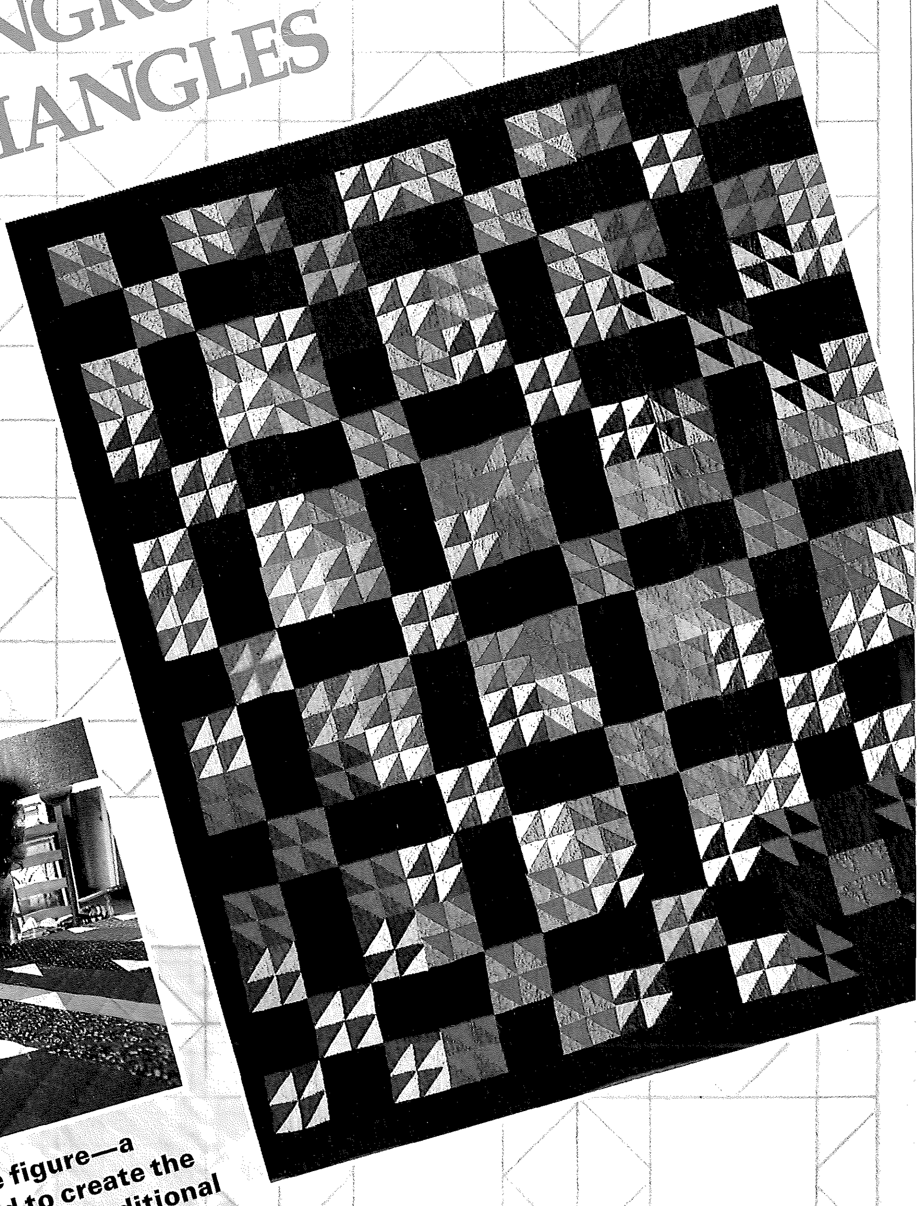
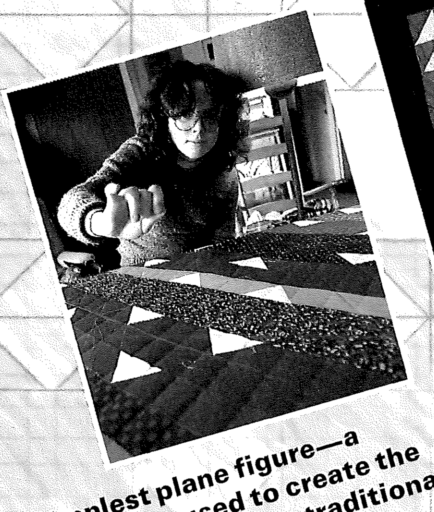
# 4 CONGRUENT TRIANGLES





The simplest plane figure—a triangle—was used to create the striking pattern of this traditional American quilt.

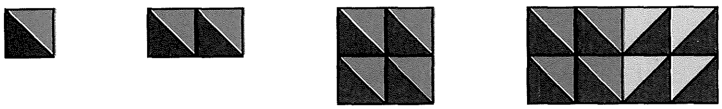
# Corresponding Parts in a Congruence

## Objectives

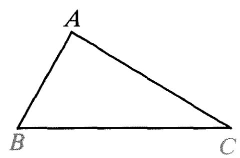
1. Identify the corresponding parts of congruent figures.
2. Prove two triangles congruent by using the SSS Postulate, the SAS Postulate, and the ASA Postulate.
3. Deduce information about segments and angles after proving that two triangles are congruent.

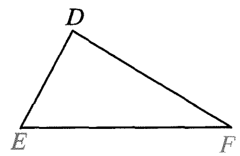
## 4-1 *Congruent Figures*

The quilt on the facing page is made up of many triangles that are all the same size and shape. These triangles are arranged to form squares and rectangles of various sizes. The diagrams below feature the pattern in the quilt. In each diagram, how many triangles with the same size and shape do you see? How many squares? How many rectangles?



Whenever two figures have the same size and shape, they are called **congruent.** You are already familiar with congruent segments (segments that have equal lengths) and congruent angles (angles that have equal measures). In this chapter you will learn about congruent triangles.




Triangles *ABC* and *DEF* are congruent. If you mentally slide $\triangle ABC$ to the right, you can fit it exactly over $\triangle DEF$ by matching up the vertices like this:

$$A \longleftrightarrow D \qquad B \longleftrightarrow E \qquad C \longleftrightarrow F$$

The sides and angles will then match up like this:

| *Corresponding angles* | *Corresponding sides* |
|---|---|
| $\angle A \longleftrightarrow \angle D$ | $\overline{AB} \longleftrightarrow \overline{DE}$ |
| $\angle B \longleftrightarrow \angle E$ | $\overline{BC} \longleftrightarrow \overline{EF}$ |
| $\angle C \longleftrightarrow \angle F$ | $\overline{AC} \longleftrightarrow \overline{DF}$ |

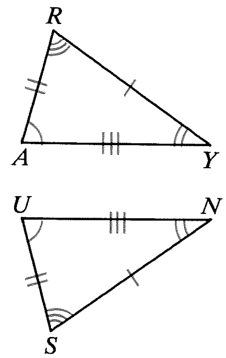
Do you see that the following statements are true?

(1) Since congruent triangles have the same shape, their corresponding angles are congruent.

(2) Since congruent triangles have the same size, their corresponding sides are congruent.

We have the following definition for *congruent triangles.*

Two triangles are **congruent** if and only if their vertices can be matched up so that the *corresponding parts* (angles and sides) of the triangles are congruent.




The congruent parts of the triangles shown are marked alike. Imagine sliding $\triangle SUN$ up until $\overline{UN}$ falls on $\overline{AY}$ and then flipping $\triangle SUN$ over so that point $S$ falls on point $R$. The vertices are matched like this:

$$S \longleftrightarrow R \qquad U \longleftrightarrow A \qquad N \longleftrightarrow Y$$

$\triangle SUN$ fits over $\triangle RAY$. The corresponding parts are congruent, and the triangles are congruent.

When referring to congruent triangles, we name their corresponding vertices in the same order. For the triangles shown,

$\triangle SUN$ is congruent to $\triangle RAY$.
$\triangle SUN \cong \triangle RAY$

The following statements about these triangles are also correct, since corresponding vertices of the triangles are named in the same order.

$$\triangle NUS \cong \triangle YAR \qquad \triangle SNU \cong \triangle RYA$$

Suppose you are given that $\triangle XYZ \cong \triangle ABC$. From the definition of congruent triangles you know, for example, that

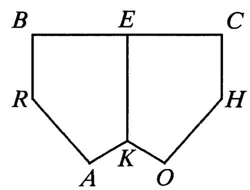
$$\overline{XY} \cong \overline{AB} \quad \text{and} \quad \angle X \cong \angle A.$$

When the definition of congruent triangles is used to justify either of these statements, the wording commonly used is

Corresponding parts of congruent triangles are congruent,

which is often written:

Corr. parts of $\cong$ $\triangle$s are $\cong$.



*BRAKE* ≅ *CHOKE*

Two *polygons* are **congruent** if and only if their vertices can be matched up so that their corresponding parts are congruent. Just as for triangles, there are many ways to list the congruence between the two pentagons at the right so that corresponding vertices are written in the same order.

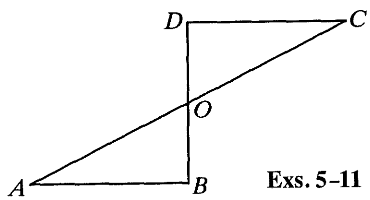
Notice that side $\overline{KE}$ of pentagon *BRAKE* corresponds to side $\overline{KE}$ of pentagon *CHOKE*. $\overline{KE}$ is called a *common side* of the two pentagons.

## Classroom Exercises

**Suppose you know that $\triangle FIN \cong \triangle WEB$.**

1. Name the three pairs of corresponding sides.
2. Name the three pairs of corresponding angles.
3. Is it correct to say $\triangle NIF \cong \triangle BEW$?
4. Is it correct to say $\triangle INF \cong \triangle EWB$?

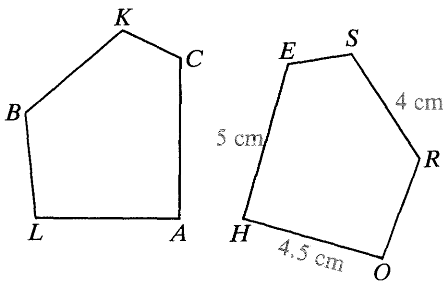
**The two triangles shown are congruent. Complete.**



**Exs. 5–11**

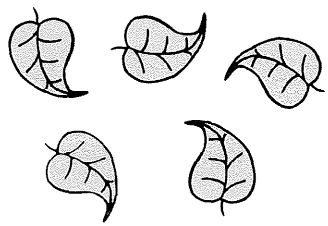
5. $\triangle ABO \cong$ ____?____ 6. $\angle A \cong$ ____?____
7. $\overline{AO} \cong$ ____?____ 8. $BO =$ ____?____
9. Can you deduce that $O$ is the midpoint of any segment? Explain.
10. Explain how you can deduce that $\overline{DC} \parallel \overline{AB}$.
11. Suppose you know that $\overline{DB} \perp \overline{DC}$. Explain how you can deduce that $\overline{DB} \perp \overline{BA}$.

**The pentagons shown are congruent. Complete.**

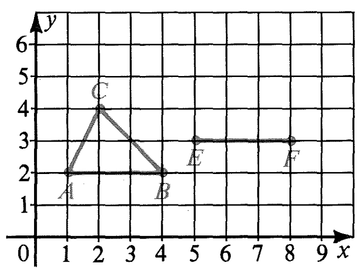



12. $B$ corresponds to ____?____.
13. $BLACK \cong$ ____?____
14. ____?____ $= m\angle E$
15. $KB =$ ____?____ cm
16. If $\overline{CA} \perp \overline{LA}$, name two right angles in the figures.

17. The five leaves shown are all congruent, but one differs from the others. Which one is different and how?



18. **a.** Name the coordinates of points $A$, $B$, and $C$.
    **b.** Name the coordinates of a point $D$ such that $\triangle ABC \cong \triangle ABD$.
19. Name the coordinates of a point $G$ such that $\triangle ABC \cong \triangle EFG$. Is there another location for $G$ such that $\triangle ABC \cong \triangle EFG$?
20. Name the coordinates of two possible points $H$ such that $\triangle ABC \cong \triangle FEH$.

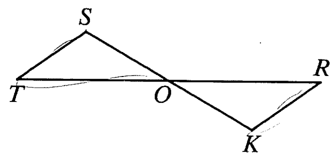


## Written Exercises

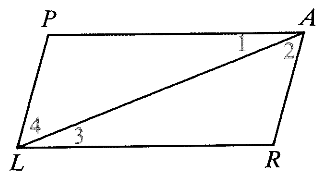
**Suppose $\triangle BIG \cong \triangle CAT$. Complete.**

A

1. $\angle G \cong \underline{\ ?\ }$
2. $\underline{\ ?\ } = m\angle A$
3. $BI = \underline{\ ?\ }$
4. $\underline{\ ?\ } \cong \overline{AT}$
5. $\triangle IGB \cong \underline{\ ?\ }$
6. $\underline{\ ?\ } \cong \triangle CTA$
7. If $\triangle DEF \cong \triangle RST$, $m\angle D = 100$, and $m\angle F = 40$, name four congruent angles.
8. Is the statement "Corresponding parts of congruent triangles are congruent" based on a definition, postulate, or theorem?
9. Suppose $\triangle LXR \cong \triangle FNE$. List six congruences that can be justified by the following reason: Corr. parts of $\cong$ ▲ are $\cong$.
10. The two triangles shown are congruent. Complete.
    a. $\triangle STO \cong \underline{\ ?\ }$
    b. $\angle S \cong \underline{\ ?\ }$ because $\underline{\ ?\ }$.
    c. $\overline{SO} \cong \underline{\ ?\ }$ because $\underline{\ ?\ }$.
    Then point $O$ is the midpoint of $\underline{\ ?\ }$.
    d. $\angle T \cong \underline{\ ?\ }$ because $\underline{\ ?\ }$.
    Then $\overline{ST} \parallel \overline{RK}$ because $\underline{\ ?\ }$.




11. The two triangles shown are congruent. Complete.
    a. $\triangle PAL \cong \underline{\ ?\ }$
    b. $\overline{PA} \cong \underline{\ ?\ }$
    c. $\angle 1 \cong \underline{\ ?\ }$ because $\underline{\ ?\ }$.
    Then $\overline{PA} \parallel \underline{\ ?\ }$ because $\underline{\ ?\ }$.
    d. $\angle 2 \cong \underline{\ ?\ }$ because $\underline{\ ?\ }$.
    Then $\underline{\ ?\ } \parallel \underline{\ ?\ }$ because $\underline{\ ?\ }$.




**Plot the given points on graph paper. Draw $\triangle FAT$. Locate point $C$ so that $\triangle FAT \cong \triangle CAT$.**

12. $F(1, 2)$ $A(4, 7)$ $T(4, 2)$
13. $F(7, 5)$ $A(-2, 2)$ $T(5, 2)$

**Plot the given points on graph paper. Draw $\triangle ABC$ and $\triangle DEF$. Copy and complete the statement $\triangle ABC \cong \underline{\ ?\ }$.**

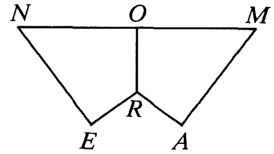
B

14. $A(-1, 2)$ $B(4, 2)$ $C(2, 4)$ $D(5, -1)$ $E(7, 1)$ $F(10, -1)$
15. $A(-7, -3)$ $B(-2, -3)$ $C(-2, 0)$ $D(0, 1)$ $E(5, 1)$ $F(0, -2)$
16. $A(-3, 1)$ $B(2, 1)$ $C(2, 3)$ $D(4, 3)$ $E(6, 3)$ $F(6, 8)$
17. $A(1, 1)$ $B(8, 1)$ $C(4, 3)$ $D(3, -7)$ $E(5, -3)$ $F(3, 0)$

**Plot the given points on graph paper. Draw $\triangle ABC$ and $\overline{DE}$. Find two locations of point $F$ such that $\triangle ABC \cong \triangle DEF$.**

18. $A(1, 2)$ $B(4, 2)$ $C(2, 4)$ $D(6, 4)$ $E(6, 7)$
19. $A(-1, 0)$ $B(-5, 4)$ $C(-6, 1)$ $D(1, 0)$ $E(5, 4)$

**$\overline{OR}$ is a common side of two congruent quadrilaterals.**




**Exs. 20, 21**

**20.** Complete: quad. $NERO \cong$ quad. $\underline{\ ?\ }$

**21.** In your own words explain why each of the following statements must be true.

**a.** $O$ is the midpoint of $\overline{NM}$.

**b.** $\angle NOR \cong \angle MOR$

**c.** $\overline{RO} \perp \overline{NM}$

**22.** Accurately draw each triangle described. Predict whether your triangle will be congruent to your classmates'.

**a.** In $\triangle RST$, $RS = 4$ cm, $m\angle S = 45$, and $ST = 6$ cm.

**b.** In $\triangle UVW$, $m\angle U = 30$, $UV = 5$ cm, and $m\angle V = 100$.

**c.** In $\triangle DEF$, $m\angle D = 30$, $m\angle E = 68$, and $m\angle F = 82$.

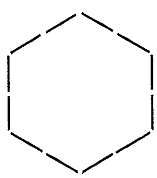
**d.** In $\triangle XYZ$, $XY = 3$ cm, $YZ = 5$ cm, and $XZ = 6$ cm. (Try for a reasonably accurate drawing. You may find it helpful to cut a thin strip of paper for each side, then form the triangle.)

**23.** Does congruence of triangles have the reflexive property? the symmetric property? the transitive property?

C **24.** Suppose you are given a scalene triangle and a point $P$ on some line $l$. How many triangles are there with one vertex at $P$, another vertex on $l$, and each triangle congruent to the given triangle?

## Challenge

Twelve toothpicks are arranged as shown to form a regular hexagon.



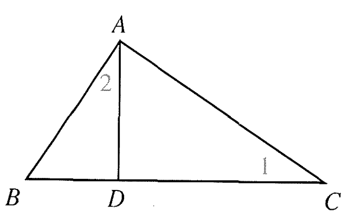
**a.** Copy the figure and show how six more toothpicks of the same size could be used to divide it into three congruent regions.

**b.** Keeping two of the toothpicks from part (a) in the same place and moving four, use the six toothpicks to divide the figure into two congruent regions.

