thus $OA = OB$. Since any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment (Theorem 4-6, page 153), O is on the perpendicular bisector of $\overline{AB}$. Since $OA = OB = OC$, point O is equidistant from A, B, and C.

The following theorems will be proved in Chapter 13.

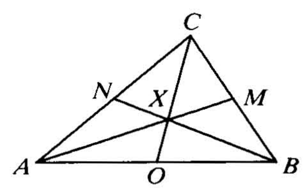
Theorem 10-3

The lines that contain the altitudes of a triangle intersect in a point.

Theorem 10-4

The medians of a triangle intersect in a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

According to Theorem 10-4, if $\overline{AM}$, $\overline{BN}$, and $\overline{CO}$ are medians of $\triangle ABC$, then:



$$AX = \tfrac{2}{3}AM$$
$$XN = \tfrac{1}{3}BN$$
$$CX:XO:CO = 2:1:3$$

The points of intersection described in the theorems in this section are sometimes called the *incenter* (point where the angle bisectors meet), *circumcenter* (point where the perpendicular bisectors meet), *orthocenter* (point where the altitudes meet), and *centroid* (point where the medians meet).

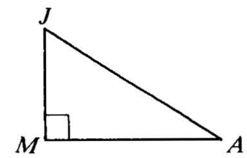
Classroom Exercises

1. Draw, if possible, a triangle in which the perpendicular bisectors of the sides intersect in a point with the location described.
a. A point inside the triangle **b.** A point outside the triangle
c. A point on the triangle

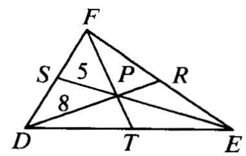
2. Repeat Exercise 1, but work with angle bisectors.

3. Is there some kind of triangle such that the perpendicular bisector of each side is also an angle bisector, a median, and an altitude?

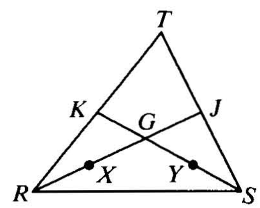
4. $\triangle JAM$ is a right triangle.
a. Is $\overline{JM}$ an altitude of $\triangle JAM$?
b. Name another altitude shown.
c. In what point do the three altitudes of $\triangle JAM$ meet?
d. Where do the perpendicular bisectors of the sides of $\triangle JAM$ meet?
e. Does your answer to (d) agree with Theorem 10-2?



5. The medians of $\triangle DEF$ are shown. Find the lengths indicated.
a. $EP = \underline{\ ?\ }$ **b.** $PR = \underline{\ ?\ }$
c. If $FT = 9$, then $PT = \underline{\ ?\ }$ and $FP = \underline{\ ?\ }$.



6. Given: $\overline{RJ}$ and $\overline{SK}$ are medians of $\triangle RST$;
X and Y are the midpoints of $\overline{RG}$ and $\overline{SG}$.
a. How are $\overline{XY}$ and $\overline{RS}$ related? Why?
b. How are $\overline{KJ}$ and $\overline{RS}$ related? Why?
c. How are $\overline{KJ}$ and $\overline{XY}$ related? Why?
d. What special kind of quadrilateral is $XYJK$? Why?
e. Why does $XG = GJ$?
f. Explain why $RG = \frac{2}{3}RJ$.



Written Exercises

A **1.** Draw a triangle such that the lines containing the three altitudes intersect in a point with the location described.
a. A point inside the triangle **b.** A point outside the triangle
c. A point on the triangle

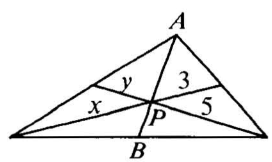
Exercises 2–5 refer to the diagram in which the medians of a triangle are shown.

2. Find the values of x and y.

3. If $AB = 6$, then $BP = \underline{\ ?\ }$ and $AP = \underline{\ ?\ }$.

4. If $AB = 7$, then $BP = \underline{\ ?\ }$ and $AP = \underline{\ ?\ }$.

5. If $PB = 1.9$, then $AP = \underline{\ ?\ }$ and $AB = \underline{\ ?\ }$.



6. Use a ruler and a protractor to draw a regular pentagon. Then construct the perpendicular bisectors of the five sides.

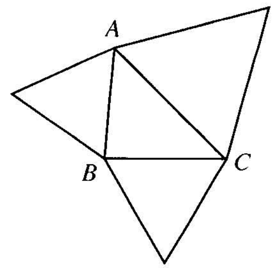
7. Draw a regular pentagon as in Exercise 6. Construct the angle bisectors.

8. Draw any large $\triangle ABC$ and construct equilateral triangles on each of the sides as shown.

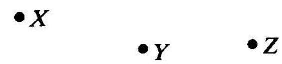
a. In each of the three equilateral triangles, construct any two medians and find their point of intersection.

b. Draw the three segments connecting these three points of intersection.

c. What appears to be true about the triangle you drew in part (b)?



