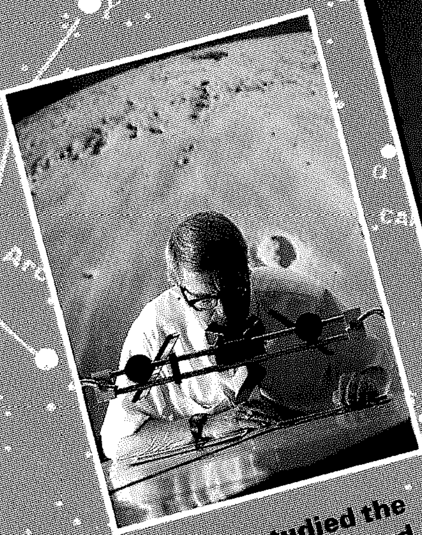
1 POINTS, LINES, PLANES, AND ANGLES



As ancient people studied the heavens, they saw and named many patterns of points, lines, and angles formed by the stars. Although modern astronomers use sophisticated observatories and equipment, they still base their calculations on geometric principles that have been known for many centuries.

Some Basic Figures

Objectives

1. Use the term *equidistant*.
2. Use the undefined terms *point*, *line*, and *plane*.
3. Draw representations of points, lines, and planes.
4. Use the terms *collinear*, *coplanar*, and *intersection*.

1-1 *A Game and Some Geometry*

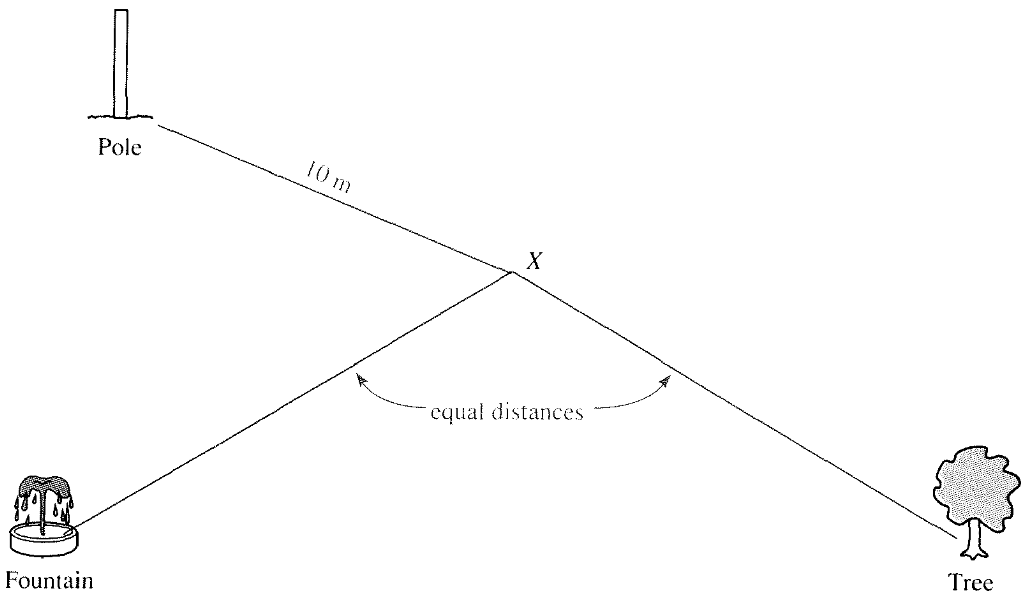
Suppose that you and Pat are partners in a game in which you must locate various clues to win. You are told to pick up your next clue at a point that

1. is as far from the fountain as from the oak tree

   and

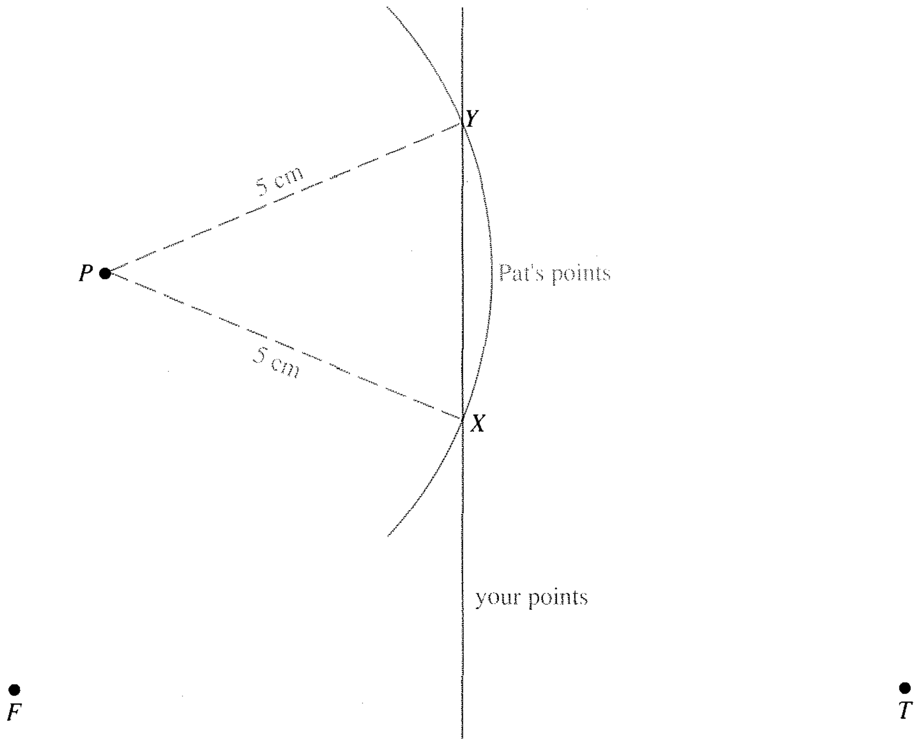
2. is 10 m (meters) from the flag pole.

You locate X, which satisfies both requirements, but grumble because there simply isn't any clue to be found at X.



Then Pat realizes that there may be a different location that satisfies both requirements. (Before reading on, see if you can find another point that meets requirements 1 and 2.)

Suppose that you concentrate on points that satisfy requirement 1 while Pat works on points that meet requirement 2. In the diagram below, 1 cm represents 2 m, so the blue arc shows points that are 10 m from the pole. Each red point is equally distant, or *equidistant*, from *F* and *T*. Point *Y*, as well as *X*, meets both requirements. You and Pat find your clue at *Y* and proceed with the game.



The game discussed above involves *points* and *distances*. When you approach the game systematically, you use *lines* and *circles*. Understanding the properties of geometric figures like these is an important part of geometry. The rest of this chapter will deal with the most basic figures of geometry.

Classroom Exercises

For Exercises 1–8 refer to the diagram above.

1. Suppose that the diagram showed, in blue, *all* the points that are 5 cm from *P*. What geometric figure would the points form?
2. In a more complete diagram, would there be a red point 15 cm from both *F* and *T*? How many such points?
3. It appears as if points *P*, *X*, and *T* might lie on a straight line. Use a ruler or the edge of a sheet of paper to see if they do.