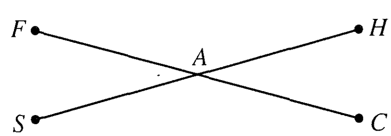
## Mixed Review Exercises

**Write proofs in two-column form.**

**1.** Given: $\overline{AD} \perp \overline{BC}$; $\overline{BA} \perp \overline{AC}$
Prove: $\angle 1 \cong \angle 2$




**2.** Given: $\overline{FC}$ and $\overline{SH}$ bisect each other at $A$; $FC = SH$
Prove: $SA = AC$

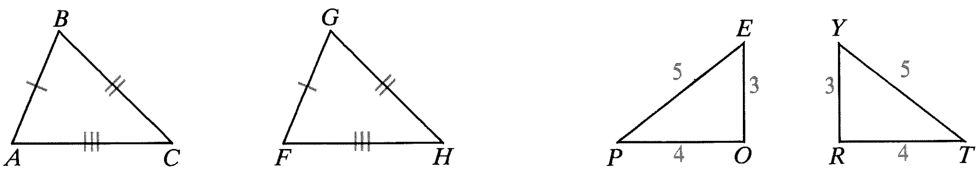


# 4-2 *Some Ways to Prove Triangles Congruent*

If two triangles are congruent, the six parts of one triangle are congruent to the six corresponding parts of the other triangle. If you are not sure whether two triangles are congruent, however, it is not necessary to compare all six parts. As you saw in Written Exercise 22 of the preceding section, sometimes three pairs of congruent corresponding parts will guarantee that two triangles are congruent. The following postulates give you three ways to show that two triangles are congruent by comparing only three pairs of corresponding parts.

## Postulate 12 *SSS Postulate*

**If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.**




By the SSS Postulate, $\triangle ABC \cong \triangle FGH$ and $\triangle POE \cong \triangle TRY$.

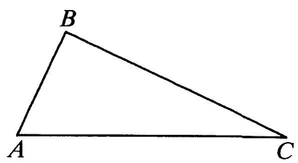
Sometimes it is helpful to describe the parts of a triangle in terms of their relative positions.

$\overline{AB}$ is *opposite* $\angle C$.

$\overline{AB}$ is *included* between $\angle A$ and $\angle B$.

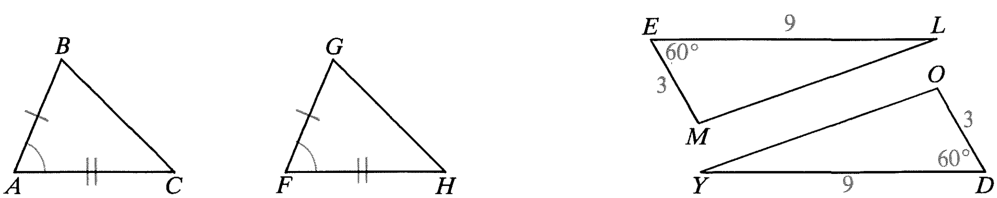
$\angle A$ is *opposite* $\overline{BC}$.

$\angle A$ is *included* between $\overline{AB}$ and $\overline{AC}$.




## Postulate 13 *SAS Postulate*

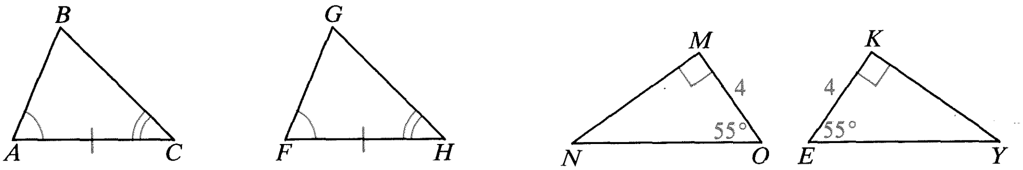
**If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.**




By the SAS Postulate, $\triangle ABC \cong \triangle FGH$ and $\triangle MEL \cong \triangle ODY$.

## Postulate 14 *ASA Postulate*

**If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.**

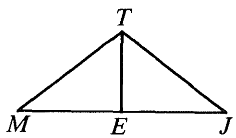



By the ASA Postulate, $\triangle ABC \cong \triangle FGH$ and $\triangle MON \cong \triangle KEY$.

**Example** Supply the missing statements and reasons in the following proof.

Given: $E$ is the midpoint of $\overline{MJ}$;
$\overline{TE} \perp \overline{MJ}$

Prove: $\triangle MET \cong \triangle JET$




**Proof:**

| Statements | Reasons |
|---|---|
| 1. $E$ is the midpoint of $\overline{MJ}$. | 1. Given |
| 2. _?_ $\cong$ _?_ | 2. Def. of midpoint |
| 3. $\overline{TE} \perp \overline{MJ}$ | 3. _?_ |
| 4. $\angle MET \cong \angle JET$ | 4. _?_ |
| 5. $\overline{TE} \cong$ _?_ | 5. _?_ |
| 6. $\triangle MET \cong \triangle JET$ | 6. _?_ |

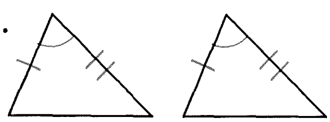
**Solution**

Statement 2 $\overline{ME} \cong \overline{JE}$
Reason 3 Given
Reason 4 If two lines are $\perp$, then they form $\cong$ adj. $\angle$s.
Statement 5 $\overline{TE}$
Reason 5 Reflexive Prop.
Reason 6 SAS Postulate

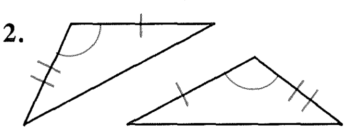
## Classroom Exercises

**Does the SAS Postulate justify that the two triangles are congruent?**

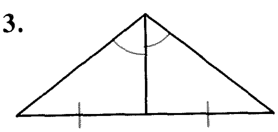
**1.**



**2.**

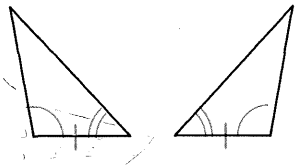


**3.**

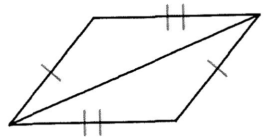


**Can the two triangles be proved congruent? If so, what postulate can be used?**

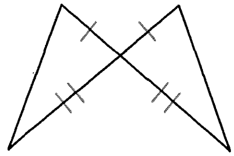
**4.**



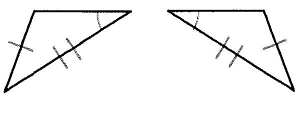
**5.**



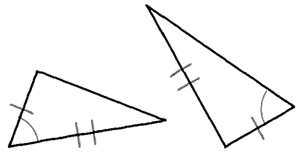
**6.**



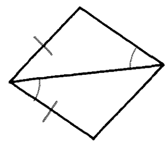
**7.**



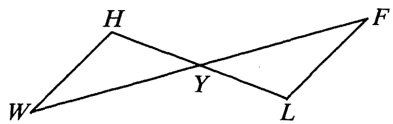
**8.**



**9.**

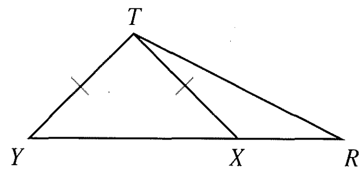


**10.** Explain how you would prove the following.
Given: $\overline{HY} \cong \overline{LY}$;
$\overline{WH} \parallel \overline{LF}$
Prove: $\triangle WHY \cong \triangle FLY$




**11. a.** List two pairs of congruent corresponding sides and one pair of congruent corresponding angles in $\triangle YTR$ and $\triangle XTR$.

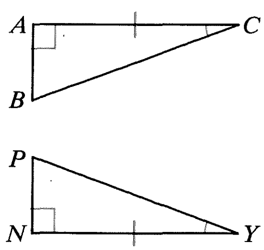
**b.** Notice that, in each triangle, you listed two sides and a *nonincluded* angle. Do you think that SSA is enough to guarantee that two triangles are congruent?



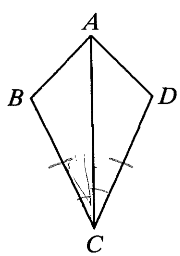

## Written Exercises

**Decide whether you can deduce by the SSS, SAS, or ASA Postulate that another triangle is congruent to $\triangle ABC$. If so, write the congruence and name the postulate used. If not, write *no congruence can be deduced.***

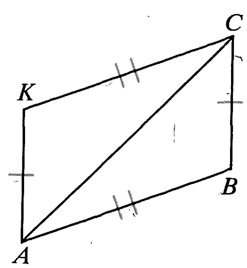
A **1.**



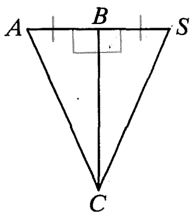

**2.**



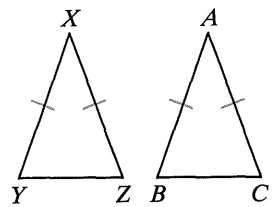

**3.**



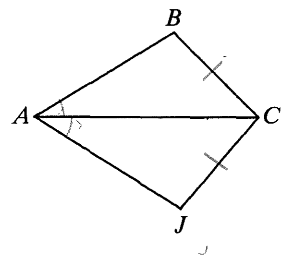
**4.**



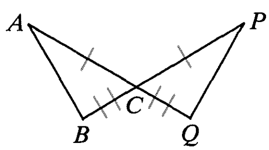

**5.**




**6.**




**7.**

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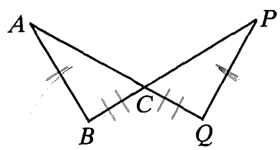
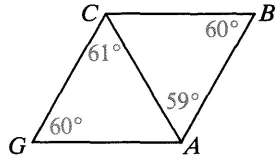



**9.**

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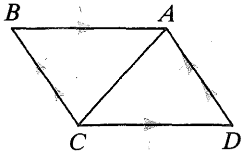
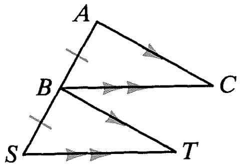
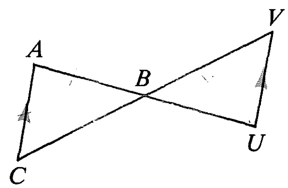


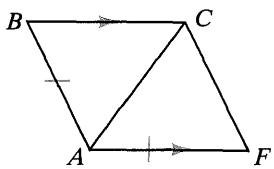

**11.**



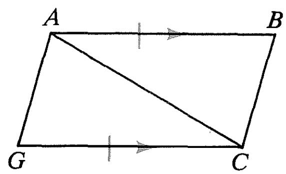

**12.**



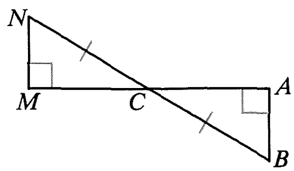

**13.**



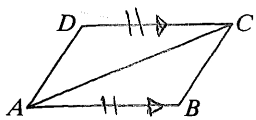

**14.**




**15.**



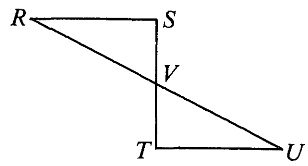

**16.** Supply the missing reasons.
Given: $\overline{AB} \parallel \overline{DC}$; $\overline{AB} \cong \overline{DC}$
Prove: $\triangle ABC \cong \triangle CDA$




**Proof:**

| Statements | Reasons |
|---|---|
| 1. $\overline{AB} \cong \overline{DC}$ | 1. ? |
| 2. $\overline{AC} \cong \overline{AC}$ | 2. ? |
| 3. $\overline{AB} \parallel \overline{DC}$ | 3. ? |
| 4. $\angle BAC \cong \angle DCA$ | 4. ? |
| 5. $\triangle ABC \cong \triangle CDA$ | 5. ? |

**17.** Supply the missing statements and reasons.
Given: $\overline{RS} \perp \overline{ST}$; $\overline{TU} \perp \overline{ST}$;
$V$ is the midpoint of $\overline{ST}$.
Prove: $\triangle RSV \cong \triangle UTV$



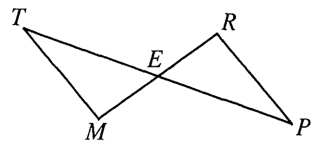

**Proof:**

| Statements | Reasons |
|---|---|
| 1. $\overline{RS} \perp \overline{ST}$; $\overline{TU} \perp \overline{ST}$ | 1. ? |
| 2. $m\angle S = 90$; $m\angle$ ? $= 90$ | 2. ? |
| 3. $\angle S \cong \angle T$ | 3. ? |
| 4. $V$ is the midpoint of $\overline{ST}$. | 4. ? |
| 5. $\overline{SV} \cong$ ? | 5. ? |
| 6. $\angle RVS \cong \angle$ ? | 6. ? |
| 7. $\triangle$ ? $\cong \triangle$ ? | 7. ? |

**Write proofs in two-column form.**

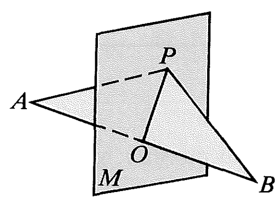
B **18.** Given: $\overline{TM} \cong \overline{PR}$; $\overline{TM} \parallel \overline{RP}$
Prove: $\triangle TEM \cong \triangle PER$

**19.** Given: $E$ is the midpoint of $\overline{TP}$;
$E$ is the midpoint of $\overline{MR}$.
Prove: $\triangle TEM \cong \triangle PER$




**20.** Given: Plane $M$ bisects $\overline{AB}$; $\overline{PA} \cong \overline{PB}$
Prove: $\triangle POA \cong \triangle POB$

**21.** Given: Plane $M$ bisects $\overline{AB}$; $\overline{PO} \perp \overline{AB}$
Prove: $\triangle POA \cong \triangle POB$




**Draw and label a diagram. List, in terms of the diagram, what is given and what is to be proved. Then write a two-column proof.**

**22.** In an isosceles triangle, if the angle between the congruent sides is bisected, then two congruent triangles are formed.

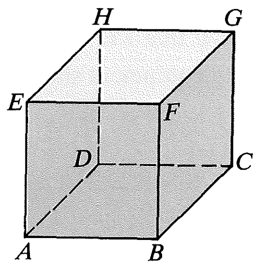
**23.** In an isosceles triangle, if a segment is drawn from the vertex of the angle between the congruent sides to the midpoint of the opposite side, then congruent triangles are formed.

**24.** If a line perpendicular to $\overline{AB}$ passes through the midpoint of $\overline{AB}$, and segments are drawn from any other point on that line to $A$ and $B$, then two congruent triangles are formed.

**25.** If pentagon $ABCDE$ is equilateral and has right angles at $B$ and $E$, then diagonals $\overline{AC}$ and $\overline{AD}$ form congruent triangles.

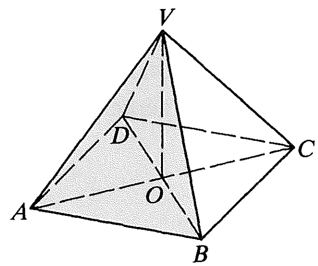
**Copy each three-dimensional figure and with colored pencils outline the triangles listed. What postulate proves that these triangles are congruent?**

C **26.**




Given: Cube whose faces are congruent squares
Show: $\triangle ABF$, $\triangle BCG$

**27.**

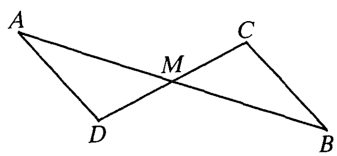



Given: Pyramid with square base; $VA = VB = VC = VD$
Show: $\triangle VAB$, $\triangle VBC$

# 4-3 *Using Congruent Triangles*

Our goal in the preceding section was to prove that two triangles are congruent. Our goal in this section is to deduce information about segments or angles once we have shown that they are corresponding parts of congruent triangles.

**Example 1**



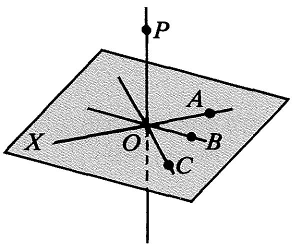

Given: $\overline{AB}$ and $\overline{CD}$ bisect each other at $M$.
Prove: $\overline{AD} \parallel \overline{BC}$

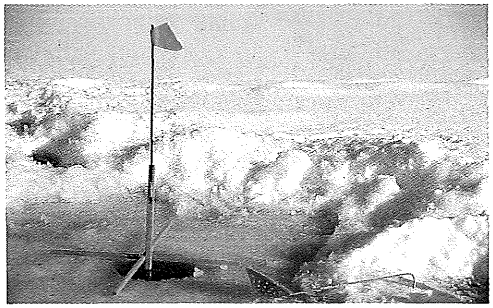
**Plan for Proof:** You can prove $\overline{AD} \parallel \overline{BC}$ if you can show that alternate interior angles $\angle A$ and $\angle B$ are congruent. You will know that $\angle A$ and $\angle B$ are congruent if they are corresponding parts of congruent triangles. The diagram suggests that you try to prove $\triangle AMD \cong \triangle BMC$.

**Proof:**

| Statements | Reasons |
| --- | --- |
| 1. $\overline{AB}$ and $\overline{CD}$ bisect each other at $M$. | 1. Given |
| 2. $M$ is the midpoint of $\overline{AB}$ and of $\overline{CD}$. | 2. Def. of a bisector of a segment |
| 3. $\overline{AM} \cong \overline{MB}$; $\overline{DM} \cong \overline{MC}$ | 3. Def. of midpoint |
| 4. $\angle AMD \cong \angle BMC$ | 4. Vertical $\angle$s are $\cong$. |
| 5. $\triangle AMD \cong \triangle BMC$ | 5. SAS Postulate |
| 6. $\angle A \cong \angle B$ | 6. Corr. parts of $\cong$ $\triangle$s are $\cong$. |
| 7. $\overline{AD} \parallel \overline{BC}$ | 7. If two lines are cut by a transversal and alt. int. $\angle$s are $\cong$, then the lines are $\parallel$. |

Some proofs require the idea of a line perpendicular to a plane. **A line and a plane are perpendicular** if and only if they intersect and the line is perpendicular to all lines in the plane that pass through the point of intersection. Suppose you are given $\overleftrightarrow{PO} \perp$ plane $X$. Then you know that $\overleftrightarrow{PO} \perp \overleftrightarrow{OA}$, $\overleftrightarrow{PO} \perp \overleftrightarrow{OB}$, $\overleftrightarrow{PO} \perp \overleftrightarrow{OC}$, $\overline{PO} \perp \overline{OC}$, and so on. The ice-fishing equipment shown below suggests a line perpendicular to a plane.