B **9.** Three towns, located as shown, plan to build one recreation center to serve all three towns. They decide that the fair thing to do is to build the hall equidistant from the three towns. Comment about the wisdom of the plan.



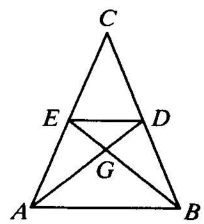
10. See Exercise 9. Locate three towns so that it isn't possible to find a spot equidistant from the three towns.

11. In the figure, $\overline{AD}$ and $\overline{BE}$ are congruent medians of $\triangle ABC$.

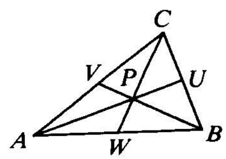
a. Explain why $GD = GE$.

b. $GA = \underline{\ ?\ }$

c. Name three angles congruent to $\angle GAB$.



$\overline{AU}$, $\overline{BV}$, and $\overline{CW}$ are the medians of $\triangle ABC$.

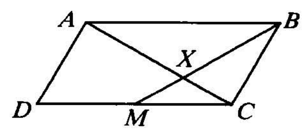


12. If $AP = x^2$ and $PU = 2x$, then $x = \underline{\ ?\ }$.

13. If $BP = y^2 + 1$ and $PV = y + 2$, then $y = \underline{\ ?\ }$ or $y = \underline{\ ?\ }$.

14. If $CW = 2z^2 - 5z - 12$ and $CP = z^2 - 15$, then $z = \underline{\ ?\ }$ and $PW = \underline{\ ?\ }$.

15. $ABCD$ is a parallelogram with M the midpoint of $\overline{CD}$. If $\overline{BM}$ intersects $\overline{AC}$ at X, prove that $CX = \frac{1}{3}AC$. (*Hint*: Draw $\overline{BD}$.)

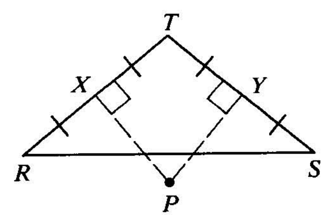


16. Prove that if two of the medians of a triangle are congruent, then the triangle is isosceles.

C **17.** In the plane figure, point P is equidistant from R, S, and T. Describe the location of the following points in the plane.

a. Points farther from both R and S than from T

b. Points closer to both R and S than to T



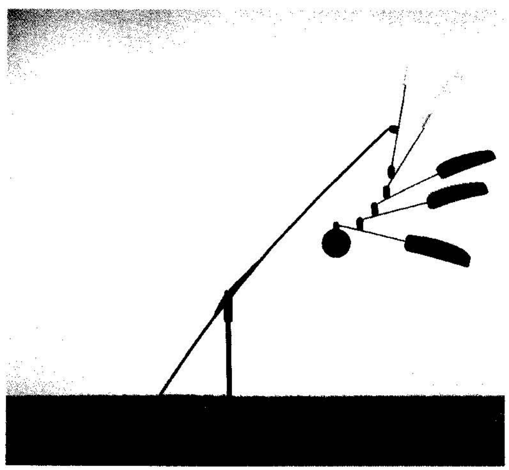
Self-Test 1

1. Draw any $\overline{CD}$. Construct the perpendicular bisector of $\overline{CD}$.
2. Construct a 60° angle, $\angle RST$, and its bisector, $\overrightarrow{SQ}$.
3. Draw a large acute $\triangle ABC$. Then construct altitude $\overline{AD}$ from vertex A.
4. Draw line t and choose any point P that is not on line t. Construct $\overleftrightarrow{PQ} \parallel t$.
5. Draw any $\overline{AB}$. Construct rectangle $JKLM$ so that $JK = 2AB$ and $KL = AB$.
6. Name four types of concurrent lines, rays, or segments that are associated with triangles.
7. The perpendicular bisectors of the sides of a right triangle intersect in a point located at _?_.
8. The medians of equilateral $\triangle ABC$ intersect at point X. If $\overline{AD}$ is a median and $AB = 12$, then $AX =$ _?_ and $XD =$ _?_.

Application — *Center of Gravity*

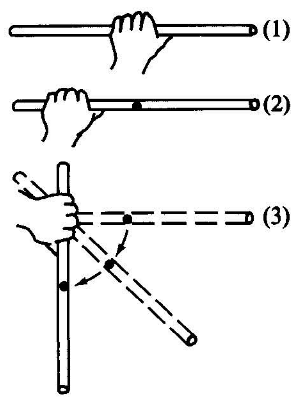
The *center of gravity* of an object is the point where the weight of the object is focused. If you lift or support an object, you can do this most easily under its center of gravity.

A mobile is either hung or supported at its center of gravity. In planning a mobile, a sculptor must take into account the centers of gravity of the component parts.