4. It looks as if P might be equidistant from F and X. Is it?

5. Suppose Pat spoke of a line l joining F and T while you thought of a line n joining F and T. Is it better to say that l and n are two different lines, or to say that we have one line with two different names?

6. Point X is equidistant from F and T. Furthermore, point Y is equidistant from F and T. Does that mean that X and Y are equally distant from F?

7. Suppose you were asked to find a point 5 cm from P, 5 cm from F, and 5 cm from T. Is there such a point?

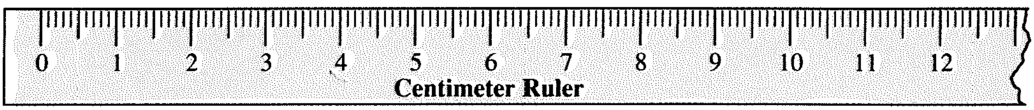
8. Do you believe there is any point that is equidistant from P, F, and T?

Written Exercises

A **1.** Copy and complete the table. Refer to the diagrams on pages 1 and 2.

Distance between	*Diagram distance*	*Ground distance*
X and P	5 cm	10 m
X and F	7 cm	? m
X and T	? cm	? m
Y and F	? cm	19 m
F and T	12 cm	? m

For Exercises 2–4 use a centimeter ruler. If you don't have a centimeter ruler, you may use the centimeter ruler shown below as a guide. Either open your compass to the appropriate distance or mark the appropriate distance on the edge of a sheet of paper.



2. Copy the points F, T, and P from the diagram on page 2. If you lay your paper over the page, you can see through the paper well enough to get the points.

a. Draw a line to indicate all points equidistant from F and T.

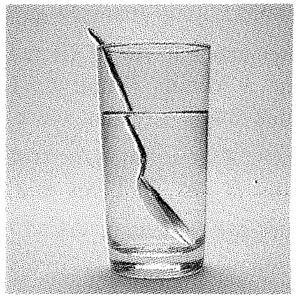
b. Draw a circle to indicate points 6 cm from P. If you don't have a compass, draw as well as you can freehand.

c. How many points are equidistant from F and T, and are also 6 cm from P?

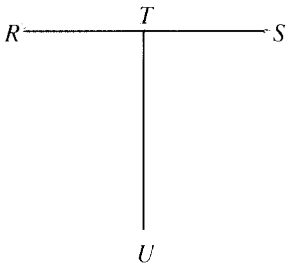
3. Repeat Exercise 2, but use 2 cm instead of 6 cm.

4. There is a distance you could use in parts (b) and (c) of Exercise 2 that would lead to the answer *one point* in part (c). Estimate that distance.

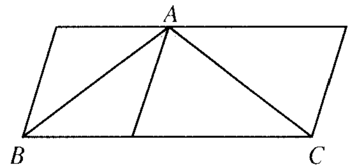
The spoon in the photograph appears to be broken because light rays bend as they go from air to water. As in the photograph, your eyes may mislead you in some of the exercises that follow, but you are asked to make estimates. You may want to check your estimates by measuring.



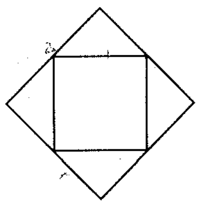
5. Which is greater, the distance from R to S or the distance from T to U?



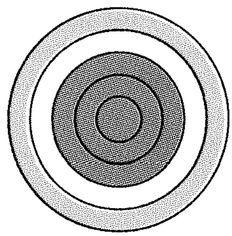
6. Which is greater, the distance from A to B or the distance from A to C?



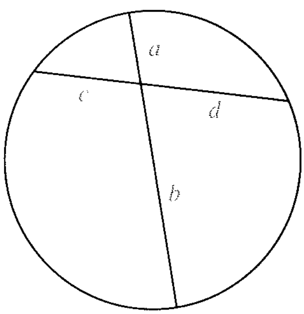
B **7.** How does the area of the outer square compare with the area of the inner square?



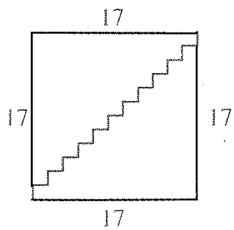
8. Compare the areas of the red and blue regions. (Area of circle $= \pi r^2$.)



9. In the diagram a, b, c, and d are lengths. Which is greater, the product ab or the product cd?

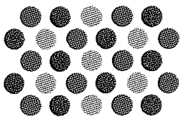


10. A path between opposite vertices of the square is made up of hundreds of horizontal and vertical segments. (The diagram shows a simplified version.) What is the best approximation to the length of the path—24, 34, 44, or more than 44?

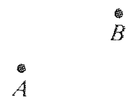


1-2 Points, Lines, and Planes

When you look at a color television picture, how many different colors do you see? Actually, the picture is made up of just three colors—red, green, and blue. Most color television screens are covered with more than 300,000 colored dots, as shown in the enlarged diagram below. Each dot glows when it is struck by an electron beam. Since the dots are so small, and so close together, your eye sees a whole image rather than individual dots.

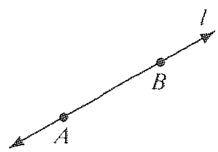


Each dot on a television screen suggests the simplest figure studied in geometry—a *point*. Although a point doesn't have any size, it is often represented by a dot that does have some size. You usually name points by capital letters. Points A and B are pictured at the right.



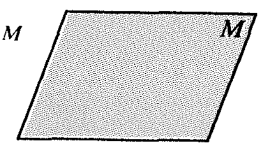
All geometric figures consist of points. One familiar geometric figure is a *line*, which extends in two directions without ending. Although a picture of a line has some thickness, the line itself has no thickness.

Often a line is referred to by a single lower-case letter, such as *line l*. If you know that a line contains the points A and B, you can also call it *line AB* (denoted $\overleftrightarrow{AB}$) or *line BA* ($\overleftrightarrow{BA}$).

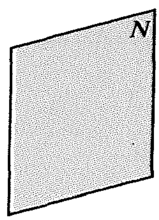


A geometric *plane* is suggested by a floor, wall, or table top. Unlike a table top, a plane extends without ending and has no thickness. Although a plane has no edges, we usually picture a plane by drawing a four-sided figure as shown below. We often label a plane with a capital letter.

Plane M



Plane N



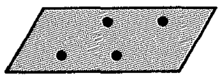
In geometry, the terms *point*, *line*, and *plane* are accepted as intuitive ideas and are not defined. These *undefined terms* are then used in the definitions of other terms, such as those at the top of the next page.

Space is the set of all points. **Collinear points** are points all in one line.

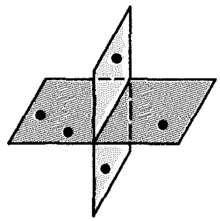
Collinear points

Noncollinear points

Coplanar points are points all in one plane.