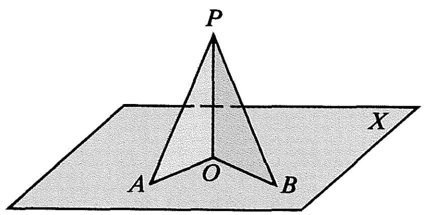


**Example 2**

Given: $\overline{PO} \perp$ plane $X$;
$\overline{AO} \cong \overline{BO}$

Prove: $\overline{PA} \cong \overline{PB}$




**Plan for Proof:** You can prove $\overline{PA} \cong \overline{PB}$ if you can show that these segments are corresponding parts of congruent triangles. The diagram suggests that you try to prove $\triangle POA \cong \triangle POB$.

**Proof:**

| Statements | Reasons |
|---|---|
| 1. $\overline{PO} \perp$ plane $X$ | 1. Given |
| 2. $\overline{PO} \perp \overline{OA}$; $\overline{PO} \perp \overline{OB}$ | 2. Def. of a line perpendicular to a plane |
| 3. $m\angle POA = 90$; $m\angle POB = 90$ | 3. Def. of $\perp$ lines |
| 4. $\angle POA \cong \angle POB$ | 4. Def. of $\cong$ $\angle$s |
| 5. $\overline{AO} \cong \overline{BO}$ | 5. Given |
| 6. $\overline{PO} \cong \overline{PO}$ | 6. Reflexive Prop. |
| 7. $\triangle POA \cong \triangle POB$ | 7. SAS Postulate |
| 8. $\overline{PA} \cong \overline{PB}$ | 8. Corr. parts of $\cong$ $\triangle$s are $\cong$. |

## A Way to Prove Two Segments or Two Angles Congruent

1. Identify two triangles in which the two segments or angles are corresponding parts.
2. Prove that the triangles are congruent.
3. State that the two parts are congruent, using the reason

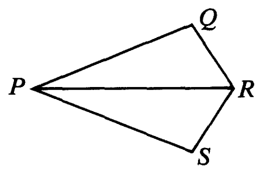
   Corr. parts of $\cong$ △s are $\cong$.

## Classroom Exercises

**Describe your plan for proving the following.**

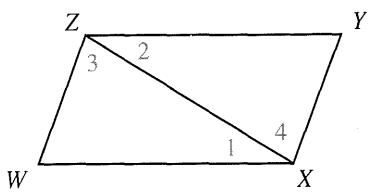
**1.** Given: $\overleftrightarrow{PR}$ bisects $\angle QPS$; $\overline{PQ} \cong \overline{PS}$
Prove: $\angle Q \cong \angle S$

**2.** Given: $\overleftrightarrow{PR}$ bisects $\angle QPS$ and $\angle QRS$
Prove: $\overline{RQ} \cong \overline{RS}$

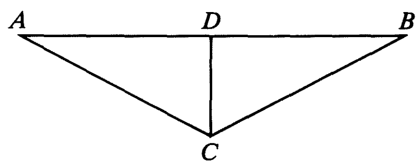



**3.** Given: $\overline{WX} \cong \overline{YZ}$; $\overline{ZW} \cong \overline{XY}$
Prove: $\overline{WX} \parallel \overline{ZY}$

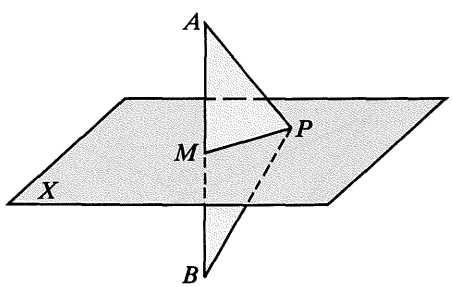
**4.** Given: $\overline{ZW} \parallel \overline{YX}$; $\overline{ZW} \cong \overline{XY}$
Prove: $\overline{ZY} \parallel \overline{WX}$




**5.** Given: $\overline{CD} \perp \overline{AB}$;
$D$ is the midpoint of $\overline{AB}$.
Prove: $\overline{CA} \cong \overline{CB}$



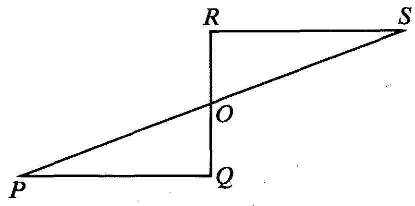

**6.** Given: $M$ is the midpoint of $\overline{AB}$;
plane $X \perp \overline{AB}$ at $M$.
What can you deduce about $\overline{AP}$ and $\overline{BP}$? Describe a plan for proving that your conclusion is correct.



## Written Exercises

**Copy and complete the proof.**

A **1.** Given: $\angle P \cong \angle S$;
$O$ is the midpoint of $\overline{PS}$.
Prove: $O$ is the midpoint of $\overline{RQ}$.



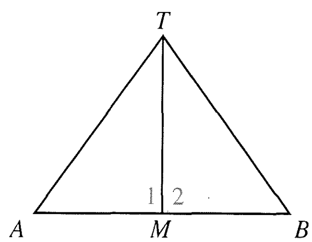

**Proof:**

| Statements | Reasons |
|---|---|
| 1. $\angle P \cong \angle S$ | 1. ? |
| 2. $O$ is the midpoint of $\overline{PS}$. | 2. ? |
| 3. $\overline{PO} \cong \overline{SO}$ | 3. ? |
| 4. $\angle POQ \cong \angle SOR$ | 4. ? |
| 5. $\triangle POQ \cong \triangle SOR$ | 5. ? |
| 6. $\overline{QO} \cong \overline{RO}$ | 6. ? |
| 7. $O$ is the midpoint of $\overline{RQ}$. | 7. ? |

**The statements in Exercise 2 might be used as statements in a proof but they are given out of order. Find an appropriate order for the statements. (There may be more than one correct order.)**

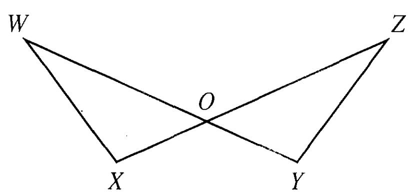
**2.** Given: $\overline{AM} \cong \overline{BM}$; $\overline{TM} \perp \overline{AB}$
Prove: $\overline{AT} \cong \overline{BT}$

(a) $\overline{AM} \cong \overline{BM}$
(b) $\triangle AMT \cong \triangle BMT$
(c) $\angle 1 \cong \angle 2$
(d) $\overline{AT} \cong \overline{BT}$
(e) $\overline{TM} \perp \overline{AB}$
(f) $\overline{TM} \cong \overline{TM}$

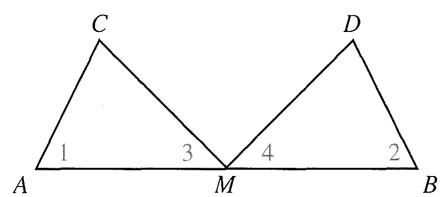



**Write proofs in two-column form.**

**3.** Given: $\overline{WO} \cong \overline{ZO}$; $\overline{XO} \cong \overline{YO}$
Prove: $\angle W \cong \angle Z$



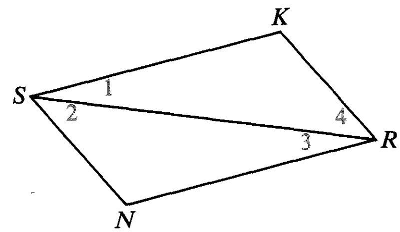

**4.** Given: $M$ is the midpoint of $\overline{AB}$;
$\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$
Prove: $\overline{AC} \cong \overline{BD}$



**5.** Prove the following statement: If both pairs of opposite sides of a quadrilateral are parallel, then they are also congruent.

Given: $\overline{SK} \parallel \overline{NR}$; $\overline{SN} \parallel \overline{KR}$

Prove: $\overline{SK} \cong \overline{NR}$; $\overline{SN} \cong \overline{KR}$




**6.** Prove the converse of the statement in Exercise 5: If both pairs of opposite sides of a quadrilateral are congruent, then they are also parallel.

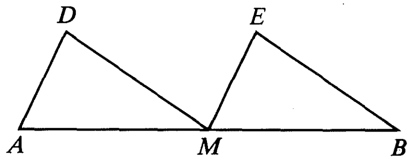
Given: $\overline{SK} \cong \overline{NR}$; $\overline{SN} \cong \overline{KR}$

Prove: $\overline{SK} \parallel \overline{NR}$; $\overline{SN} \parallel \overline{KR}$

**Write proofs in two-column form.**

**7.** Given: $\overline{AD} \parallel \overline{ME}$; $\overline{MD} \parallel \overline{BE}$;
$M$ is the midpoint of $\overline{AB}$.

Prove: $\overline{MD} \cong \overline{BE}$



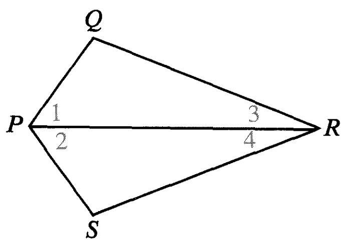

B **8.** Given: $M$ is the midpoint of $\overline{AB}$;
$\overline{AD} \cong \overline{ME}$; $\overline{AD} \parallel \overline{ME}$

Prove: $\overline{MD} \parallel \overline{BE}$

**In Exercises 9 and 10 you are given more information than you need. For each exercise state one piece of given information that you do not need for the proof. Then give a two-column proof that does not use that piece of information.**

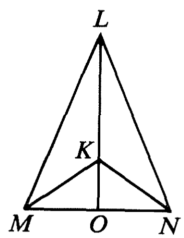
**9.** Given: $\overline{PQ} \cong \overline{PS}$; $\overline{QR} \cong \overline{SR}$;
$\angle 1 \cong \angle 2$

Prove: $\angle 3 \cong \angle 4$



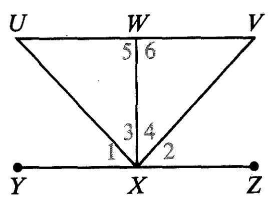

**10.** Given: $\overline{LM} \cong \overline{LN}$; $\overline{KM} \cong \overline{KN}$;
$\overrightarrow{KO}$ bisects $\angle MKN$.

Prove: $\overrightarrow{LO}$ bisects $\angle MLN$.

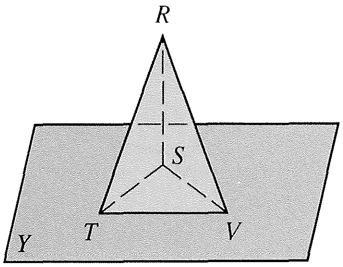



**11.** Given: $\overline{WX} \perp \overline{YZ}$; $\angle 1 \cong \angle 2$; $\overline{UX} \cong \overline{VX}$
Which one(s) of the following statements *must* be true?
(1) $\overline{XW} \perp \overline{UV}$ (2) $\overline{UV} \parallel \overline{YZ}$ (3) $\overline{VX} \perp \overline{UX}$

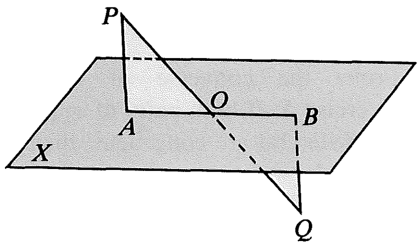
**12.** Given: $\overline{WX} \perp \overline{UV}$; $\overline{WX} \perp \overline{YZ}$; $\overline{WU} \cong \overline{WV}$
Prove whatever you can about angles 1, 2, 3, and 4.



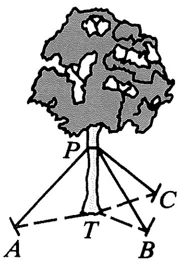
**13.** Given: $\overline{RS} \perp$ plane $Y$;
$\angle TRS \cong \angle VRS$
Prove: $\triangle RTV$ is isosceles.




**14.** Given: $\overline{PA} \perp$ plane $X$; $\overline{QB} \perp$ plane $X$;
$O$ is the midpoint of $\overline{AB}$.
Prove: $O$ is the midpoint of $\overline{PQ}$.




**15.** A young tree on level ground is supported at $P$ by three wires of equal length. The wires are staked to the ground at points $A$, $B$, and $C$, which are equally distant from the base of the tree, $T$. Explain in a paragraph how you can prove that the angles the wires make with the ground are all congruent.




**C** **16.** Napoleon, on a river bank, wanted to know the width of the stream. A young soldier faced directly across the stream and adjusted the visor of his cap until the tip of the visor was in line with his eye and the opposite bank. Next he did an about-face and noted the spot on the ground now in line with his eye and visor-tip. He paced off the distance to this spot, made his report, and earned a promotion. What postulate is this method based on? Draw a diagram to help you explain.

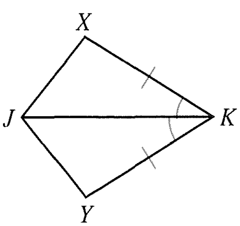
# Self-Test 1

**Given: $\triangle KOP \cong \triangle MAT$**

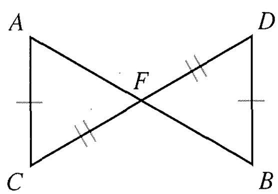
1. What can you conclude about $\angle P$? Why?
2. Name three pairs of corresponding sides.

**Decide whether the two triangles must be congruent. If so, write the congruence and name the postulate used. If not, write *no congruence can be deduced.***

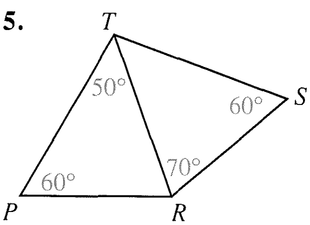
**3.**




**4.**




**5.**

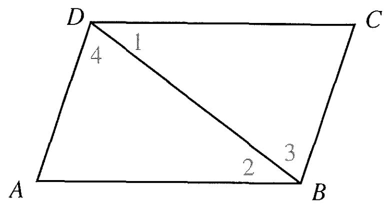


**Write proofs in two-column form.**

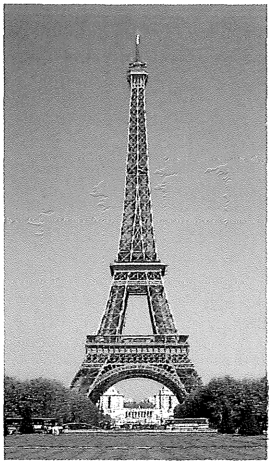
**6.** Given: $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$
Prove: $\triangle ADB \cong \triangle CBD$

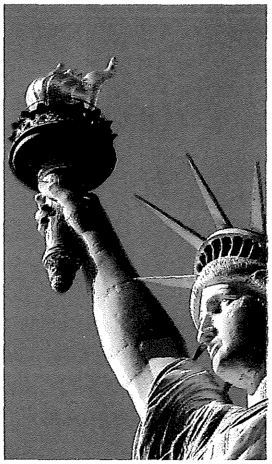
**7.** Given: $\overline{CD} \cong \overline{AB}$; $\overline{CB} \cong \overline{AD}$
Prove: $\angle 1 \cong \angle 2$

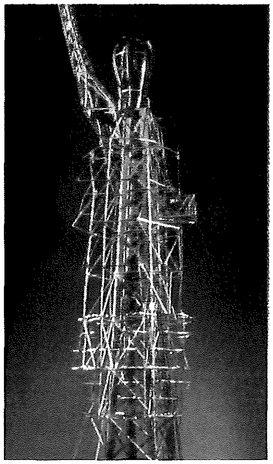
**8.** Given: $\overline{AD} \parallel \overline{BC}$; $\overline{AD} \cong \overline{CB}$
Prove: $\overline{DC} \parallel \overline{AB}$




## Application — *Bracing With Triangles*







The two famous landmarks pictured above have much in common. They were completed within a few years of each other, the Eiffel Tower in 1889 and the Statue of Liberty in 1886. The French engineer Gustave Eiffel designed both the tower's sweeping form and the complex structure that supports Liberty's copper skin. And both designs gain strength from the rigidity of the triangular shape.

The strength of triangular bracing is related to the SSS Postulate, which tells us that a triangle with given sides can have only one shape. A rectangle formed by four bars joined at their ends can flatten into a parallelogram, but the structural triangle cannot be deformed except by bending or stretching the bars.

The Eiffel Tower's frame is tied together by a web of triangles. A portion of the statue's armature is shown in the photograph at the right. The inner tower of wide members is strengthened by double diagonal bracing. A framework of lighter members, also joined in triangular patterns, surrounds this core.

Structural engineers use geometry in designing bridges, towers, and large-span roofs. See what you can find out about Eiffel's bridges and about the work of some of the other great modern builders.

## Explorations

**These exploratory exercises can be done using a computer with a program that draws and measures geometric figures.**

Draw several isosceles triangles. For each triangle, measure all sides and angles. What do you notice?
What is the relationship between the congruent sides and some of the angles?

Draw several triangles with two congruent angles. Measure all sides.
What do you notice?
What is the relationship between the congruent angles and some of the sides?

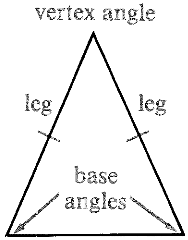
# Some Theorems Based on Congruent Triangles

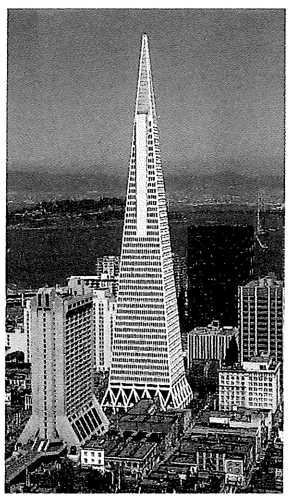
### Objectives

1. Apply the theorems and corollaries about isosceles triangles.
2. Use the AAS Theorem to prove two triangles congruent.
3. Use the HL Theorem to prove two right triangles congruent.
4. Prove that two overlapping triangles are congruent.

## 4-4 *The Isosceles Triangle Theorems*

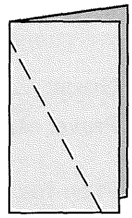
The photograph shows the Transamerica Pyramid in San Francisco. Each of its four faces is an isosceles triangle, with two congruent sides. These congruent sides are called **legs** and the third side is called the **base.** The angles at the base are called *base angles* and the angle opposite the base is called the *vertex angle* of the isosceles triangle.