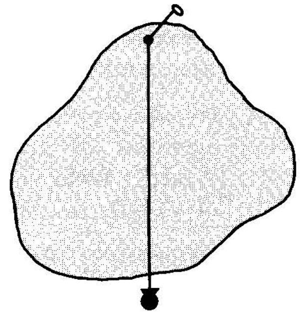


If an object is not supported under its center of gravity, it becomes unstable. Suppose you hold a heavy bar in one hand. If you support it near the center of gravity, it will be easy to hold (Figure 1). To support it at one end requires more effort (Figure 2), since the pole tends to turn until the center of gravity is directly below the point of support (Figure 3).

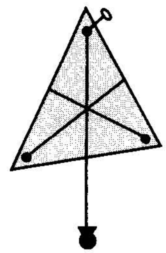
The center of gravity may be inside an object or outside of it. The center of gravity of an ice cube is in the middle of the ice, but the center of gravity of an automobile tire is not in a part of the tire itself.



Exercises



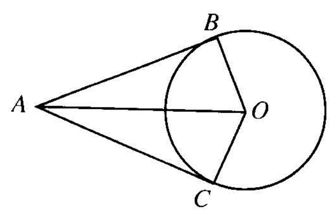
1. For this experiment, cut out a large, irregularly shaped piece of cardboard.
   a. Near the edge, poke a hole just large enough to allow the cardboard to rotate freely when pinned through the hole.
   b. Pin the cardboard through the hole to a suitable wall surface. The piece of cardboard will position itself so that its center of gravity is as low as possible. This means that it will lie on a vertical line through the point of suspension. To find this line, tie a weighted string to the pin. Then draw on the cardboard the line determined by the string.
   c. Repeat parts (a) and (b) but use a different hole. The center of gravity of the cardboard ought to lie on both of the lines you have drawn and should therefore be their point of intersection. The cardboard should balance if supported at this point.



2. Cut out a piece of cardboard in the shape of a large scalene triangle.
   a. Follow the steps of Exercise 1 using three holes, one near each of the three vertices.
   b. If you worked carefully, all three lines drawn intersect in one point, the center of gravity of the cardboard. This point is also referred to as the *center of mass* or the *centroid* of the cardboard. Study the lines you have drawn and explain why in geometry the point of intersection of the medians of a triangle is called the *centroid of the triangle*.
3. Do you think that the center of gravity of a parallelogram is the point where the diagonals intersect? Use the technique of Exercise 1 to test this idea.

Mixed Review Exercises

$\overline{AB}$ is tangent to $\odot O$ at B. Complete.



1. If the radius of $\odot O$ is 5 and $AO = 13$, then $AB = \underline{\ ?\ }$.
2. If $m\angle ACO = 90$ and $AB = 10$, then $\overline{AC}$ is $\underline{\ ?\ }$ to $\odot O$ at C and $AC = \underline{\ ?\ }$.
3. A triangle circumscribed about a circle intersects the circle in how many points?
4. Quad. $QRST$ is inscribed in a circle. If $m\angle Q = 39$, find $m\angle S$.

Explorations

These exploratory exercises can be done using a computer with a program that draws and measures geometric figures.

1. Inscribe a circle D inside a $\triangle ABC$. Draw $\overline{DA}$, $\overline{DB}$, and $\overline{DC}$. Compare the measures of $\angle ABD$ and $\angle ABC$, $\angle ACD$ and $\angle ACB$, $\angle BAD$ and $\angle BAC$. What do you notice? What type of lines intersect at the center of a circle inscribed in a triangle?
2. Circumscribe a circle D about a $\triangle ABC$. Draw perpendicular segments from D to $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, intersecting the sides at E, F, and G, respectively. Compare the lengths of $\overline{AE}$ and $\overline{AB}$, $\overline{BF}$ and $\overline{BC}$, and $\overline{CG}$ and $\overline{CA}$. What do you notice? What type of lines intersect at the center of a circle circumscribed about a triangle?

More Constructions

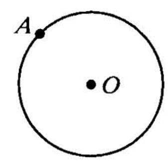
Objectives

1. Perform seven additional basic constructions.
2. Use the basic constructions in original construction exercises.

10-4 *Circles*

Construction 8

Given a point on a circle, construct the tangent to the circle at the given point.

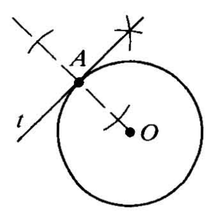


Given: Point A on $\odot O$

Construct: The tangent to $\odot O$ at A

Procedure:

1. Draw $\overrightarrow{OA}$.
2. Construct the line perpendicular to $\overrightarrow{OA}$ at A. Call it t.



Line t is tangent to $\odot O$ at A.

Justification: Because t is perpendicular to radius $\overline{OA}$ at A, t is tangent to $\odot O$.