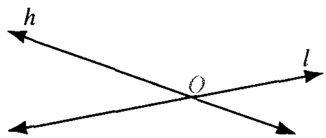
Coplanar points



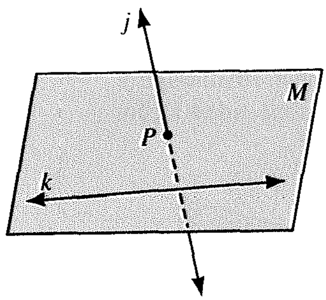
Noncoplanar points

Some expressions commonly used to describe relationships between points, lines, and planes follow. In these expressions, *intersects* means "meets" or "cuts." The **intersection** of two figures is the set of points that are in both figures. Dashes in the diagrams indicate parts hidden from view in figures in space.

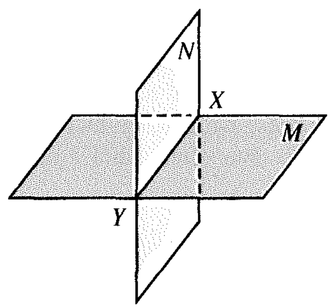
A is in l, or A is on l.
l contains A.
l passes through A.



l and h intersect in O.
l and h intersect at O.
O is the intersection of l and h.



k and P are in M.
M contains k and P.
j intersects M at P.
P is the intersection of j and M.



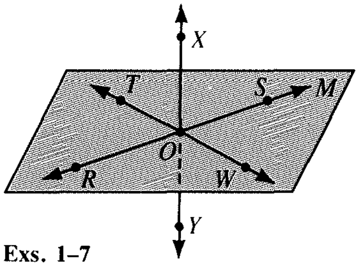
M and N intersect in $\overleftrightarrow{XY}$.
$\overleftrightarrow{XY}$ is the intersection of M and N.
$\overleftrightarrow{XY}$ is in M and N.
M and N contain $\overleftrightarrow{XY}$.

In this book, whenever we refer, for example, to "two points" or "three lines," we will mean *different* points or lines (or other geometric figures).

Classroom Exercises

Classify each statement as true or false.

1. $\overleftrightarrow{XY}$ intersects plane M at point O.
2. Plane M intersects $\overleftrightarrow{XY}$ in more than one point.
3. T, O, and R are collinear.
4. X, O, and Y are collinear.
5. R, O, S, and W are coplanar.
6. R, S, T, and X are coplanar.
7. R, X, O, and Y are coplanar.

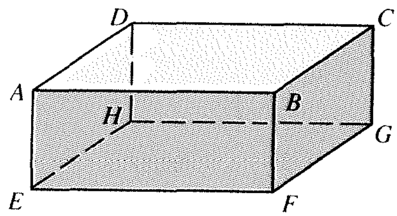


Exs. 1–7

8. Does a plane have edges?
9. Can a given point be in two lines? in ten lines?
10. Can a given line be in two planes? in ten planes?

Name a fourth point that is in the same plane as the given points.

11. A, B, C 12. E, F, H 13. D, C, H
14. A, D, E 15. B, E, F 16. B, G, C



Exs. 11–20

The plane that contains the top of the box can be called plane *ABCD*.

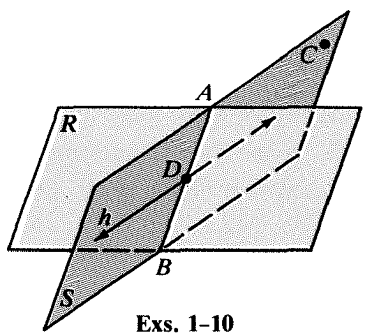
17. Are there any points in $\overleftrightarrow{CG}$ besides C and G?
18. Are there more than four points in plane $ABCD$?
19. Name the intersection of planes $ABFE$ and $BCGF$.
20. Name two planes that do not intersect.

Written Exercises

Classify each statement as true or false.

A

1. $\overleftrightarrow{AB}$ is in plane R. 2. S contains $\overleftrightarrow{AB}$.
3. R and S contain D. 4. D is on line h.
5. h is in S. 6. h is in R.
7. Plane R intersects plane S in $\overleftrightarrow{AB}$.
8. Point C is in R and S.
9. A, B, and C are collinear.
10. A, B, C, and D are coplanar.



Exs. 1–10

11. Make a sketch showing four coplanar points such that three, but not four, of them are collinear.

12. Make a sketch showing four points that are not coplanar.

A plane can be named by three or more noncollinear points it contains. In Chapter 12 you will study *pyramids* like the one shown at the right below.

13. Name five planes that contain sides of the pyramid shown.

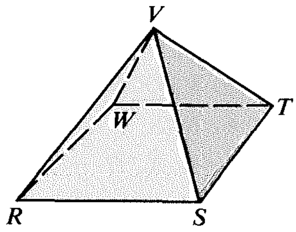
14. Of the five planes containing sides of the pyramid, are there any that do not intersect?

15. Name three lines that intersect at point R.

16. Name two planes that intersect in $\overleftrightarrow{ST}$.

17. Name three planes that intersect at point S.

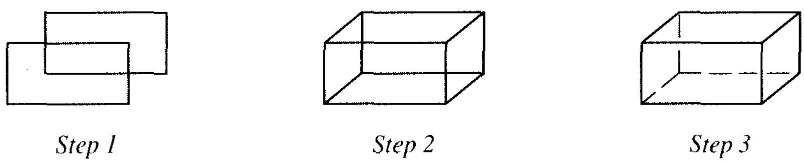
18. Name a line and a plane that intersect in a point.



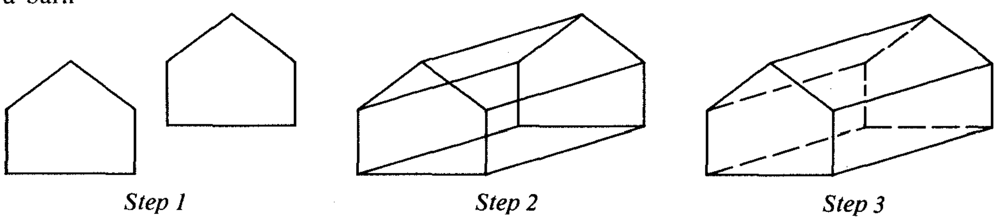
Exs. 13–18

Follow the steps shown to draw the figure named.

19. a rectangular solid or box



20. a barn



Note: After drawing more figures in space, you will probably be able to go directly from Step 1 to Step 3.

21. Name two planes that intersect in $\overleftrightarrow{FG}$.

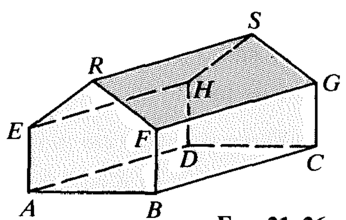
22. Name three lines that intersect at point E.