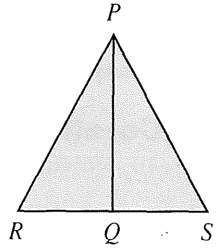
You can use the steps described below to form an isosceles triangle. Refer to the diagrams shown.

(1) Fold a sheet of paper in half.
(2) Cut off a double-thickness corner piece along the dashed line.
(3) Open the corner piece and lay it flat. You will have a triangle, which is labeled $\triangle PRS$ in the diagram. The fold line is labeled $\overline{PQ}$.
(4) Since $\overline{PR}$ and $\overline{PS}$ were formed by the same cut line, you can conclude that they are congruent segments and that $\triangle PRS$ is isosceles.



Since $\triangle PRQ$ fits exactly over $\triangle PSQ$ when you fold along $\overline{PQ}$, you can also conclude the following about isosceles $\triangle PRS$:

$\angle PRS \cong \angle PSR$
$\overline{PQ}$ bisects $\angle RPS$.
$\overline{PQ}$ bisects $\overline{RS}$.
$\overline{PQ} \perp \overline{RS}$ at $Q$.
$\triangle PQR \cong \triangle PQS$



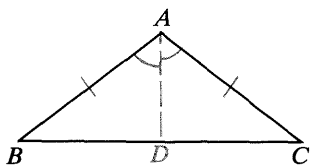

These observations suggest some of the following results.

**Theorem 4-1** *The Isosceles Triangle Theorem*

**If two sides of a triangle are congruent, then the angles opposite those sides are congruent.**

Given: $\overline{AB} \cong \overline{AC}$

Prove: $\angle B \cong \angle C$




**Plan for Proof:** You can show that $\angle B$ and $\angle C$ are corresponding parts of congruent triangles if you draw an auxiliary line that will give you such triangles. For example, draw the bisector of $\angle A$.

Theorem 4-1 is often stated as follows: Base angles of an isosceles triangle are congruent. The following corollaries of Theorem 4-1 will be discussed as classroom exercises.

## Corollary 1
**An equilateral triangle is also equiangular.**

## Corollary 2
**An equilateral triangle has three 60° angles.**

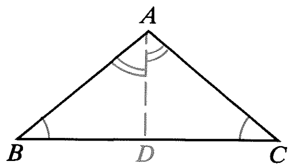
## Corollary 3
**The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.**

## Theorem 4-2

**If two angles of a triangle are congruent, then the sides opposite those angles are congruent.**

Given: $\angle B \cong \angle C$

Prove: $\overline{AB} \cong \overline{AC}$



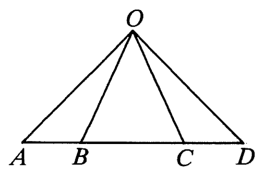

**Plan for Proof:** You can show that $\overline{AB}$ and $\overline{AC}$ are corresponding parts of congruent triangles. Draw the bisector of $\angle A$ as your auxiliary line, show that $\angle ADB \cong \angle ADC$, and use ASA.

## Corollary

**An equiangular triangle is also equilateral.**

Notice that Theorem 4-2 is the converse of Theorem 4-1, and the corollary of Theorem 4-2 is the converse of Corollary 1 of Theorem 4-1.

## Classroom Exercises

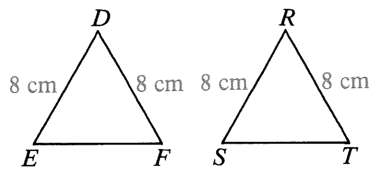



**1.** If $\triangle AOD$ is isosceles, with $\overline{OA} \cong \overline{OD}$, then $\angle$ __?__ $\cong$ $\angle$ __?__.

**2.** If $\triangle BOC$ is isosceles, with $\overline{OB} \cong \overline{OC}$, then $\angle$ __?__ $\cong$ $\angle$ __?__.

**3.** If $\triangle AOD$ is an isosceles right triangle with right $\angle AOD$, then the measure of $\angle A$ is __?__.

**4.** Given the triangles at the right, which of the following can you conclude are true?

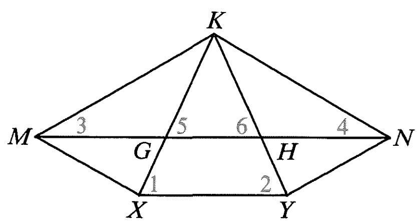



**a.** $\angle D \cong \angle R$  **b.** $\overline{DE} \cong \overline{DF}$

**c.** $\overline{DF} \cong \overline{RT}$  **d.** $\angle E \cong \angle F$

**e.** $\angle E \cong \angle S$  **f.** $\angle S \cong \angle T$

**Given the two congruent angles, name two segments that must be congruent.**




**5.** $\angle 1 \cong \angle 2$

**6.** $\angle 3 \cong \angle 4$

**7.** $\angle 5 \cong \angle 6$

**8.** Is the statement "$\overline{MK} \cong \overline{NK}$ if and only if $\angle 3 \cong \angle 4$" true or false?

**9.** Explain how Corollary 1 follows from Theorem 4-1.

**10.** Explain how Corollary 2 follows from Corollary 1.

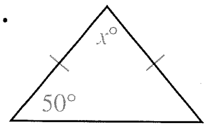
**11.** Explain how Corollary 3 follows from Theorem 4-1.

**12.** Explain how the Corollary follows from Theorem 4-2.

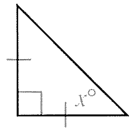
## Written Exercises

**Find the value of $x$.**

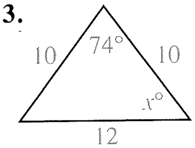
A **1.**



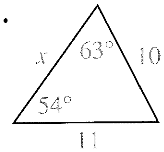

**2.**



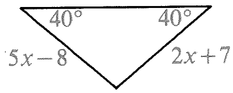

**3.**



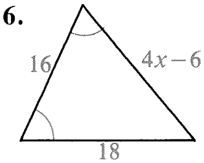

**4.**



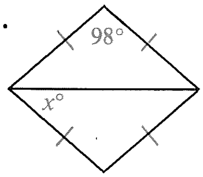

**5.**



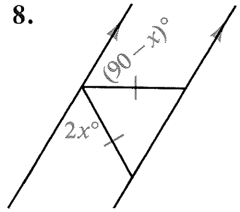

**6.**




**7.**

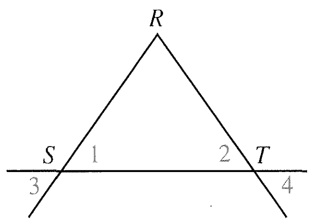



**8.**

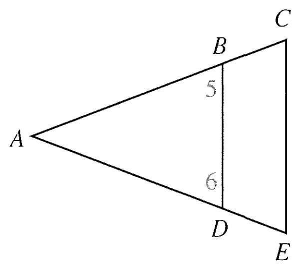



**For each exercise place the statements in an appropriate order for a proof. (There may be more than one correct order.)**

**9.** Given: $\overline{RS} \cong \overline{RT}$
Prove: $\angle 3 \cong \angle 4$
(a) $\angle 3 \cong \angle 4$
(b) $\angle 3 \cong \angle 1$; $\angle 2 \cong \angle 4$
(c) $\overline{RS} \cong \overline{RT}$
(d) $\angle 1 \cong \angle 2$




**10.** Given: $\overline{BD} \parallel \overline{CE}$; $\angle 5 \cong \angle 6$
Prove: $\overline{AC} \cong \overline{AE}$
(a) $\overline{BD} \parallel \overline{CE}$
(b) $\overline{AC} \cong \overline{AE}$
(c) $\angle 5 \cong \angle C$; $\angle 6 \cong \angle E$
(d) $\angle 5 \cong \angle 6$
(e) $\angle C \cong \angle E$



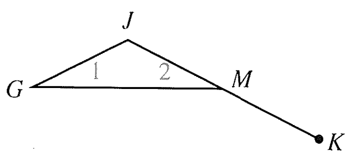

**Write proofs in two-column form.**

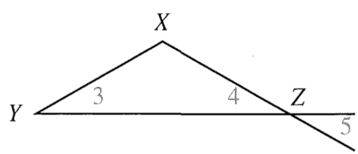
**11.** Theorem 4-1

**12.** Theorem 4-2

**13.** Given: $M$ is the midpoint of $\overline{JK}$;
$\angle 1 \cong \angle 2$
Prove: $\overline{JG} \cong \overline{MK}$

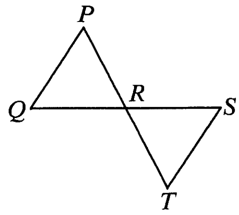
**14.** Given: $\overline{XY} \cong \overline{XZ}$
Prove: $\angle 3 \cong \angle 5$



**B** **15.** Given: $\overline{PQ} \cong \overline{PR}$; $\overline{TR} \cong \overline{TS}$
Which one(s) of the following *must* be true?
(1) $\overline{ST} \parallel \overline{QP}$ (2) $\overline{ST} \cong \overline{QP}$ (3) $\angle T \cong \angle P$

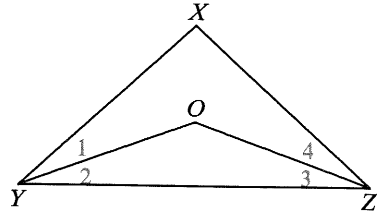
**16.** Given: $\angle S \cong \angle T$; $\overline{ST} \parallel \overline{QP}$
Which one(s) of the following *must* be true?
(1) $\angle P \cong \angle Q$ (2) $PR = QR$
(3) $R$ is the midpoint of $\overline{PT}$.




**Write proofs in two-column form.**

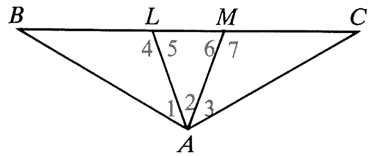
**17.** Given: $\overline{XY} \cong \overline{XZ}$; $\overline{OY} \cong \overline{OZ}$
Prove: $m\angle 1 = m\angle 4$

**18.** Given: $\overline{XY} \cong \overline{XZ}$;
$\overrightarrow{YO}$ bisects $\angle XYZ$;
$\overrightarrow{ZO}$ bisects $\angle XZY$.
Prove: $\overline{YO} \cong \overline{ZO}$




**19.** Given: $\overline{AB} \cong \overline{AC}$; $\overline{AL}$ and $\overline{AM}$ trisect $\angle BAC$.
(This means $\angle 1 \cong \angle 2 \cong \angle 3$.)
Prove: $\overline{AL} \cong \overline{AM}$

**20.** Given: $\angle 4 \cong \angle 7$; $\angle 1 \cong \angle 3$
Prove: $\triangle ABC$ is isosceles.

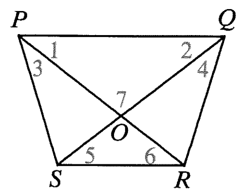



**21.** Given: $\overline{OP} \cong \overline{OQ}$; $\angle 3 \cong \angle 4$
Prove: $\angle 5 \cong \angle 6$

**22.** Given: $\overline{PO} \cong \overline{QO}$; $\overline{RO} \cong \overline{SO}$

**a.** If you are also given that $m\angle 1 = 40$, find the measures of $\angle 2$, $\angle 7$, $\angle 5$, and $\angle 6$. Then decide whether $\overline{PQ}$ must be parallel to $\overline{SR}$.

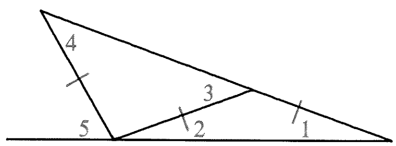
**b.** Repeat part (a), but use $m\angle 1 = k$.



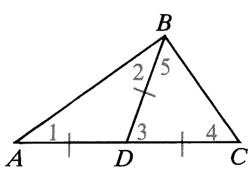

**23.** Complete.

**a.** If $m\angle 1 = 20$, then $m\angle 3 =$ __?__, $m\angle 4 =$ __?__, and $m\angle 5 =$ __?__.

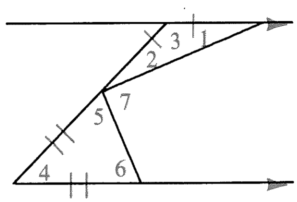
**b.** If $m\angle 1 = x$, then $m\angle 3 =$ __?__, $m\angle 4 =$ __?__, and $m\angle 5 =$ __?__.




**24.** **a.** If $m\angle 1 = 35$, find $m\angle ABC$.
**b.** If $m\angle 1 = k$, find $m\angle ABC$.




**25.** **a.** If $m\angle 1 = 23$, find $m\angle 7$.
**b.** If $m\angle 1 = k$, find $m\angle 7$.



**26.** Draw an isosceles $\triangle ABC$ whose vertex angle, $\angle A$, has measure 80.

**a.** Draw $\overrightarrow{AX}$, the bisector of an exterior angle at $A$. Is $\overrightarrow{AX} \parallel \overline{BC}$? Explain.

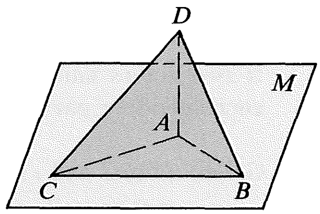
**b.** Would your answer change if the measure of $\angle A$ changed?

**Find the values of *x* and *y*.**

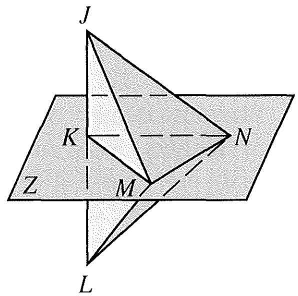
**27.** In equiangular $\triangle ABC$, $AB = 4x - y$, $BC = 2x + 3y$, and $AC = 7$.

**28.** In equilateral $\triangle DEF$, $m\angle D = x + y$ and $m\angle E = 2x - y$.

**29.** In $\triangle JKL$, $\overline{JK} \cong \overline{KL}$, $m\angle J = 2x - y$, $m\angle K = 2x + 2y$, and $m\angle L = x + 2y$.




**30.** Given: $\triangle ABC$ in plane $M$; $D$ not in plane $M$;
$\angle ACB \cong \angle ABC$; $\angle DCB \cong \angle DBC$
Name a pair of congruent triangles.
Prove that your answer is correct.



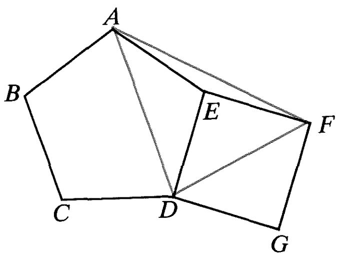

**31.** Given: $\overline{JL} \perp$ plane $Z$;
$\triangle KMN$ is isosceles, with $\overline{KM} \cong \overline{KN}$.

**a.** Prove that two other triangles are isosceles.

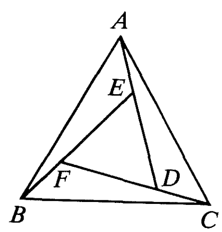
**b.** Must these two isosceles triangles be congruent? Explain.

**32.** Draw an isosceles triangle and then join the midpoints of its sides to form another triangle. What can you deduce about this second triangle? Explain.

**C** **33.** $ABCDE$ is a regular pentagon and $DEFG$ is a square. Find the measures of $\angle EAF$, $\angle AFD$, and $\angle DAF$.

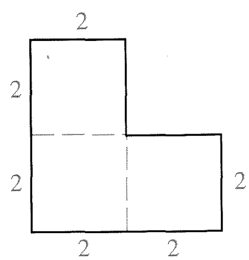



**34.** Given: $\triangle ABC$ is equilateral;
$\angle CAD \cong \angle ABE \cong \angle BCF$
Prove something interesting about $\triangle DEF$.




## Challenge

The figure shown at the right can be dissected into three congruent pieces, as shown by the dashed lines. Can you dissect the figure into **(a)** two congruent pieces? **(b)** four congruent pieces?



# 4-5 *Other Methods of Proving Triangles Congruent*

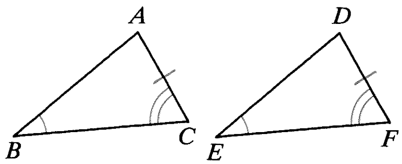
The SSS, SAS, and ASA Postulates give us three methods of proving triangles congruent. In this section we will develop two other methods.

## Theorem 4-3 *AAS Theorem*

**If two angles and a non-included side of one triangle are congruent to the corresponding parts of another triangle, then the triangles are congruent.**

Given: $\triangle ABC$ and $\triangle DEF$; $\angle B \cong \angle E$;
$\angle C \cong \angle F$; $\overline{AC} \cong \overline{DF}$

Prove: $\triangle ABC \cong \triangle DEF$

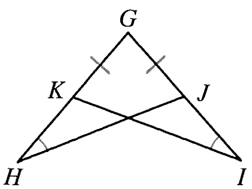



**Plan for Proof:** You can prove the triangles congruent if you can apply one of the SSS, SAS, or ASA Postulates. You can use the ASA Postulate if you first show that $\angle A \cong \angle D$. To do that, use the fact that the other two angles of $\triangle ABC$ are congruent to the other two angles of $\triangle DEF$.

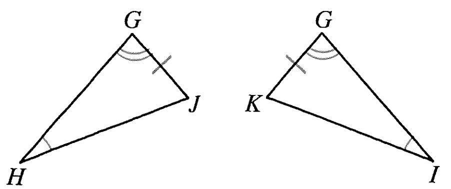
Do you see overlapping triangles in the photograph? Sometimes you want to prove that certain overlapping triangles are congruent. For example, suppose you have the following problem:

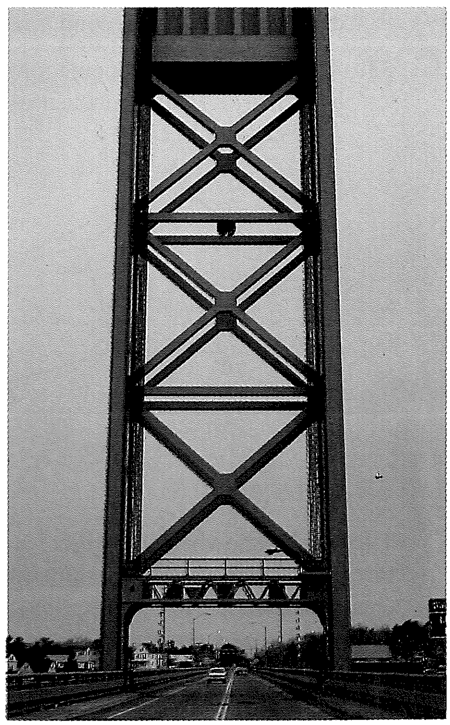
Given: $\overline{GJ} \cong \overline{GK}$;
$\angle H \cong \angle I$

Prove: $\triangle GHJ \cong \triangle GIK$

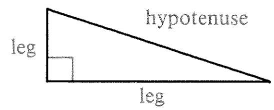



You may find it helps you visualize the congruence if you redraw the two triangles, as shown below. Now you can see that since $\angle G$ is common to both triangles, the triangles must be congruent by the AAS Theorem.



Our final method of proving triangles congruent applies only to right triangles. In a right triangle the side opposite the right angle is called the **hypotenuse** (hyp.). The other two sides are called **legs.**



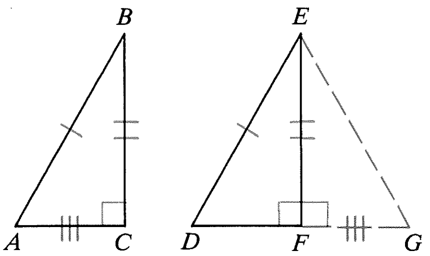

A proof in two-column form for the next theorem would be too long and involved. The proof shown below is written instead in *paragraph form*, which emphasizes the *key steps* in the proof. You will learn to write paragraph proofs in the next section.

## Theorem 4-4 *HL Theorem*

**If the hypotenuse and a leg of one right triangle are congruent to the corresponding parts of another right triangle, then the triangles are congruent.**

Given: $\triangle ABC$ and $\triangle DEF$;
$\angle C$ and $\angle F$ are right $\angle$s;
$\overline{AB} \cong \overline{DE}$ (hypotenuses);
$\overline{BC} \cong \overline{EF}$ (legs)

Prove: $\triangle ABC \cong \triangle DEF$




**Proof:**

By the Ruler Postulate there is a point $G$ on the ray opposite to $\overrightarrow{FD}$ such that $\overline{FG} \cong \overline{CA}$. Draw $\overline{GE}$. Because $\angle DFE$ is a right angle, $\angle GFE$ is also a right angle. $\triangle ABC \cong \triangle GEF$ by the SAS Postulate. Then $\overline{AB} \cong \overline{GE}$. Since $\overline{DE} \cong \overline{AB}$, we have $\overline{DE} \cong \overline{GE}$. In isosceles $\triangle DEG$, $\angle G \cong \angle D$. Since $\triangle ABC \cong \triangle GEF$, $\angle A \cong \angle G$. Then $\angle A \cong \angle D$. Finally, $\triangle ABC \cong \triangle DEF$ by the AAS Theorem.

Recall from Exercise 22 on page 121 and Exercise 11 on page 124 that AAA and SSA correspondences do not guarantee congruent triangles. We can now summarize the methods available for proving triangles congruent.

## Summary of Ways to Prove Two Triangles Congruent

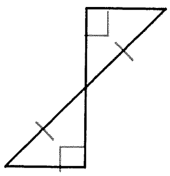
| | | | | |
|---|---|---|---|---|
| All triangles: | SSS | SAS | ASA | AAS |
| Right triangles: | HL | | | |

Which of these methods are postulates and which are theorems?

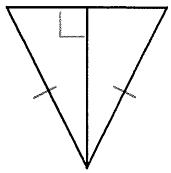
## Classroom Exercises

**State which congruence method(s) can be used to prove the triangles congruent. If no method applies, say *none*.**

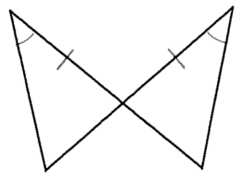
**1.**



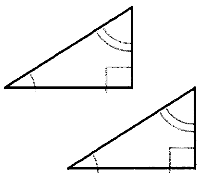
**2.**



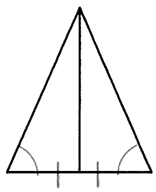
**3.**



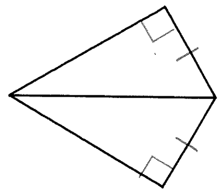
**4.**



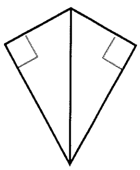
**5.**



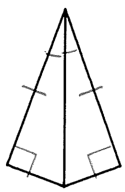
**6.**



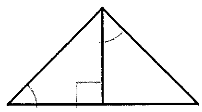
**7.**



**8.**

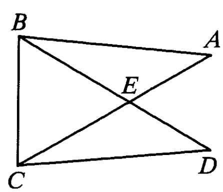


**9.**

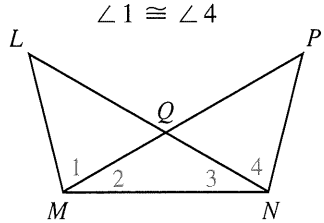


**For each diagram, name a pair of overlapping triangles. Tell whether the triangles are congruent by the SSS, SAS, ASA, AAS, or HL method.**

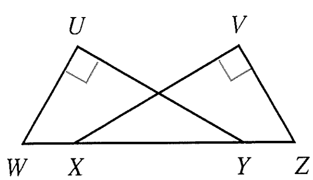
**10.** Given: $\overline{AB} \cong \overline{DC}$; $\overline{AC} \cong \overline{DB}$



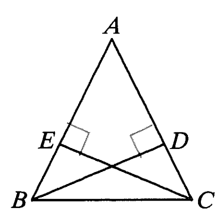

**11.** Given: $\angle 2 \cong \angle 3$; $\angle 1 \cong \angle 4$




**12.** Given: $\overline{WU} \cong \overline{ZV}$; $WX = YZ$; $\angle U$ and $\angle V$ are rt. $\angle$s.




**13.** Given: $\angle ABC \cong \angle ACB$; $\overline{AE} \perp \overline{EC}$; $\overline{AD} \perp \overline{DB}$



**14.** To prove that right triangles are congruent, some geometry books also use the methods stated below. For each method, draw two right triangles that appear to be congruent. Mark the given information on your triangles. Use your marks to determine which of our methods (SSS, SAS, ASA, AAS, or HL) could be used instead of each method listed.

**a. Leg-Leg Method** (LL) If two legs of one right triangle are congruent to the two legs of another right triangle, then the triangles are congruent.

**b. Hypotenuse-Acute Angle Method** (HA) If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle of another right triangle, then the triangles are congruent.

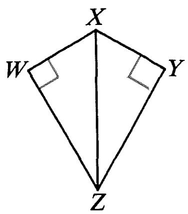
**c. Leg-Acute Angle Method** (LA) If a leg and an acute angle of one right triangle are congruent to the corresponding parts in another right triangle, then the triangles are congruent.

## Written Exercises

A **1.** Supply the missing statements and reasons.

Given: $\angle W$ and $\angle Y$ are rt. $\angle$s;
$\overline{WX} \cong \overline{YX}$

Prove: $\overline{WZ} \cong \overline{YZ}$



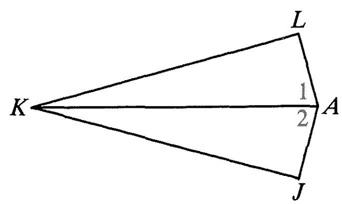

**Proof:**

| Statements | Reasons |
|---|---|
| 1. $\angle W$ and $\angle Y$ are rt. $\angle$s. | 1. ? |
| 2. $\triangle XWZ$ and $\triangle XYZ$ are rt. $\triangle$s. | 2. ? |
| 3. $\overline{WX} \cong \overline{YX}$ | 3. ? |
| 4. ? | 4. Reflexive Prop. |
| 5. $\triangle XWZ \cong$ ? | 5. ? |
| 6. ? | 6. ? |

**2.** Place the statements in an appropriate order for a proof.

Given: $\overline{KL} \perp \overline{LA}$; $\overline{KJ} \perp \overline{JA}$;
$\overrightarrow{AK}$ bisects $\angle LAJ$.

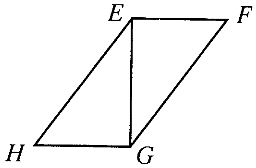
Prove: $\overline{LK} \cong \overline{JK}$



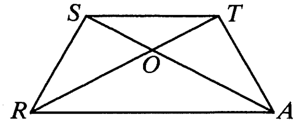

(a) $\overline{KL} \perp \overline{LA}$; $\overline{KJ} \perp \overline{JA}$
(b) $\angle 1 \cong \angle 2$
(c) $\overrightarrow{AK}$ bisects $\angle LAJ$.
(d) $m\angle L = 90$; $m\angle J = 90$
(e) $\overline{LK} \cong \overline{JK}$
(f) $\angle L \cong \angle J$
(g) $\triangle LKA \cong \triangle JKA$
(h) $\overline{KA} \cong \overline{KA}$

**Write proofs in two-column form.**

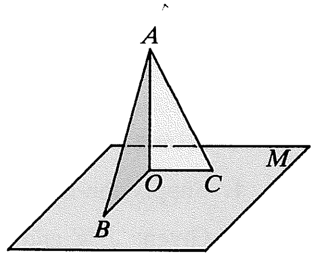
**3.** Given: $\overline{EF} \perp \overline{EG}$; $\overline{HG} \perp \overline{EG}$; $\overline{EH} \cong \overline{GF}$
Prove: $\angle H \cong \angle F$




**4.** Given: $\overline{RT} \cong \overline{AS}$; $\overline{RS} \cong \overline{AT}$
Prove: $\angle TSA \cong \angle STR$




**Use the information given in each exercise to name the method (SSS, SAS, ASA, AAS, or HL) you could use to prove $\triangle AOB \cong \triangle AOC$. You need not write the proofs.**



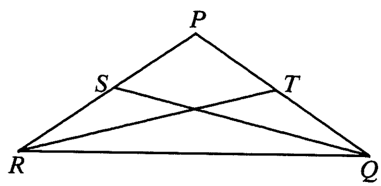

**5.** Given: $\overline{AO} \perp$ plane $M$; $\overline{BO} \cong \overline{CO}$

**6.** Given: $\overline{AO} \perp$ plane $M$; $\angle B \cong \angle C$

**7.** Given: $\overline{AO} \perp$ plane $M$; $\overline{AB} \cong \overline{AC}$

B **8.** Given: $\overline{AB} \cong \overline{AC}$; $\overline{OB} \cong \overline{OC}$
   **a.** Is it possible to prove that $\angle AOB \cong \angle AOC$?
   **b.** Is it possible to prove that $\angle AOB$ and $\angle AOC$ are right angles?

**9.** In many proofs you may find that different methods can be used. You may not know in advance which method will be better. There are *two* possible pairs of overlapping triangles that could be used in this proof. To compare the two methods, write a two-column proof for each plan.




Given: $\overline{PR} \cong \overline{PQ}$; $\overline{SR} \cong \overline{TQ}$
Prove: $\overline{QS} \cong \overline{RT}$
   **a.** **Plan for Proof:** Show that $\triangle RQS \cong \triangle QRT$ by SAS.
   **b.** **Plan for Proof:** Show that $\triangle PQS \cong \triangle PRT$ by SAS.

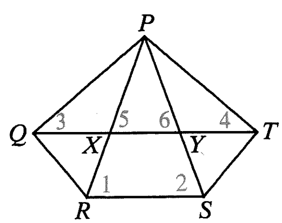
**10. a.** Draw an isosceles $\triangle RST$ with $\overline{RT} \cong \overline{ST}$. Let $M$ be the midpoint of $\overline{ST}$ and $N$ be the midpoint of $\overline{RT}$. Draw $\overline{RM}$ and $\overline{SN}$ and label their common point $O$. Now draw $\overline{NM}$.
   **b.** Name four *pairs* of congruent triangles.

**Tell which pairs of congruent parts and what method (SSS, SAS, ASA, AAS, or HL) you would use to prove the triangles are congruent.**

**11.** Given: $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$; $\overline{QR} \cong \overline{TS}$
$\triangle QPR \cong \triangle TPS$ by what method?

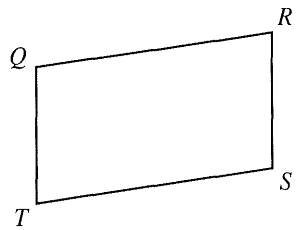
**12.** Given: $\angle 3 \cong \angle 4$; $\angle 5 \cong \angle 6$
$\triangle PQX \cong \triangle PTY$ by what method?

**13.** Given: $\angle 3 \cong \angle 4$; $\angle 5 \cong \angle 6$
$\triangle QPY \cong \triangle TPX$ by what method?

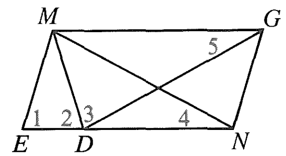


**Write proofs in two-column form.**

**14.** Given: $\angle R \cong \angle T$; $\overline{RS} \parallel \overline{QT}$
Prove: $\overline{RS} \cong \overline{TQ}$
(*Hint*: What auxiliary line can you draw to form congruent triangles?)




**15.** Given: $\angle 1 \cong \angle 2 \cong \angle 3$; $\overline{EN} \cong \overline{DG}$
Prove: $\angle 4 \cong \angle 5$




**For Exercises 16–19 draw and label a diagram. List, in terms of the diagram, what is given and what is to be proved. Then write a two-column proof.**

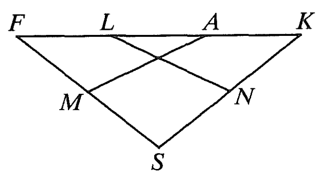
**16.** In two congruent triangles, if segments are drawn from two corresponding vertices perpendicular to the opposite sides, then those segments are congruent.

**17.** If segments are drawn from the endpoints of the base of an isosceles triangle perpendicular to the opposite legs, then those segments are congruent.

**18.** If $\angle A$ and $\angle B$ are the base angles of isosceles $\triangle ABC$, and the bisector of $\angle A$ meets $\overline{BC}$ at $X$ and the bisector of $\angle B$ meets $\overline{AC}$ at $Y$, then $\overline{AX} \cong \overline{BY}$.

**19.** If segments are drawn from the midpoints of the legs of an isosceles triangle perpendicular to the base, then those segments are congruent.

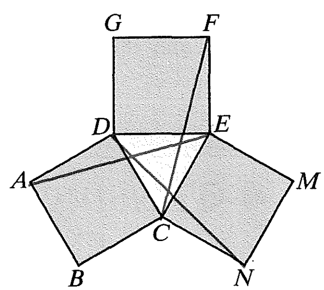
**20.** Write a detailed plan for proof.
Given: $\overline{FL} \cong \overline{AK}$;
$\overline{SF} \cong \overline{SK}$;
$M$ is the midpoint of $\overline{SF}$;
$N$ is the midpoint of $\overline{SK}$.
Prove: $\overline{AM} \cong \overline{LN}$




**Write proofs in two-column form. Use the facts that the sides of a square are all congruent and that the angles of a square are all right angles.**

C **21.** The diagram shows three squares and an equilateral triangle.
Prove: $\overline{AE} \cong \overline{FC} \cong \overline{ND}$

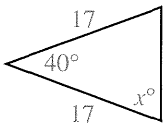
**22.** Use the results of Exercise 21 to prove that $\triangle FAN$ is equilateral.



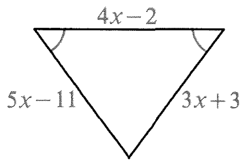
# Self-Test 2

**Find the value of $x$.**

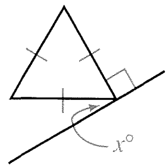
**1.**




**2.**

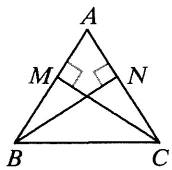



**3.**




**4.** Given: $\overline{AB} \cong \overline{AC}$; $\overline{BN} \perp \overline{AC}$; $\overline{CM} \perp \overline{AB}$
Explain how you could prove that $\triangle ABN \cong \triangle ACM$.

**5.** Given: $\overline{MB} \cong \overline{NC}$; $\overline{BN} \perp \overline{AC}$; $\overline{CM} \perp \overline{AB}$
Prove: $\overline{CM} \cong \overline{BN}$




# More about Proof in Geometry

## Objectives

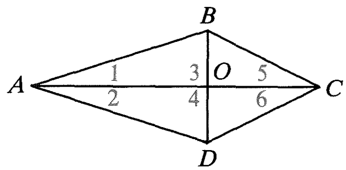
1. Prove two triangles congruent by first proving two other triangles congruent.
2. Apply the definitions of the median and the altitude of a triangle and the perpendicular bisector of a segment.
3. State and apply the theorem about a point on the perpendicular bisector of a segment, and the converse.
4. State and apply the theorem about a point on the bisector of an angle, and the converse.

## 4-6 *Using More than One Pair of Congruent Triangles*

Sometimes two triangles that you want to prove congruent have common parts with two *other* triangles that you can easily prove congruent. You may then be able to use corresponding parts of these other triangles to prove the original triangles congruent.

**Example**

Given: $\angle 1 \cong \angle 2$; $\angle 5 \cong \angle 6$
Prove: $\overline{AC} \perp \overline{BD}$




**Plan for Proof:** It may be helpful here to *reason backward* from what you want to prove. You can show $\overline{AC} \perp \overline{BD}$ if you can show that $\angle 3 \cong \angle 4$. You can prove $\angle 3 \cong \angle 4$ if you can prove that the angles are corresponding parts of congruent triangles. To prove $\triangle ABO \cong \triangle ADO$, you need $\overline{AB} \cong \overline{AD}$. You can prove this congruence by proving that $\triangle ABC \cong \triangle ADC$. You should prove this congruence first.

**Proof:**

| Statements | Reasons |
|---|---|
| 1. $\angle 1 \cong \angle 2$; $\angle 5 \cong \angle 6$ | 1. Given |
| 2. $\overline{AC} \cong \overline{AC}$ | 2. Reflexive Property |
| 3. $\triangle ABC \cong \triangle ADC$ | 3. ASA Postulate |
| 4. $\overline{AB} \cong \overline{AD}$ | 4. Corr. parts of $\cong$ $\triangle$s are $\cong$. |
| 5. $\overline{AO} \cong \overline{AO}$ | 5. Reflexive Property |
| 6. $\triangle ABO \cong \triangle ADO$ | 6. SAS Postulate (Steps 1, 4, and 5) |
| 7. $\angle 3 \cong \angle 4$ | 7. Corr. parts of $\cong$ $\triangle$s are $\cong$. |
| 8. $\overline{AC} \perp \overline{BD}$ | 8. If two lines form $\cong$ adj. $\angle$s, then the lines are $\perp$. |

If you were to outline this two-column proof, you might pick out the following *key steps*.