23. Name three planes that intersect at point B.

24. a. Are points A, D, and C collinear?
b. Are points A, D, and C coplanar?

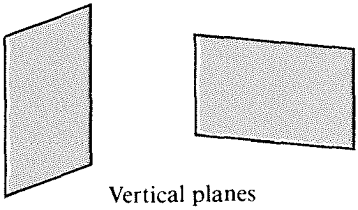
25. a. Are points R, S, G, and F coplanar?
b. Are points R, S, G, and C coplanar?

26. a. Name two planes that do not intersect.
b. Name two other planes that do not intersect.

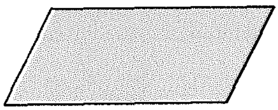


Exs. 21–26

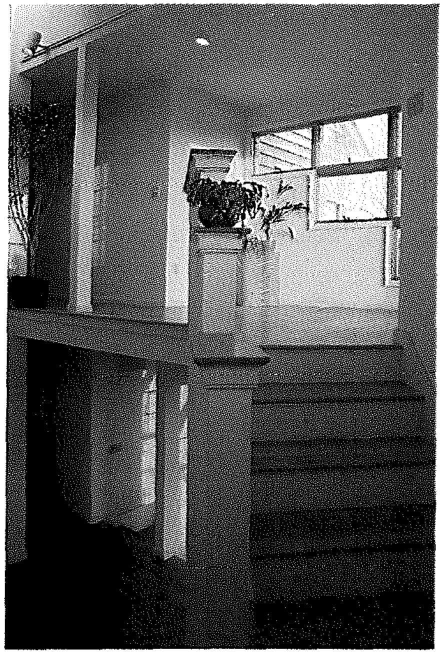
You can think of the ceiling and floor of a room as parts of *horizontal planes*. The walls are parts of *vertical planes*. Vertical planes are represented by figures like those shown in which two sides are vertical. A horizontal plane is represented by a figure like that shown, with two sides horizontal and no sides vertical.



Vertical planes



Horizontal plane



B **27.** Can two horizontal planes intersect?

28. a. Can two vertical planes intersect?

b. Suppose a line is known to be in a vertical plane. Does the line have to be a vertical line?

Sketch and label the figures described. Use dashes for hidden parts.

29. Vertical line l intersects a horizontal plane M at point O.

30. Horizontal plane P contains two lines k and n that intersect at point A.

31. Horizontal plane Q and vertical plane N intersect.

32. Vertical planes X and Y intersect in $\overleftrightarrow{AB}$.

33. Point P is not in plane N. Three lines through point P intersect N in points A, B, and C.

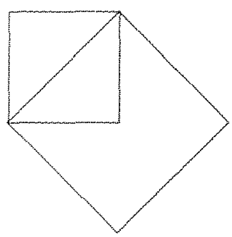
C **34.** Three vertical planes intersect in a line.

35. A vertical plane intersects two horizontal planes in lines l and n.

36. Three planes intersect in a point.

Challenge

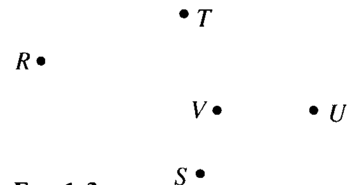
If the area of the red square is 1 square unit, what is the area of the blue square? Give a convincing argument.



Self-Test 1

Name the point that appears to satisfy the description.

1. Equidistant from R and S
2. Equidistant from S and U
3. Equidistant from U and T

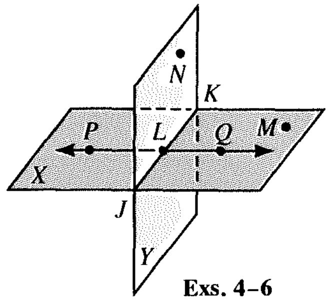


Exs. 1–3

Classify each statement as true or false.

4. Plane Y and $\overleftrightarrow{PQ}$ intersect in point L.
5. Points J, K, L, and N are coplanar.
6. Points J, L, and Q are collinear.

7. Draw a vertical plane Z intersecting a horizontal line l in a point T.



Exs. 4–6

Algebra Review: *Linear Equations*

Find the value of the variable.

1. $c + 5 = 12$
2. $8 + c = 13$
3. $c - 5 = 12$
4. $7 - z = 13$
5. $15 - z = 0$
6. $4x = 28$
7. $3x = 15$
8. $7x = -35$
9. $-5x = -5$
10. $\frac{1}{3}a = 2$
11. $\frac{3}{4}a = 9$
12. $\frac{4}{5}a = -20$
13. $-2b = 6$
14. $-3b = -9$
15. $-9b = 2$
16. $42 = 6k$
17. $5 = 10k$
18. $-16 = -4k$
19. $12 = \frac{e}{2}$
20. $-9 = \frac{e}{3}$
21. $5 = -\frac{e}{3}$
22. $2p + 5 = 13$
23. $3p - 5 = 13$
24. $4p + 2 = 22$
25. $60 = 6t + 12$
26. $12 = 3r - 9$
27. $55 = 7s - 8$
28. $8x + 2x = 90$
29. $8x - 2x = 90$
30. $x + 9x = 5$
31. $(2g - 15) + g = 9$
32. $3u + (u - 2) = 10$
33. $(w - 20) + 5w = 28$
34. $3x = 2x - 17$
35. $5y = 3y + 26$
36. $7z = 180 - 2z$
37. $12 + 3b = 2 + 5b$
38. $4c + 23 = 9c - 7$
39. $7h + (90 - h) = 210$
40. $5x + (180 - x) = 300$
41. $(4f + 5) + (5f + 40) = 180$
42. $(3g - 4) + (4g + 10) = 90$
43. $2(4d + 4) = d + 1$
44. $2(d + 5) = 3(d - 2)$
45. $180 - x = 3(90 - x)$
46. $3(180 - y) = 2(90 - y)$

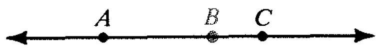
Definitions and Postulates

Objectives

1. Use symbols for lines, segments, rays, and distances; find distances.
2. Name angles and find their measures.
3. State and use the Segment Addition Postulate and the Angle Addition Postulate.
4. Recognize what you can conclude from a diagram.
5. Use postulates and theorems relating points, lines, and planes.

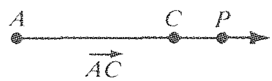
1-3 *Segments, Rays, and Distance*

In the diagram, point B is *between* points A and C. Note that B must lie on $\overleftrightarrow{AC}$.



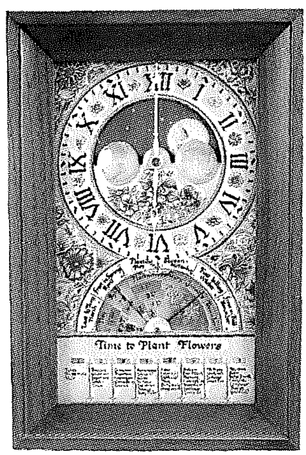
Segment AC, denoted $\overline{AC}$, consists of points A and C and all points that are between A and C. Points A and C are called the *endpoints* of $\overline{AC}$.