**Key steps of proof:**

1. $\triangle ABC \cong \triangle ADC$ (ASA Postulate)
2. $\overline{AB} \cong \overline{AD}$ (Corr. parts of $\cong$ $\triangle$s are $\cong$.)
3. $\triangle ABO \cong \triangle ADO$ (SAS Postulate)
4. $\angle 3 \cong \angle 4$ (Corr. parts of $\cong$ $\triangle$s are $\cong$.)
5. $\overline{AC} \perp \overline{BD}$ (If two lines form $\cong$ adj. $\angle$s, then the lines are $\perp$.)

In mathematics a proof is often given in paragraph form rather than in two-column form. A *paragraph proof* usually focuses on the key ideas and omits details that the writer thinks will be clear to the reader. The following paragraph proof might be given for the example above.

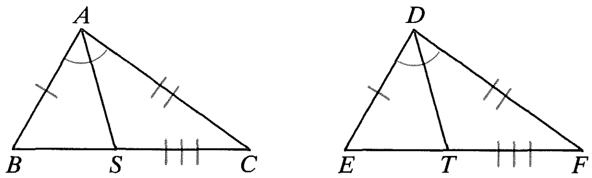
**Paragraph proof:**

$\triangle ABC \cong \triangle ADC$ by the ASA Postulate. Therefore, corresponding parts $\overline{AB}$ and $\overline{AD}$ are congruent. $\overline{AB}$ and $\overline{AD}$ are also corresponding parts of $\triangle ABO$ and $\triangle ADO$, which can now be proved congruent by the SAS Postulate. So corresponding parts $\angle 3$ and $\angle 4$ are congruent, and $\overline{AC} \perp \overline{BD}$.

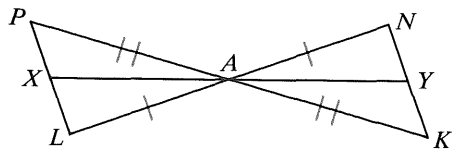
## Classroom Exercises

**In Exercises 1–3 you are given a diagram that is marked with given information. Give the reason for each key step of the proof.**

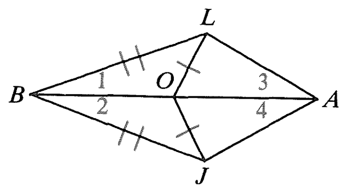
1. Prove: $\overline{AS} \cong \overline{DT}$
   Key steps of proof:
   **a.** $\triangle ABC \cong \triangle DEF$
   **b.** $\angle C \cong \angle F$
   **c.** $\triangle ACS \cong \triangle DFT$
   **d.** $\overline{AS} \cong \overline{DT}$



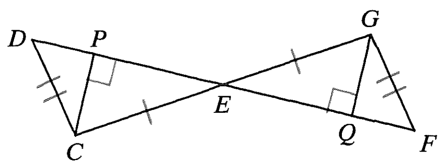

2. Prove: $\overline{AX} \cong \overline{AY}$
   Key steps of proof:
   **a.** $\triangle PAL \cong \triangle KAN$
   **b.** $\angle L \cong \angle N$
   **c.** $\triangle LAX \cong \triangle NAY$
   **d.** $\overline{AX} \cong \overline{AY}$




3. Prove: $\angle 3 \cong \angle 4$
   Key steps of proof:
   **a.** $\triangle LOB \cong \triangle JOB$
   **b.** $\angle 1 \cong \angle 2$
   **c.** $\triangle LBA \cong \triangle JBA$
   **d.** $\angle 3 \cong \angle 4$



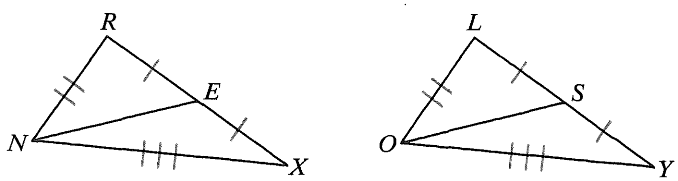

4. Suggest a plan for proving that $\angle D \cong \angle F$.



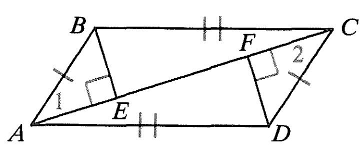

## Written Exercises

**In Exercises 1–6 you are given a diagram that is marked with given information. Give the reason for each key step of the proof.**

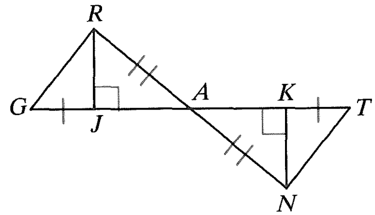
A 1. Prove: $\overline{NE} \cong \overline{OS}$
   Key steps of proof:
   **a.** $\triangle RNX \cong \triangle LOY$
   **b.** $\angle X \cong \angle Y$
   **c.** $\triangle NEX \cong \triangle OSY$
   **d.** $\overline{NE} \cong \overline{OS}$



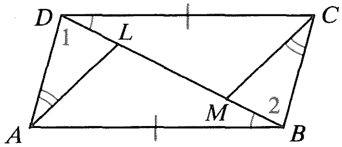

2. Prove: $\overline{BE} \cong \overline{DF}$
   Key steps of proof:
   **a.** $\triangle ABC \cong \triangle CDA$
   **b.** $\angle 1 \cong \angle 2$
   **c.** $\triangle ABE \cong \triangle CDF$
   **d.** $\overline{BE} \cong \overline{DF}$



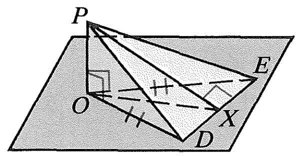
**3.** Prove: $\angle G \cong \angle T$
Key steps of proof:
**a.** $\triangle RAJ \cong \triangle NAK$
**b.** $\overline{RJ} \cong \overline{NK}$
**c.** $\triangle GRJ \cong \triangle TNK$
**d.** $\angle G \cong \angle T$



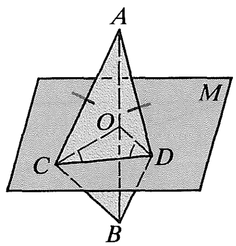

**4.** Prove: $\overline{AL} \cong \overline{CM}$
Key steps of proof:
**a.** $\triangle ABD \cong \triangle CDB$
**b.** $\overline{AD} \cong \overline{CB}$; $\angle 1 \cong \angle 2$
**c.** $\triangle ADL \cong \triangle CBM$
**d.** $\overline{AL} \cong \overline{CM}$



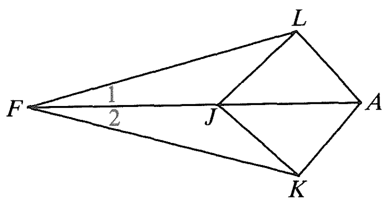

**5.** Prove: $\overline{DX} \cong \overline{EX}$
Key steps of proof:
**a.** $\triangle POD \cong \triangle POE$
**b.** $\overline{PD} \cong \overline{PE}$
**c.** $\triangle PDX \cong \triangle PEX$
**d.** $\overline{DX} \cong \overline{EX}$



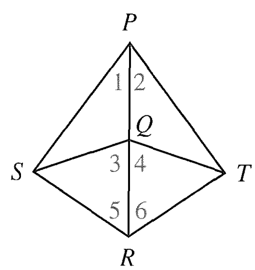

**6.** Prove: $\angle CBA \cong \angle DBA$
Key steps of proof:
**a.** $\overline{OC} \cong \overline{OD}$
**b.** $\triangle CAO \cong \triangle DAO$
**c.** $\angle CAO \cong \angle DAO$
**d.** $\triangle CAB \cong \triangle DAB$
**e.** $\angle CBA \cong \angle DBA$




**B** **7.** Given: $\overline{LF} \cong \overline{KF}$; $\overline{LA} \cong \overline{KA}$
Prove: $\overline{LJ} \cong \overline{KJ}$
**a.** List the key steps of a proof.
**b.** Write a proof in two-column form.

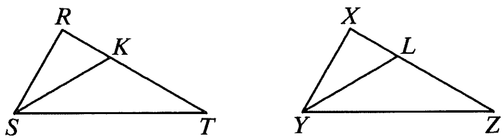



**8.** Given: $\overline{PR}$ bisects $\angle SPT$ and $\angle SRT$.
Prove: $\overline{PR}$ bisects $\angle SQT$.
**a.** List the key steps of a proof.
**b.** Write a proof in paragraph form.

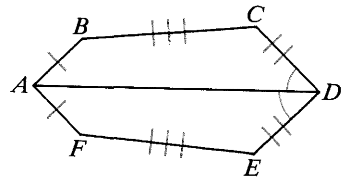


**Write proofs in the form specified by your teacher (two-column form, paragraph form, or a list of key steps).**

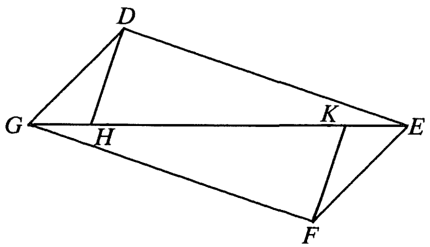
**9.** Given: $\triangle RST \cong \triangle XYZ$;
$\overrightarrow{SK}$ bisects $\angle RST$;
$\overrightarrow{YL}$ bisects $\angle XYZ$.
Prove: $\overline{SK} \cong \overline{YL}$



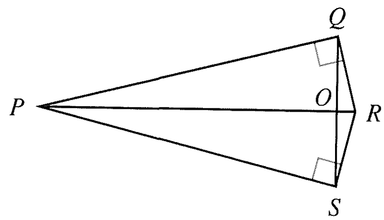

**10.** Given: Congruent parts as marked in the diagram.
Prove: $\angle B \cong \angle F$
(*Hint*: First draw two auxiliary lines.)




**11.** Given: $\overline{DE} \cong \overline{FG}$; $\overline{GD} \cong \overline{EF}$;
$\angle HDE$ and $\angle KFG$ are rt. $\angle$s.
Prove: $\overline{DH} \cong \overline{FK}$




**12.** Given: $\overline{PQ} \perp \overline{QR}$;
$\overline{PS} \perp \overline{SR}$;
$\overline{PQ} \cong \overline{PS}$
Prove: $O$ is the midpoint of $\overline{QS}$.

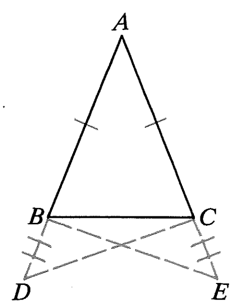



**13.** Draw two line segments, $\overline{KL}$ and $\overline{MN}$, that bisect each other at $O$. Mark a point $P$ on $\overline{KN}$ and let $Q$ be the point where $\overrightarrow{PO}$ intersects $\overline{ML}$. Prove that $O$ is the midpoint of $\overline{PQ}$. (First state what is given and what is to be proved.)

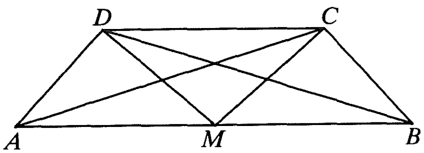
**14.** This figure is like the one that Euclid used to prove that the base angles of an isosceles triangle are congruent (our Theorem 4-1). Write a paragraph proof following the key steps shown below.

Given: $\overline{AB} \cong \overline{AC}$;
$\overline{AB}$ and $\overline{AC}$ are extended so $\overline{BD} \cong \overline{CE}$.
Prove: $\angle ABC \cong \angle ACB$

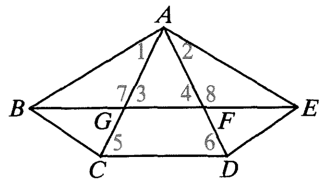
Key steps of proof:
1. $\triangle DAC \cong \triangle EAB$
2. $\triangle DBC \cong \triangle ECB$
3. $\angle DBC \cong \angle ECB$
4. $\angle ABC \cong \angle ACB$



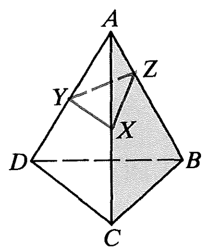
**C** **15.** Given: $\overline{AM} \cong \overline{MB}$; $\overline{AD} \cong \overline{BC}$;
$\angle MDC \cong \angle MCD$
Prove: $\overline{AC} \cong \overline{BD}$




**16.** Given: $\angle 1 \cong \angle 2$;
$\angle 3 \cong \angle 4$;
$\angle 5 \cong \angle 6$
Prove: $\overline{BC} \cong \overline{ED}$




**17.** $A$, $B$, $C$, and $D$ are noncoplanar. $\triangle ABC$, $\triangle ACD$, and $\triangle ABD$ are equilateral. $X$ and $Y$ are midpoints of $\overline{AC}$ and $\overline{AD}$. $Z$ is a point on $\overline{AB}$. What kind of triangle is $\triangle XYZ$? Explain.




## Mixed Review Exercises

**1.** Write the Isosceles Triangle Theorem (Theorem 4-1) and its converse (Theorem 4-2) as a single biconditional statement.

**Complete each statement with the word *always*, *sometimes*, or *never*.**

**2.** Two isosceles triangles with congruent bases are _?_ congruent.

**3.** Two isosceles triangles with congruent vertex angles are _?_ congruent.

**4.** Two equilateral triangles with congruent bases are _?_ congruent.

**Draw a diagram for each of the following.**

**5. a.** $M$ is between $A$ and $B$.
**b.** $M$ is the midpoint of $\overline{AB}$.

**6. a.** $\overline{XY}$ bisects $\overline{CD}$.
**b.** $\overrightarrow{XY}$ bisects $\angle CXD$.

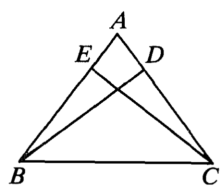
**7. a.** acute scalene $\triangle JKL$
**b.** obtuse scalene $\triangle JKL$

**8. a.** acute isosceles $\triangle XYZ$
**b.** obtuse isosceles $\triangle XYZ$

**9. a.** right scalene $\triangle RST$
**b.** right isosceles $\triangle RST$

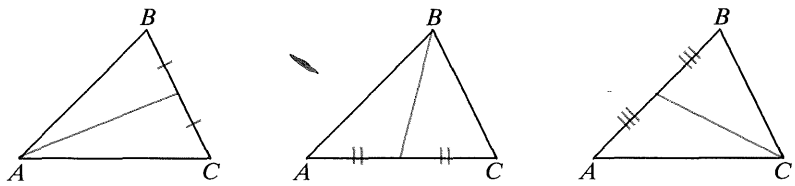
**10. a.** equilateral $\triangle EFG$
**b.** equiangular $\triangle EFG$

**11.** Write a proof in two-column form.
Given: $\overline{BE} \cong \overline{CD}$; $\overline{BD} \cong \overline{CE}$
Prove: $\triangle ABC$ is isosceles.

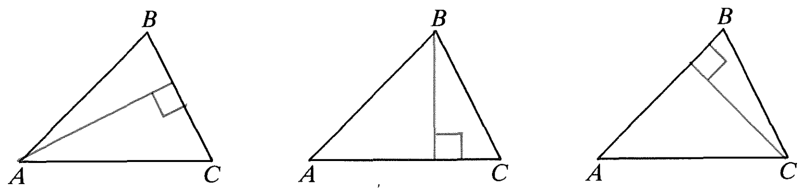


# 4-7 Medians, Altitudes, and Perpendicular Bisectors

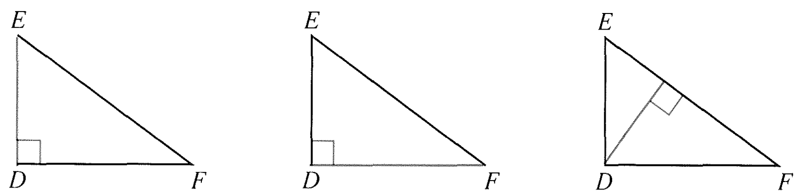
A **median** of a triangle is a segment from a vertex to the midpoint of the opposite side. The three medians of $\triangle ABC$ are shown below in red.



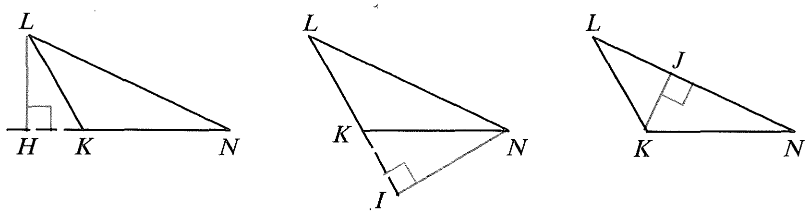
An **altitude** of a triangle is the perpendicular segment from a vertex to the line that contains the opposite side. In an acute triangle, the three altitudes are all inside the triangle.



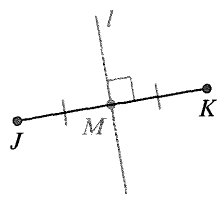
In a right triangle, two of the altitudes are parts of the triangle. They are the legs of the right triangle. The third altitude is inside the triangle.



In an obtuse triangle, two of the altitudes are outside the triangle. For obtuse $\triangle KLN$, $\overline{LH}$ is the altitude from $L$, and $\overline{NI}$ is the altitude from $N$.



A **perpendicular bisector** of a segment is a line (or ray or segment) that is perpendicular to the segment at its midpoint. In the figure at the right, line $l$ is a perpendicular bisector of $\overline{JK}$.




In a given plane, there is exactly one line perpendicular to a segment at its midpoint. We speak of *the* perpendicular bisector of a segment in such a case.

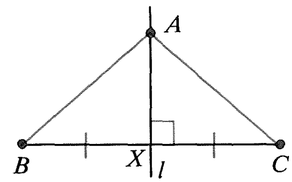
Proofs of the following theorems are left as Exercises 14 and 15.

## Theorem 4-5

**If a point lies on the perpendicular bisector of a segment, then the point is equidistant from the endpoints of the segment.**

Given: Line $l$ is the perpendicular bisector of $\overline{BC}$; $A$ is on $l$.

Prove: $AB = AC$

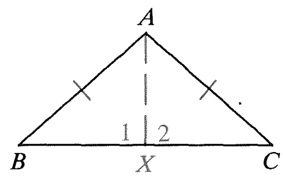



## Theorem 4-6

**If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of the segment.**

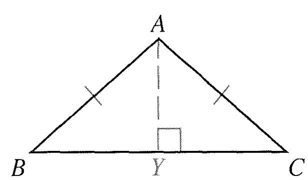
Given: $AB = AC$

Prove: $A$ is on the perpendicular bisector of $\overline{BC}$.




**Plan for Proof:** The perpendicular bisector of $\overline{BC}$ must contain the midpoint of $\overline{BC}$ and be perpendicular to $\overline{BC}$. Draw an auxiliary line containing $A$ that has one of these properties and prove that it has the other property as well. For example, first draw a segment from $A$ to the midpoint $X$ of $\overline{BC}$. You can show that $\overline{AX} \perp \overline{BC}$ if you can show that $\angle 1 \cong \angle 2$. Since these angles are corresponding parts of two triangles, first show that $\triangle AXB \cong \triangle AXC$.

In the proof of Theorem 4-6 other auxiliary lines could have been chosen instead. For example, we can draw the altitude to $\overline{BC}$ from $A$, meeting $\overline{BC}$ at a point $Y$ as shown in the diagram at the right. Here, since $\overline{AY} \perp \overline{BC}$ we need to prove that $\overline{YB} \cong \overline{YC}$. Either method can be used to prove Theorem 4-6.

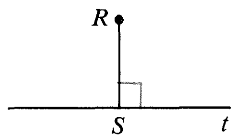


**Example** Suppose you know that line $l$ is the perpendicular bisector of $\overline{RS}$. What can you deduce if you also know that

**a.** $P$ lies on $l$?
**b.** there is a point $Q$ such that $QR = 7$ and $QS = 7$?

**Solution** **a.** $PR = PS$ (Theorem 4-5)
**b.** $Q$ lies on $l$. (Theorem 4-6)

The **distance from a point to a line** (or plane) is defined to be the length of the perpendicular segment from the point to the line (or plane). Since $\overline{RS} \perp t$, $RS$ is the distance from $R$ to line $t$.



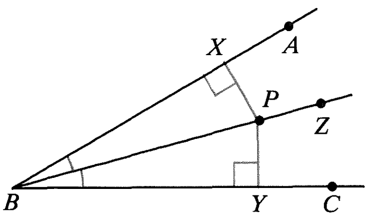

In Exercises 16 and 17 you will prove the following theorems, which are similar to Theorems 4-5 and 4-6.

## Theorem 4-7

**If a point lies on the bisector of an angle, then the point is equidistant from the sides of the angle.**

Given: $\overrightarrow{BZ}$ bisects $\angle ABC$; $P$ lies on $\overrightarrow{BZ}$;
$\overline{PX} \perp \overrightarrow{BA}$; $\overline{PY} \perp \overrightarrow{BC}$

Prove: $PX = PY$

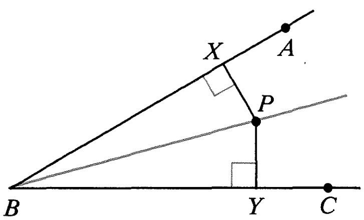



## Theorem 4-8

**If a point is equidistant from the sides of an angle, then the point lies on the bisector of the angle.**

Given: $\overline{PX} \perp \overrightarrow{BA}$; $\overline{PY} \perp \overrightarrow{BC}$;
$PX = PY$

Prove: $\overrightarrow{BP}$ bisects $\angle ABC$.



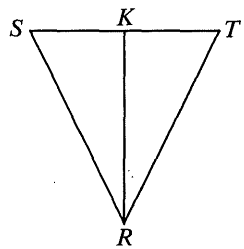

Theorem 4-5 and its converse, Theorem 4-6, can be combined into a single biconditional statement. The same is true for Theorems 4-7 and 4-8.

A point is on the perpendicular bisector of a segment if and only if it is equidistant from the endpoints of the segment.

A point is on the bisector of an angle if and only if it is equidistant from the sides of the angle.

## Classroom Exercises

**Complete.**



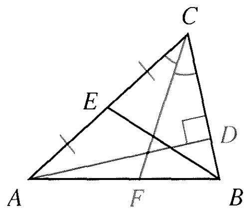

**1.** If $K$ is the midpoint of $\overline{ST}$, then $\overline{RK}$ is called a(n) $\underline{\ ?\ }$ of $\triangle RST$.

**2.** If $\overline{RK} \perp \overline{ST}$, then $\overline{RK}$ is called a(n) $\underline{\ ?\ }$ of $\triangle RST$.

**3.** If $K$ is the midpoint of $\overline{ST}$ and $\overline{RK} \perp \overline{ST}$, then $\overline{RK}$ is called a(n) $\underline{\ ?\ }$ of $\overline{ST}$.

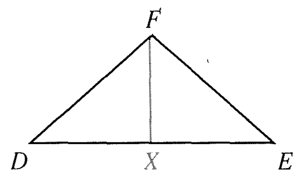
**4.** If $\overline{RK}$ is both an altitude and a median of $\triangle RST$, then:
**a.** $\triangle RSK \cong \triangle RTK$ by $\underline{\ ?\ }$. **b.** $\triangle RST$ is a(n) $\underline{\ ?\ }$ triangle.

**5.** If $R$ is on the perpendicular bisector of $\overline{ST}$, then $R$ is equidistant from $\underline{\ ?\ }$ and $\underline{\ ?\ }$. Thus $\underline{\ ?\ } = \underline{\ ?\ }$.




**6.** Refer to $\triangle ABC$ and name each of the following.
**a.** a median of $\triangle ABC$
**b.** an altitude of $\triangle ABC$
**c.** a bisector of an angle of $\triangle ABC$

**7.** Draw $\overline{XY}$. Label its midpoint $Q$.
**a.** Select a point $P$ equidistant from $X$ and $Y$. Draw $\overline{PX}$, $\overline{PY}$, and $\overline{PQ}$.
**b.** What postulate justifies the statement $\triangle PQX \cong \triangle PQY$?
**c.** What reason justifies the statement $\angle PQX \cong \angle PQY$?
**d.** What reason justifies the statement $\overleftrightarrow{PQ} \perp \overline{XY}$?
**e.** What name for $\overleftrightarrow{PQ}$ best describes the relationship between $\overleftrightarrow{PQ}$ and $\overline{XY}$?

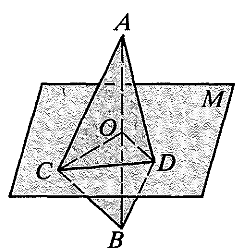



**8.** Given: $\triangle DEF$ is isosceles with $DF = EF$;
$\overline{FX}$ bisects $\angle DFE$.
**a.** Would the median drawn from $F$ to $\overline{DE}$ be the same segment as $\overline{FX}$?
**b.** Would the altitude drawn from $F$ to $\overline{DE}$ be the same segment as $\overline{FX}$?

**9.** What kind of triangle has three angle bisectors that are also altitudes and medians?

**10.** Given: $\overrightarrow{NO}$ bisects $\angle N$.
What can you conclude from each of the following additional statements?
**a.** $P$ lies on $\overrightarrow{NO}$.
**b.** The distance from a point $Q$ to each side of $\angle N$ is 13.

**11.** Plane $M$ is the *perpendicular bisecting plane* of $\overline{AB}$ at $O$ (that is, $M$ is the plane that is perpendicular to $\overline{AB}$ at its midpoint, $O$). Points $C$ and $D$ also lie in plane $M$. List three pairs of congruent triangles and tell which congruence method can be used to prove each pair congruent.



## Written Exercises

A **1. a.** Draw a large scalene triangle $ABC$. Carefully draw the bisector of $\angle A$, the altitude from $A$, and the median from $A$. These three should all be different.

**b.** Draw a large isosceles triangle $ABC$ with vertex angle $A$. Carefully draw the bisector of $\angle A$, the altitude from $A$, and the median from $A$. Are these three different?

**2.** Draw a large obtuse triangle. Then draw its three altitudes in color.

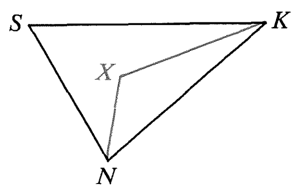
**3.** Draw a right triangle. Then draw its three altitudes in color.

**4.** Draw a large acute scalene triangle. Then draw the perpendicular bisectors of its three sides.

**5.** Draw a large scalene right triangle. Then draw the perpendicular bisectors of its three sides and tell whether they appear to meet in a point. If so, where is this point?

**6.** Cut out any large triangle. Fold the two sides of one angle of the triangle together to form the angle bisector. Use the same method to form the bisectors of the other two angles. What do you notice?

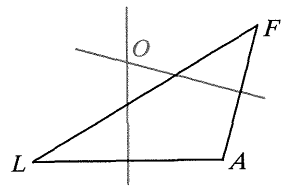
**Complete each statement.**




**7.** If $X$ is on the bisector of $\angle SKN$, then $X$ is equidistant from _?_ and _?_.

**8.** If $X$ is on the bisector of $\angle SNK$, then $X$ is equidistant from _?_ and _?_.

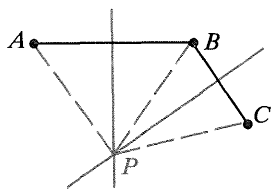
**9.** If $X$ is equidistant from $\overline{SK}$ and $\overline{SN}$, then $X$ lies on the _?_.




**10.** If $O$ is on the perpendicular bisector of $\overline{LA}$, then $O$ is equidistant from _?_ and _?_.

**11.** If $O$ is on the perpendicular bisector of $\overline{AF}$, then $O$ is equidistant from _?_ and _?_.

**12.** If $O$ is equidistant from $L$ and $F$, then $O$ lies on the _?_.




**13.** Given: $P$ is on the perpendicular bisector of $\overline{AB}$;
$P$ is on the perpendicular bisector of $\overline{BC}$.
Prove: $PA = PC$

**Use the diagrams on pages 153 and 154 to prove the following theorems.**

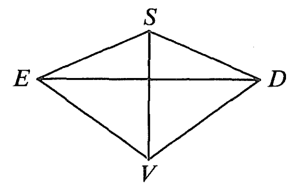
B **14.** Theorem 4-5

**15.** Theorem 4-6

**16.** Theorem 4-7

**17.** Theorem 4-8

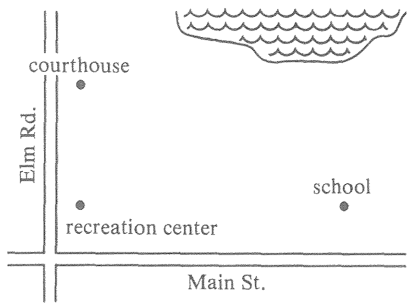
**18.** Given: $S$ is equidistant from $E$ and $D$;
$V$ is equidistant from $E$ and $D$.
Prove: $\overleftrightarrow{SV}$ is the perpendicular bisector of $\overline{ED}$.



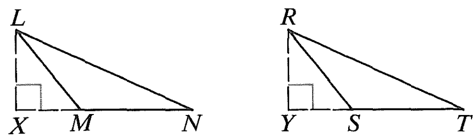

**19. a.** A town wants to build a beach house on the lake front equidistant from the recreation center and the school. Copy the diagram and show the point $B$ where the beach house should be located.

**b.** The town also wants to build a boat-launching site that is equidistant from Elm Road and Main Street. Find the point $L$ where it should be built.

**c.** On your diagram, locate the spot $F$ for a flagpole that is to be the same distance from the recreation center, the school, and the courthouse.

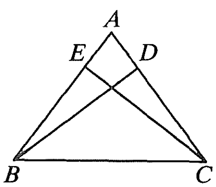



**20.** Given: $\triangle LMN \cong \triangle RST$;
$\overline{LX}$ and $\overline{RY}$ are altitudes.
Prove: $\overline{LX} \cong \overline{RY}$




**21. a.** Given: $\overline{AB} \cong \overline{AC}$; $\overline{BD} \perp \overline{AC}$; $\overline{CE} \perp \overline{AB}$
Prove: $\overline{BD} \cong \overline{CE}$

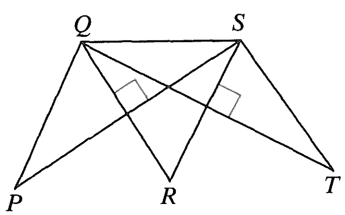
**b.** The result you proved in part (a) can be stated as a theorem about certain altitudes. State this theorem in your own words.



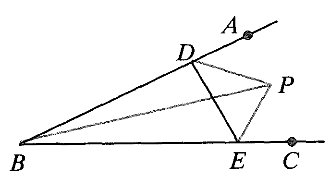

**22.** Prove that the medians drawn to the legs of an isosceles triangle are congruent. Write the proof in two-column form.

**For Exercises 23–27 write proofs in paragraph form. (*Hint*: You can use theorems from this section to write fairly short proofs for Exercises 23 and 24.)**

**23.** Given: $\overleftrightarrow{SR}$ is the $\perp$ bisector of $\overline{QT}$;
$\overleftrightarrow{QR}$ is the $\perp$ bisector of $\overline{SP}$.
Prove: $PQ = TS$

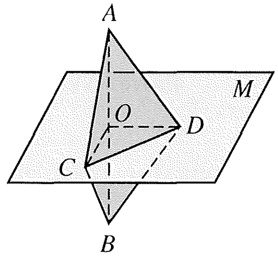



**24.** Given: $\overrightarrow{DP}$ bisects $\angle ADE$;
$\overrightarrow{EP}$ bisects $\angle DEC$.
Prove: $\overrightarrow{BP}$ bisects $\angle ABC$.



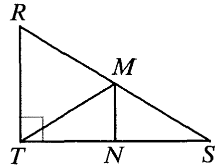
**25.** Given: Plane $M$ is the perpendicular bisecting plane of $\overline{AB}$. (That is, $\overline{AB} \perp$ plane $M$ and $O$ is the midpoint of $\overline{AB}$.)

Prove: **a.** $\overline{AD} \cong \overline{BD}$
**b.** $\overline{AC} \cong \overline{BC}$
**c.** $\angle CAD \cong \angle CBD$




**C** **26.** Given: $m\angle RTS = 90$; $\overleftrightarrow{MN}$ is the $\perp$ bisector of $\overline{TS}$.

Prove: $\overline{TM}$ is a median.



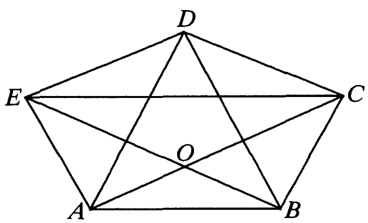

**27.** Given: $\overline{EH}$ and $\overline{FJ}$ are medians of scalene $\triangle EFG$; $P$ is on $\overrightarrow{EH}$ such that $\overline{EH} \cong \overline{HP}$; $Q$ is on $\overrightarrow{FJ}$ such that $\overline{FJ} \cong \overline{JQ}$.

Prove: **a.** $\overline{GQ} \cong \overline{GP}$
**b.** $\overline{GQ}$ and $\overline{GP}$ are both parallel to $\overline{EF}$.
**c.** $P$, $G$, and $Q$ are collinear.

**Write paragraph proofs. (In this book a star designates an exercise that is unusually difficult.)**

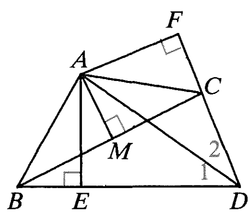
★ **28.** Given: $\overline{AE} \parallel \overline{BD}$; $\overline{BC} \parallel \overline{AD}$; $\overline{AE} \cong \overline{BC}$; $\overline{AD} \cong \overline{BD}$

Prove: **a.** $\overline{AC} \cong \overline{BE}$
**b.** $\overline{EC} \parallel \overline{AB}$




★ **29.** Given: $\overleftrightarrow{AM}$ is the $\perp$ bis. of $\overline{BC}$; $\overline{AE} \perp \overline{BD}$; $\overline{AF} \perp \overline{DF}$; $\angle 1 \cong \angle 2$

Prove: $\overline{BE} \cong \overline{CF}$




## Explorations

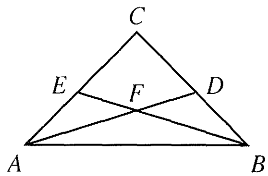
**These exploratory exercises can be done using a computer with a program that draws and measures geometric figures.**

Decide if the following statements are true or false. If you think the statement is true, give a convincing argument to support your belief. If you think the statement is false, make a sketch and give all the measurements of the triangle that you find as your counterexample. For each false statement, also discover if there are types of triangles for which the statement is true.

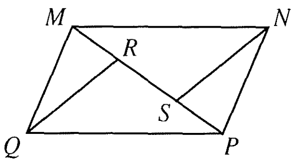
1. An angle bisector bisects the side opposite the bisected angle.
2. A median bisects the angle at the vertex from which it is drawn.
3. The length of a median is equal to half of the length of the side it bisects.

# Self-Test 3

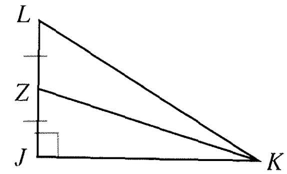
1. Suppose you wish to prove $\triangle AFE \cong \triangle BFD$. If you have already proved $\triangle ABE \cong \triangle BAD$, what corresponding parts from this second pair of congruent triangles would you use to prove the first pair of triangles congruent?




2. Given: $\triangle MPQ \cong \triangle PMN$; $\overline{MS} \cong \overline{PR}$
   Prove: $\triangle MSN \cong \triangle PRQ$




3. In $\triangle JKL$ name each of the following.
   **a.** an altitude    **b.** a median
4. Note that $ZL = ZJ$. Can you deduce that $\overrightarrow{KZ}$ bisects $\angle LKJ$?




5. $\overrightarrow{UV}$ bisects $\angle WUX$. Write the theorem that justifies the statement that $V$ is equidistant from $\overrightarrow{UW}$ and $\overrightarrow{UX}$.
6. In $\triangle ABC$, $AB = 7$ and $BC = 7$. Write the theorem that allows you to conclude that $B$ is on the perpendicular bisector of $\overline{AC}$.