Ray AC, denoted $\overrightarrow{AC}$, consists of $\overline{AC}$ and all other points P such that C is between A and P. The *endpoint* of $\overrightarrow{AC}$ is A, the point named first.

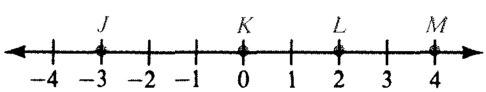


$\overrightarrow{SR}$ and $\overrightarrow{ST}$ are called **opposite rays** if S is between R and T.

The hands of the clock shown suggest opposite rays.



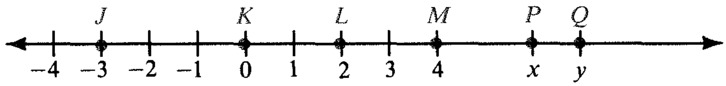
On a *number line* every point is paired with a number and every number is paired with a point. In the diagram, point J is paired with -3, the *coordinate* of J.



The **length** of $\overline{MJ}$, denoted by MJ, is the distance between point M and point J. You can find the length of a segment on a number line by subtracting the coordinates of its endpoints:

$$MJ = 4 - (-3) = 7$$

Notice that since a length must be a positive number, you subtract the lesser coordinate from the greater one. Actually, the distance between two points is the absolute value of the difference of their coordinates. When you use absolute value, the order in which you subtract coordinates doesn't matter.



$JL = |-3 - 2| = |-5| = 5$ $\qquad$ $PQ = |x - y|$

or $\qquad$ or

$JL = |2 - (-3)| = |5| = 5$ $\qquad$ $PQ = |y - x|$

There are many different ways to pair the points on a line with numbers. For example, the red coordinates shown below would give distances in centimeters. The blue coordinates would give distances in inches.

Once you have chosen a unit of measure, the distance between any two points will be the same no matter where you place the coordinate 0. For example, the black coordinates below show another way of assigning coordinates to points on the line so that distances will be measured in inches.

Using number lines involves the following basic assumptions. Statements such as these that are accepted without proof are called **postulates** or **axioms.** Notice that the Ruler Postulate below allows you to measure distances using centimeters or inches or any other convenient unit. But once a unit of measure has been chosen for a particular problem, you must use that unit throughout the problem.

Postulate 1 *Ruler Postulate*

1. The points on a line can be paired with the real numbers in such a way that any two points can have coordinates 0 and 1.
2. Once a coordinate system has been chosen in this way, the distance between any two points equals the absolute value of the difference of their coordinates.

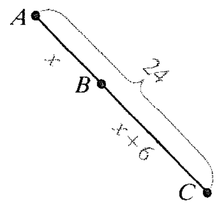
Postulate 2 *Segment Addition Postulate*

If B is between A and C, then

$$AB + BC = AC.$$

Example B is between A and C, with $AB = x$, $BC = x + 6$, and $AC = 24$. Find:

a. the value of x **b.** BC



Solution

a.
$$\begin{aligned} AB + BC &= AC \\ x + (x + 6) &= 24 \\ 2x + 6 &= 24 \\ 2x &= 18 \\ x &= 9 \end{aligned}$$

b.
$$\begin{aligned} BC &= x + 6 \\ &= 9 + 6 \\ &= 15 \end{aligned}$$

In geometry two objects that have the same size and shape are called **congruent**. For many geometric figures we can give a more precise definition of what it means to be congruent. For example, we will define congruent segments in this section, congruent angles in the next section, congruent triangles in Chapter 4, and congruent circles and arcs in Chapter 9.

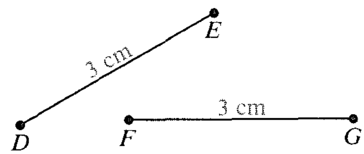
Congruent segments are segments that have equal lengths. To indicate that $\overline{DE}$ and $\overline{FG}$ have equal lengths, you write

$$DE = FG.$$

To indicate that $\overline{DE}$ and $\overline{FG}$ are congruent, you write

$$\overline{DE} \cong \overline{FG}$$

(read "$\overline{DE}$ is congruent to $\overline{FG}$"). The definition tells us that the two statements are equivalent. We will use them interchangeably.



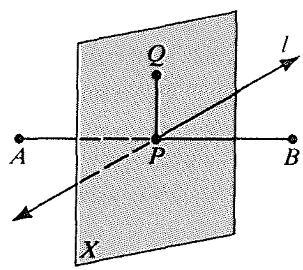
The **midpoint of a segment** is the point that divides the segment into two congruent segments. In the diagram

$$AP = PB,$$

$$\overline{AP} \cong \overline{PB},$$

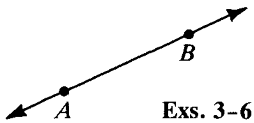
and P is the midpoint of $\overline{AB}$.

A **bisector of a segment** is a line, segment, ray, or plane that intersects the segment at its midpoint. Line l is a bisector of $\overline{AB}$. $\overline{PQ}$ and plane X also bisect $\overline{AB}$.



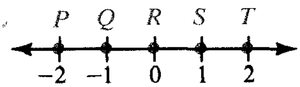
Classroom Exercises

1. Does the symbol represent a line, segment, ray, or length?
   a. $\overline{PQ}$ **b.** $\overrightarrow{PQ}$ **c.** $\overleftrightarrow{PQ}$ **d.** PQ
2. How many endpoints does a segment have? a ray? a line?
3. Is $\overline{AB}$ the same as $\overline{BA}$?
4. Is $\overrightarrow{AB}$ the same as $\overrightarrow{BA}$?
5. Is $\overleftrightarrow{AB}$ the same as $\overleftrightarrow{BA}$?
6. Is AB the same as BA?



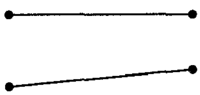
Exs. 3–6

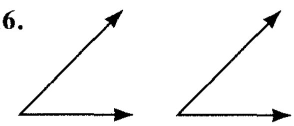
7. What is the coordinate of P? of R?
8. Name the point with coordinate 2.
9. Find each distance: **a.** RS **b.** RQ **c.** PT
10. Name three segments congruent to $\overline{PQ}$.
11. Name the ray opposite to $\overrightarrow{SP}$.
12. Name the midpoint of $\overline{PT}$.
13. **a.** What number is halfway between 1 and 2?
    b. What is the coordinate of the midpoint of $\overline{ST}$?
14. **a.** Could you list all the numbers between 1 and 2?
    b. Is there a point on the number line for every number between 1 and 2?
    c. Is there any limit to the number of points between S and T?