# Chapter Summary

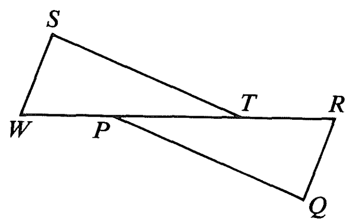
1. Congruent figures have the same size and shape. Two triangles are congruent if their corresponding sides and angles are congruent.
2. We have five ways to prove two triangles congruent:
   SSS    SAS    ASA    AAS    HL (rt. △s)
3. A common way to prove that two segments or two angles are congruent is to show that they are corresponding parts of congruent triangles.
4. A line and plane are perpendicular if and only if they intersect and the line is perpendicular to all lines in the plane that pass through the point of intersection.
5. If two sides of a triangle are congruent, then the angles opposite those sides are congruent. An equilateral triangle is also equiangular, with three $60°$ angles.

6. If two angles of a triangle are congruent, then the sides opposite those angles are congruent. An equiangular triangle is also equilateral.
7. Sometimes you can prove one pair of triangles congruent and then use corresponding parts from those triangles to prove that another pair of triangles are congruent.
8. Proofs in geometry are commonly written in two-column form, as a list of key steps, or in paragraph form.
9. Every triangle has three medians and three altitudes.
10. The perpendicular bisector of a segment is the line that is perpendicular to the segment at its midpoint.
11. A point lies on the perpendicular bisector of a segment if and only if the point is equidistant from the endpoints of the segment.
12. A point lies on the bisector of an angle if and only if the point is equidistant from the sides of the angle.

# Chapter Review

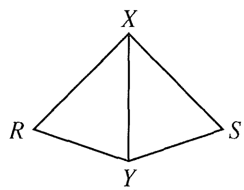
**The two triangles shown are congruent. Complete.** 4–1

1. $\triangle STW \cong$ __?__
2. $\triangle PQR \cong$ __?__
3. $\angle R \cong$ __?__
4. __?__ $= RP$



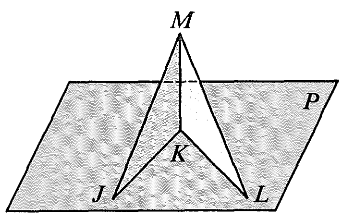

**Can you deduce from the given information that $\triangle RXY \cong \triangle SXY$? If so, what postulate can you use?** 4–2

5. Given: $\overline{RX} \cong \overline{SX}$; $\overline{RY} \cong \overline{SY}$
6. Given: $\overline{RY} \cong \overline{SY}$; $\angle R \cong \angle S$
7. Given: $\overline{XY}$ bisects $\angle RXS$ and $\angle RYS$.
8. Given: $\angle RXY \cong \angle SXY$; $\overline{RX} \cong \overline{SX}$




**Write proofs in two-column form.** 4–3

9. Given: $\overline{JM} \cong \overline{LM}$; $\overline{JK} \cong \overline{LK}$
   Prove: $\angle MJK \cong \angle MLK$
10. Given: $\angle JMK \cong \angle LMK$; $\overline{MK} \perp$ plane $P$
    Prove: $\overline{JK} \cong \overline{LK}$



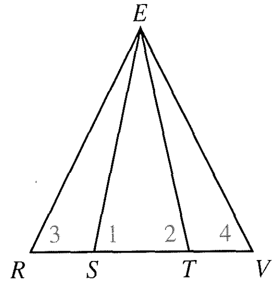
**Complete.**

**11.** If $\angle 3 \cong \angle 4$, then which segments must be congruent? 4–4

**12.** If $\triangle REV$ is an equiangular triangle, then $\triangle REV$ is also a(n) __?__ triangle.

**13.** If $\overline{ES} \cong \overline{ET}$, $m\angle 1 = 75$, and $m\angle 2 = 3x$, then $x =$ __?__.

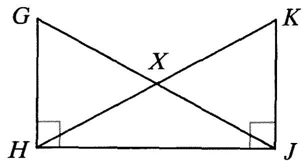
**14.** If $\angle 1 \cong \angle 2$, $ES = 3y + 5$, and $ET = 25 - y$, then $y =$ __?__.




**Write proofs in two-column form.**

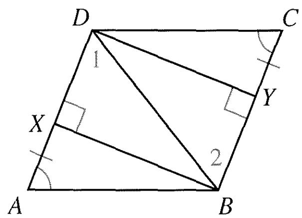
**15.** Given: $\overline{GH} \perp \overline{HJ}$; $\overline{KJ} \perp \overline{HJ}$; $\angle G \cong \angle K$ 4–5
Prove: $\triangle GHJ \cong \triangle KJH$

**16.** Given: $\overline{GH} \perp \overline{HJ}$; $\overline{KJ} \perp \overline{HJ}$; $\overline{GJ} \cong \overline{KH}$
Prove: $\overline{GH} \cong \overline{KJ}$

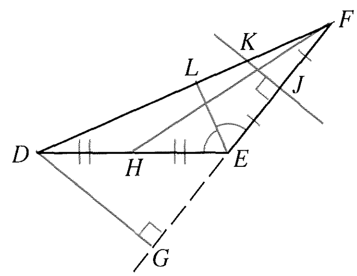



**17.** Give the reason for each key step of the proof. 4–6
Given: $\overline{AX} \cong \overline{CY}$; $\angle A \cong \angle C$; $\overline{BX} \perp \overline{AD}$; $\overline{DY} \perp \overline{BC}$
Prove: $\overline{AD} \parallel \overline{BC}$

1. $\triangle ABX \cong \triangle CDY$
2. $\overline{BX} \cong \overline{DY}$
3. $\triangle BDX \cong \triangle DBY$
4. $\angle 1 \cong \angle 2$
5. $\overline{AD} \parallel \overline{BC}$




**18.** Refer to $\triangle DEF$ and name each of the following: 4–7
**a.** an altitude
**b.** a median
**c.** the perpendicular bisector of a side of the triangle




**19.** Point $G$ lies on the perpendicular bisector of $\overline{EF}$. Write the theorem that justifies the statement that $GE = GF$.

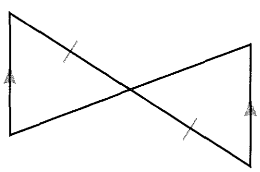
**20.** $\triangle ABC$ and $\triangle ABD$ are congruent right triangles with common hypotenuse $\overline{AB}$. Write the theorem that allows you to conclude that point $B$ lies on the bisector of $\angle DAC$.

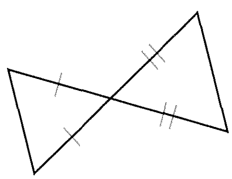
# Chapter Test

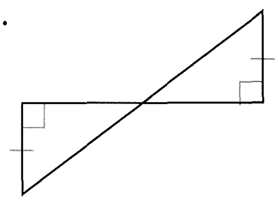
**Complete.**

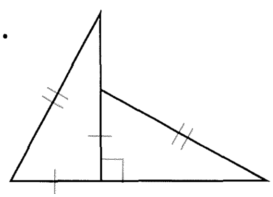
1. If $\triangle BAD \cong \triangle TOP$, then $\overline{DB} \cong$ ___?___ and $\triangle PTO \cong$ ___?___.
2. $\triangle EFG$ is isosceles, with $m\angle G = 94$. The legs are sides ___?___ and ___?___. $m\angle E =$ ___?___ (numerical answer).
3. You want to prove $\triangle ABC \cong \triangle XYZ$. You have shown $\overline{AB} \cong \overline{XY}$ and $\overline{AC} \cong \overline{XZ}$. To prove the triangles congruent by SAS you must show that ___?___ $\cong$ ___?___. To prove the triangles congruent by SSS you must show that ___?___ $\cong$ ___?___.
4. A method that can be used to prove right triangles congruent, but cannot be used with other types of triangles, is the ___?___ method.
5. $\triangle CAP$ and $\triangle TAP$ are equilateral and coplanar. $\overline{AP}$ is a common side of the two triangles. $m\angle CAT =$ ___?___ (numerical answer).
6. A segment from a vertex of a triangle to the midpoint of the opposite side is called a(n) ___?___ of the triangle.
7. A point lies on the bisector of an angle if and only if it is equidistant from ___?___.
8. If in $\triangle ABC$ $m\angle A = 50$, $m\angle C = 80$, $AC = 7x + 8$, and $BC = 38 - 3x$, then $x =$ ___?___.

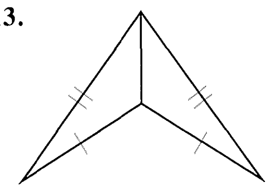
**Can two triangles be proved congruent? If so, by which method, SSS, SAS, ASA, AAS, or HL?**

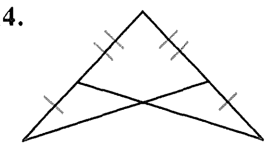
9. 

10. 

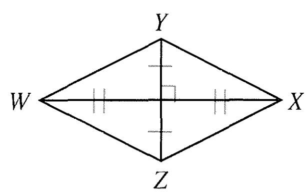
11. 

12. 

13. 

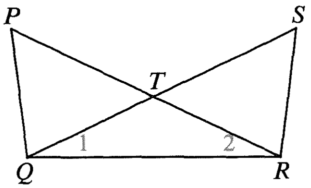
14. 

**$\overline{WX}$ and $\overline{YZ}$ are perpendicular bisectors of each other.**

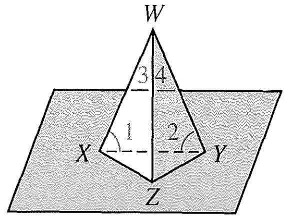



15. $W$ is equidistant from ___?___ and ___?___.
16. $Z$ is equidistant from ___?___ and ___?___.
17. Name four isosceles triangles.
18. How many pairs of congruent triangles are shown in the diagram?

**19.** Given: $\angle 1 \cong \angle 2$; $\angle PQR \cong \angle SRQ$
Prove: $\overline{PR} \cong \overline{SQ}$




**20.** Given: $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$
Prove: $\triangle ZXY$ is isosceles.




# Algebra Review: *Quadratic Equations*

**Solve each equation by factoring or by using the quadratic formula. The quadratic formula is:**

$$\text{If } ax^2 + bx + c = 0, \text{ with } a \neq 0, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

**Example** $3x^2 + 14x + 8 = 0$

**Solution 1** *By factoring*

$$3x^2 + 14x + 8 = 0$$
$$(3x + 2)(x + 4) = 0$$
$$3x + 2 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = -\frac{2}{3} \quad \text{or} \quad x = -4$$

**Solution 2** *By quadratic formula*

$$3x^2 + 14x + 8 = 0 \qquad a = 3,\ b = 14,\ c = 8$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-14 \pm \sqrt{14^2 - 4(3)(8)}}{2(3)}$$
$$x = \frac{-14 \pm \sqrt{196 - 96}}{6} = \frac{-14 \pm 10}{6}$$
$$x = -\frac{2}{3} \quad \text{or} \quad x = -4$$

**1.** $x^2 + 5x - 6 = 0$
**2.** $n^2 - 6n + 8 = 0$
**3.** $y^2 - 7y - 18 = 0$
**4.** $x^2 + 8x = 0$
**5.** $y^2 = 13y$
**6.** $2z^2 + 7z = 0$
**7.** $n^2 - 144 = 25$
**8.** $50x^2 = 200$
**9.** $50x^2 = 2$
**10.** $49z^2 = 1$
**11.** $y^2 - 6y + 9 = 0$
**12.** $x^2 - 7x + 12 = 0$
**13.** $y^2 + 8y + 12 = 0$
**14.** $t^2 + 5t = 24$
**15.** $v^2 + 25 = 10v$
**16.** $x^2 = 3x + 4$
**17.** $t^2 - t = 20$
**18.** $y^2 = 20y - 36$
**19.** $3x^2 + 3x = 4$
**20.** $15 + 4y^2 = 17y$
**21.** $x^2 + 5x + 2 = 0$
**22.** $x^2 + 2x - 1 = 0$
**23.** $x^2 - 5x + 3 = 0$
**24.** $x^2 + 3x - 2 = 0$
**25.** $(y - 5)^2 = 16$
**26.** $z^2 = 4(2z - 3)$
**27.** $x(x + 5) = 14$

**In Exercises 28–33 *x* represents the length of a segment. When a value of *x* doesn't make sense as a length, eliminate that value of *x*.**

**28.** $x(x - 50) = 0$
**29.** $x^2 - 400 = 0$
**30.** $x^2 - 17x + 72 = 0$
**31.** $2x^2 + x - 3 = 0$
**32.** $2x^2 - 7x - 4 = 0$
**33.** $6x^2 = 5x + 6$

# Preparing for College Entrance Exams

**Strategy For Success**

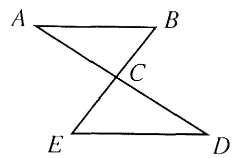
Some college entrance exam questions ask you to decide if several statements are true based on given information (see Exercises 3 and 8). In these exercises, check each statement separately and then choose the answer with the correct combination of true statements.

**Indicate the best answer by writing the appropriate letter.**

**1.** The measures of the angles of a triangle are $2x + 10$, $3x$, and $8x - 25$. The triangle is:
(**A**) obtuse (**B**) right (**C**) acute (**D**) equilateral (**E**) isosceles

**2.** A regular polygon has an interior angle of measure 120. How many vertices does the polygon have?
(**A**) 3 (**B**) 5 (**C**) 6 (**D**) 9 (**E**) 12

**3.** Plane $M$ is parallel to plane $N$. Line $l$ lies in $M$ and line $k$ lies in $N$. Which of the following statement(s) are possible?
(I) Lines $l$ and $k$ are parallel. (II) Lines $l$ and $k$ intersect.
(III) Lines $l$ and $k$ are skew.
(**A**) I only (**B**) II only (**C**) III only
(**D**) I and III only (**E**) I, II, and III




**4.** Given: $\overline{BE}$ bisects $\overline{AD}$. To prove that the triangles are congruent by the AAS method, you must show that:
(**A**) $\angle A \cong \angle E$ (**B**) $\angle A \cong \angle D$ (**C**) $\angle B \cong \angle E$
(**D**) $\angle B \cong \angle D$ (**E**) $\overline{AD}$ bisects $\overline{BE}$.

**5.** Given: $\triangle RGA$ and $\triangle PMC$ with $\overline{RG} \cong \overline{PM}$, $\overline{RA} \cong \overline{PC}$, and $\angle R \cong \angle P$. Which method could be used to prove that $\triangle RGA \cong \triangle PMC$?
(**A**) SSS (**B**) SAS (**C**) HL (**D**) ASA
(**E**) There is not enough information for a proof.

**6.** Predict the next number in the sequence, 2, 6, 12, 20, 30, 42, $\underline{\ ?\ }$.
(**A**) 52 (**B**) 54 (**C**) 56 (**D**) 58 (**E**) 60

**7.** In $\triangle JKL$, $\overline{KL} \cong \overline{JL}$, $m\angle K = 2x - 36$, and $m\angle L = x + 2$. Find $m\angle J$.
(**A**) 56 (**B**) 52 (**C**) 53 (**D**) 55 (**E**) 64

**8.** In $\triangle RST$, $\overleftrightarrow{SU}$ is the perpendicular bisector of $\overline{RT}$ and $U$ lies on $\overline{RT}$. Which statement(s) must be true?
(I) $\triangle RST$ is equilateral. (II) $\triangle RSU \cong \triangle TSU$
(III) $\overrightarrow{SU}$ is the bisector of $\angle RST$.
(**A**) I only (**B**) II only (**C**) III only
(**D**) II and III only (**E**) I, II, and III

**9.** Given: $\triangle SUN \cong \triangle TAN$. You can conclude that:
(**A**) $\angle S \cong \angle A$ (**B**) $\overline{SN} \cong \overline{TN}$ (**C**) $\angle T \cong \angle U$
(**D**) $\overline{SU} \cong \overline{TN}$ (**E**) $\overline{UN} \cong \overline{TA}$

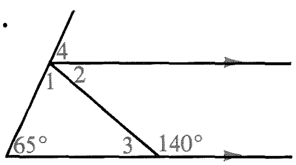
# Cumulative Review: Chapters 1–4

**Complete each sentence with the most appropriate word, phrase, or value.**

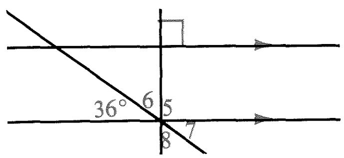
A
1. If $S$ is between $R$ and $T$, then $RS + ST = RT$ by the _?_.
2. If two parallel planes are cut by a third plane, then the lines of intersection are _?_.
3. $\overrightarrow{BD}$ bisects $\angle ABC$, $m\angle ABC = 5x - 4$, and $m\angle CBD = 2x + 10$. $\angle ABC$ is a(n) _?_ angle.
4. If two intersecting lines form congruent adjacent angles, then the lines are _?_.
5. If $\angle 1$ and $\angle 2$ are complements and $m\angle 1 = 74$, then $m\angle 2 =$ _?_.
6. Given the conditional "If $x = 9$, then $3x = 27$," its converse is _?_.
7. If the measure of each interior angle of a polygon is 144, then the polygon has _?_ sides.
8. In quadrilateral $EFGH$, $\overline{EF} \parallel \overline{HG}$, $m\angle E = y + 10$, $m\angle F = 2y - 40$, and $m\angle H = 2y - 31$. $m\angle G =$ _?_ (numerical answer)
9. If a diagonal of an equilateral quadrilateral is drawn, the two triangles formed can be proved congruent by the _?_ method.

**Find the measure of each numbered angle.**

**10.**

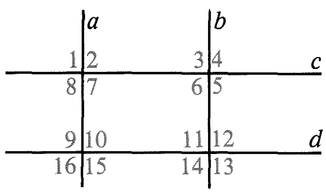



**11.**

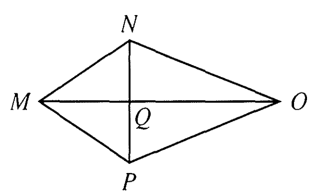



**Could the given information be used to prove that two lines are parallel? If so, which lines?**

12. $m\angle 8 + m\angle 9 = 180$
13. $\angle 1 \cong \angle 4$
14. $m\angle 2 = m\angle 6$
15. $\angle 8$ and $\angle 5$ are rt. $\angle$s.




B
16. Given: $\overline{MN} \cong \overline{MP}$; $\angle NMO \cong \angle PMO$
    Prove: $\overleftrightarrow{MO}$ is the $\perp$ bisector of $\overline{NP}$.
17. Given: $\overline{MO} \perp \overline{NP}$; $\overline{NO} \cong \overline{PO}$
    Prove: $\overline{MN} \cong \overline{MP}$




18. Write a paragraph proof: If $\overline{AX}$ is both a median and an altitude of $\triangle ABC$, then $\triangle ABC$ is isosceles.