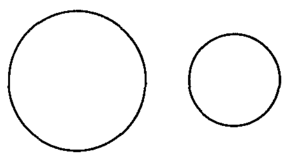


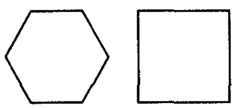
Exs. 7–14

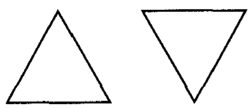
State whether the figures *appear* to be congruent (that is, appear to have the same size and shape).

15. 

16. 

17. 

18. 

19. 

20. 

21. Draw two points P and Q on a sheet of paper. Fold the paper so that fold line f contains both P and Q. Unfold the paper. Now fold so that P falls on Q. Call the second fold g. Lay the paper flat and label the intersection of f and g as point X. How are points P, Q, and X related? Explain.
22. If $AB = BC$, must point B be the midpoint of $\overline{AC}$? Explain.

The given numbers are the coordinates of two points on a number line. State the distance between the points.

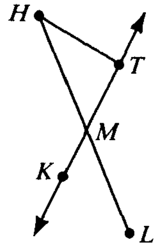
23. -2 and 6 **24.** -2 and -6 **25.** 2 and -6 **26.** 7 and -1

Written Exercises

The numbers given are the coordinates of two points on a number line. State the distance between the points.

A **1.** −6 and 9 **2.** −3 and −17 **3.** −1.2 and −5.7 **4.** −2.5 and 4.6

In the diagram, $\overline{HL}$ and $\overleftrightarrow{KT}$ intersect at the midpoint of $\overline{HL}$. Classify each statement as true or false.



Exs. 5–18

5. $\overline{LM} \cong \overline{MH}$
6. KM must equal MT.
7. $\overline{MT}$ bisects $\overline{LH}$.
8. $\overleftrightarrow{KT}$ is a bisector of $\overline{LH}$.
9. $\overrightarrow{MT}$ and $\overrightarrow{TM}$ are opposite rays.
10. $\overrightarrow{MT}$ and $\overrightarrow{MK}$ are opposite rays.
11. $\overleftrightarrow{LH}$ is the same as $\overline{HL}$.
12. $\overrightarrow{KT}$ is the same as $\overrightarrow{KM}$.
13. $\overleftrightarrow{KT}$ is the same as $\overleftrightarrow{KM}$.
14. $\overline{KT}$ is the same as $\overline{KM}$.
15. $HM + ML = HL$
16. $TM + MH = TH$
17. T is between H and M.
18. M is between K and T.

Name each of the following.

19. The point on $\overrightarrow{DA}$ whose distance from D is 2
20. The point on $\overrightarrow{DG}$ whose distance from D is 2
21. Two points whose distance from E is 2
22. The ray opposite to $\overrightarrow{BE}$
23. The midpoint of $\overline{BF}$
24. The coordinate of the midpoint of $\overline{BD}$
25. The coordinate of the midpoint of $\overline{AE}$
26. A segment congruent to $\overline{AF}$

Exs. 19–26

In Exercises 27–30 draw $\overline{CD}$ and $\overline{RS}$ so that the conditions are satisfied.

27. $\overline{CD}$ and $\overline{RS}$ intersect, but neither segment bisects the other.
28. $\overline{CD}$ and $\overline{RS}$ bisect each other.
29. $\overline{CD}$ bisects $\overline{RS}$, but $\overline{RS}$ does not bisect $\overline{CD}$.
30. $\overline{CD}$ and $\overline{RS}$ do not intersect, but $\overrightarrow{CD}$ and $\overrightarrow{RS}$ do intersect.

B **31.** In the diagram, $\overline{PR} \cong \overline{RT}$, S is the midpoint of $\overline{RT}$, $QR = 4$, and $ST = 5$. Complete.
a. RS = __?__ **b.** RT = __?__
c. PR = __?__ **d.** PQ = __?__

32. In the diagram, X is the midpoint of $\overline{VZ}$, $VW = 5$, and $VY = 20$. Find the coordinates of W, X, and Y.

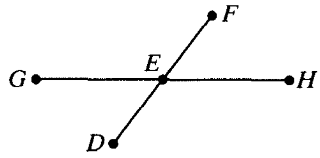
E is the midpoint of $\overline{DF}$. Find the value of x.

33. $DE = 5x + 3$, $EF = 33$

34. $DE = 45$, $EF = 5x - 10$

35. $DE = 3x$, $EF = x + 6$

36. $DE = 2x - 3$, $EF = 5x - 24$



Exs. 33–40

Find the value of y.

37. $GE = y$, $EH = y - 1$, $GH = 11$

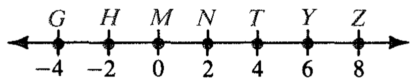
38. $GE = 3y$, $GH = 7y - 4$, $EH = 24$

Find the value of z. Then find GE and EH and state whether E is the midpoint of $\overline{GH}$.

39. $GE = z + 2$, $GH = 20$, $EH = 2z - 6$

40. $GH = z + 6$, $EH = 2z - 4$, $GE = z$

Name the graph of the given equation or inequality.



Exs. 41–45

Example **a.** $x \geq 2$ **b.** $4 \leq x \leq 6$

Solution **a.** $\overrightarrow{NT}$ **b.** $\overline{TY}$

41. $-2 \leq x \leq 2$ **42.** $x \leq 0$ **43.** $|x| \leq 4$ **44.** $|x| \geq 0$ **45.** $|x| = 0$

In Exercises 46 and 47 draw a diagram to illustrate your answer.

46. a. On $\overrightarrow{AB}$, how many points are there whose distance from point A is 3 cm?

b. On $\overleftrightarrow{AB}$, how many points are there whose distance from point A is 3 cm?

C **47.** On $\overrightarrow{AB}$, how many points are there whose distance from point B is 3 cm?

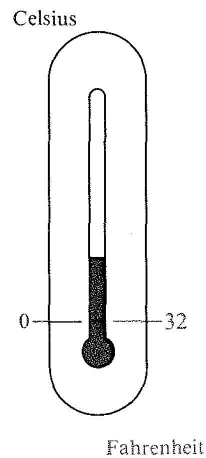
48. The Ruler Postulate suggests that there are many ways to assign coordinates to a line. The Fahrenheit and Celsius temperature scales on a thermometer indicate two such ways of assigning coordinates. A Fahrenheit temperature of 32° corresponds to a Celsius temperature of 0°. The formula, or rule, for converting a Fahrenheit temperature F into a Celsius temperature C is

$$C = \frac{5}{9}(F - 32).$$

a. What Celsius temperatures correspond to Fahrenheit temperatures of 212° and 98.6°?

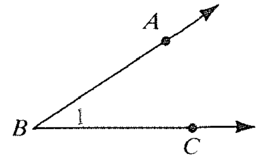
b. Solve the equation above for F to obtain a rule for converting Celsius temperatures to Fahrenheit temperatures.

c. What Fahrenheit temperatures correspond to Celsius temperatures of −40° and 2000°?



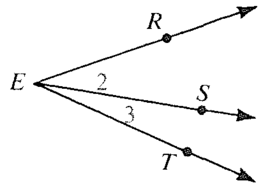
1-4 *Angles*

An **angle** ($\angle$) is the figure formed by two rays that have the same endpoint. The two rays are called the **sides** of the angle, and their common endpoint is the **vertex** of the angle.



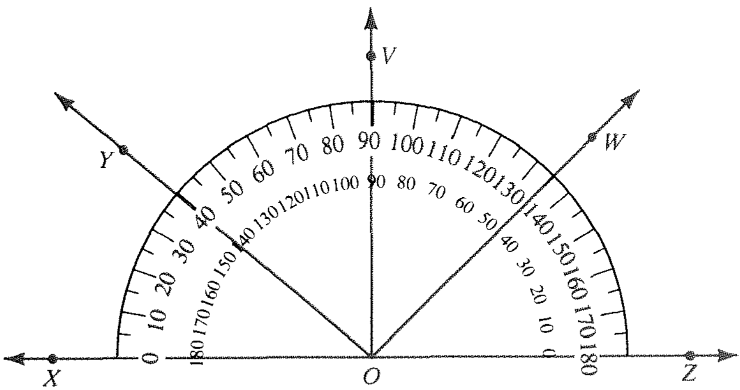
The sides of the angle shown are $\overrightarrow{BA}$ and $\overrightarrow{BC}$. The vertex is point B. The angle can be called $\angle B$, $\angle ABC$, $\angle CBA$, or $\angle 1$. If three letters are used to name an angle, the middle letter must name the vertex.

When you talk about this $\angle B$, everyone knows what angle you mean. But if you tried to talk about $\angle E$ in the diagram at the right, people wouldn't know which angle you meant. There are three angles with vertex E. To name any particular one of them you need to use either three letters or a number.



$\angle 2$ could also be called $\angle RES$ or $\angle SER$.
$\angle 3$ could also be called $\angle SET$ or $\angle TES$.
$\angle RET$ could also be called $\angle TER$.

You can use a protractor like the one shown below to find the *measure in degrees* of an angle. Although angles are sometimes measured in other units, this book will always use degree measure. Using the outer (red) scale of the protractor, you can see that $\angle XOY$ is a 40° angle. You can indicate that the (degree) measure of $\angle XOY$ is 40 by writing $m\angle XOY = 40$.



Using the inner scale of the protractor, you find that:

$m\angle YOZ = 140 \qquad m\angle WOZ = 45 \qquad m\angle YOW = 140 - 45 = 95$

Angles are classified according to their measures.

Acute angle:	Measure between 0 and 90	**Examples:**	$\angle XOY$ and $\angle VOW$
Right angle:	Measure 90	**Examples:**	$\angle XOV$ and $\angle VOZ$
Obtuse angle:	Measure between 90 and 180	**Examples:**	$\angle XOW$ and $\angle YOW$
Straight angle:	Measure 180	**Example:**	$\angle XOZ$

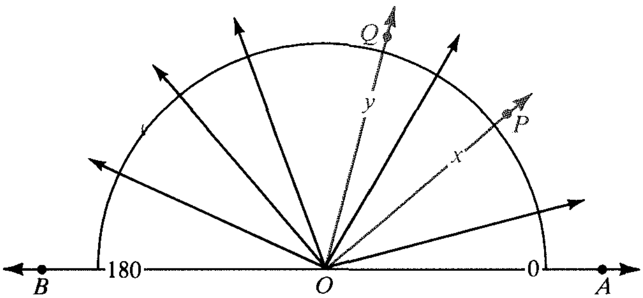
The two angle postulates below are very much like the Ruler Postulate and the Segment Addition Postulate on page 12.

Postulate 3 *Protractor Postulate*

On $\overleftrightarrow{AB}$ in a given plane, choose any point O between A and B. Consider $\overrightarrow{OA}$ and $\overrightarrow{OB}$ and all the rays that can be drawn from O on one side of $\overleftrightarrow{AB}$. These rays can be paired with the real numbers from 0 to 180 in such a way that:

a. $\overrightarrow{OA}$ is paired with 0, and $\overrightarrow{OB}$ with 180.

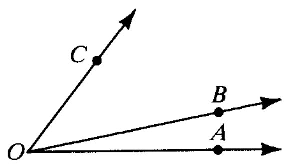
b. If $\overrightarrow{OP}$ is paired with x, and $\overrightarrow{OQ}$ with y, then $m\angle POQ = |x - y|$.



Postulate 4 *Angle Addition Postulate*

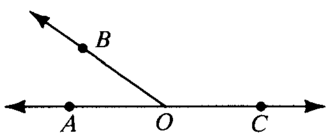
If point B lies in the interior of $\angle AOC$, then

$$m\angle AOB + m\angle BOC = m\angle AOC.$$



If $\angle AOC$ is a straight angle and B is any point not on $\overleftrightarrow{AC}$, then

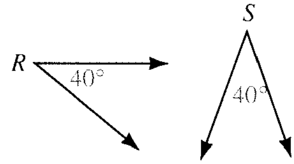
$$m\angle AOB + m\angle BOC = 180.$$



Congruent angles are angles that have equal measures. Since $\angle R$ and $\angle S$ both have measure 40, you can write

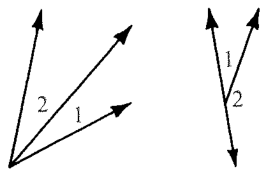
$$m\angle R = m\angle S \text{ or } \angle R \cong \angle S.$$

The definition of congruent angles tells us that these two statements are equivalent. We will use them interchangeably.

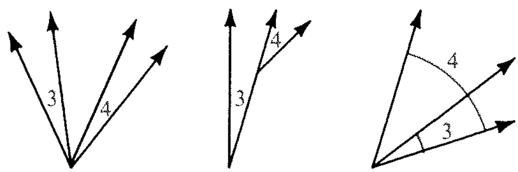


Adjacent angles (adj. ∠s) are two angles in a plane that have a common vertex and a common side but no common interior points.

$\angle 1$ and $\angle 2$ are adjacent angles.



$\angle 3$ and $\angle 4$ are not adjacent angles.

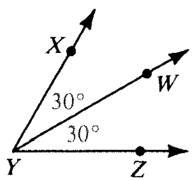


The **bisector of an angle** is the ray that divides the angle into two congruent adjacent angles. In the diagram,

$$m\angle XYW = m\angle WYZ,$$

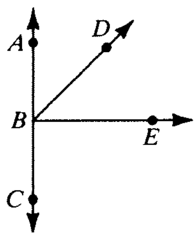
$$\angle XYW \cong \angle WYZ,$$

and $\overrightarrow{YW}$ bisects $\angle XYZ$.

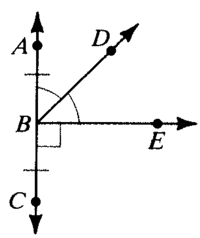


There are certain things that you can conclude from a diagram and others that you can't. The following are things you can conclude from the diagram shown below.

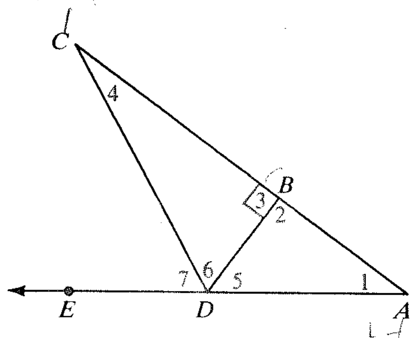
All points shown are coplanar.
$\overleftrightarrow{AB}$, $\overrightarrow{BD}$, and $\overrightarrow{BE}$ intersect at B.
A, B, and C are collinear.
B is between A and C.
$\angle ABC$ is a straight angle.
D is in the interior of $\angle ABE$.
$\angle ABD$ and $\angle DBE$ are adjacent angles.



The diagram above does *not* tell you that $\overline{AB} \cong \overline{BC}$, that $\angle ABD \cong \angle DBE$, or that $\angle CBE$ is a right angle. These three new pieces of information can be indicated in a diagram by using marks as shown at the right. Note that a small square is used to indicate a right angle (rt. ∠).



Classroom Exercises



Exs. 1-16

Name the vertex and the sides of the given angle.

1. $\angle 4$ 2. $\angle 1$ 3. $\angle 6$
4. Name all angles adjacent to $\angle 6$.
5. Name three angles that have B as the vertex.
6. How many angles have D as the vertex?

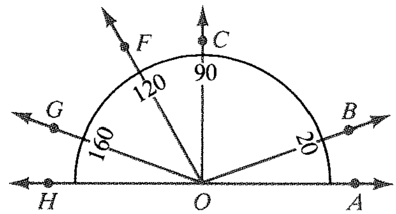
State whether the angle appears to be acute, right, obtuse, or straight. Then estimate its measure.

7. $\angle 1$ 8. $\angle 2$ 9. $\angle EDB$
10. $\angle CDB$ 11. $\angle ADC$ 12. $\angle ADE$

Complete.

13. $m\angle 7 + m\angle 6 = m\angle$ _?_ 14. $m\angle 6 + m\angle 5 = m\angle$ _?_
15. $m\angle 2 + m\angle 3 =$ _?_ 16. If $\overrightarrow{DB}$ bisects $\angle CDA$, then $\angle$ _?_ $\cong \angle$ _?_.

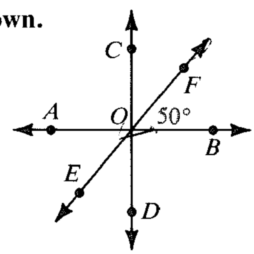
State the measure of each angle.



Exs. 17–25

17. $\angle BOC$ 18. $\angle GOH$
19. $\angle FOG$ 20. $\angle COF$
21. $\angle GOB$ 22. $\angle HOA$
23. Name four angles that are adjacent to $\angle FOG$.
24. What ray bisects which two angles?
25. Name a pair of congruent:
    a. acute angles **b.** right angles **c.** obtuse angles
26. Study a corner of your classroom where two walls and the ceiling meet. How many right angles can you see at the corner?
27. Draw an angle, $\angle AOB$, on a sheet of paper. Fold the paper so that $\overrightarrow{OA}$ falls on $\overrightarrow{OB}$. Lay the paper flat and call the fold line $\overleftrightarrow{OK}$. How is $\overrightarrow{OK}$ related to $\angle AOB$? Explain.

Given the diagram, state whether you can reach the conclusion shown.



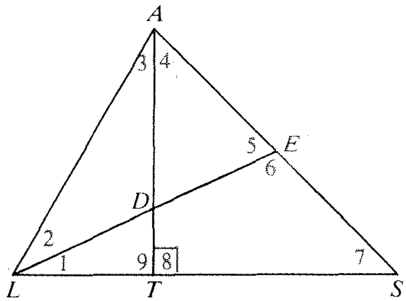
Exs. 28–38

28. $m\angle FOB = 50$ 29. $m\angle AOC = 90$
30. $m\angle DOC = 180$ 31. $AO = OB$
32. $\angle AOC \cong \angle BOC$ 33. $m\angle AOF = 130$
34. Points E, O, and F are collinear.
35. Point C is in the interior of $\angle AOF$.
36. $\angle AOE$ and $\angle AOD$ are adjacent angles.
37. $\angle AOB$ is a straight angle. 38. $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are opposite rays.

Written Exercises

A **1.** Name the vertex and the sides of $\angle 5$.

2. Name all angles adjacent to $\angle ADE$.



Exs. 1-18

State another name for the angle.

3. $\angle 1$ **4.** $\angle 3$ **5.** $\angle 5$

6. $\angle ALD$ **7.** $\angle AST$ **8.** $\angle LES$

State whether the angle appears to be acute, right, obtuse, or straight.

9. $\angle 2$ **10.** $\angle LAS$ **11.** $\angle ATL$

12. $\angle S$ **13.** $\angle LTS$ **14.** $\angle EDT$

Complete.

15. $m\angle 3 + m\angle 4 = m\angle$ _?_

16. $m\angle ALS - m\angle 2 = m\angle$ _?_

17. If $m\angle 1 = m\angle 2$, then _?_ bisects _?_.

18. $m\angle LDA + m\angle ADE =$ _?_

Without measuring, sketch each angle. Then use a protractor to check your accuracy.

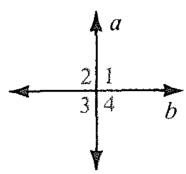
19. 90° angle **20.** 45° angle **21.** 150° angle **22.** 10° angle

Draw a line, $\overleftrightarrow{AB}$. Choose a point O between A and B. Use a protractor to investigate the following questions.

23. In the plane represented by your paper, how many lines can you draw through O that will form a 30° angle with $\overrightarrow{OB}$?

24. In the plane represented by your paper, how many lines can you draw through O that will form a 90° angle with $\overrightarrow{OB}$?

B **25.** Using a ruler, draw a large triangle. Then use a protractor to find the approximate measure of each angle and compute the sum of the three measures. Repeat this exercise for a triangle with a different shape. Did you get the same result?



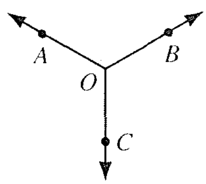
26. Find $m\angle 2$, $m\angle 3$, and $m\angle 4$ when the measure of $\angle 1$ is:
a. 90 **b.** 93

27. Express $m\angle 2$, $m\angle 3$, and $m\angle 4$ in terms of t when $m\angle 1 = t$.

28. A careless person wrote, using the figure shown,

$$m\angle AOB + m\angle BOC = m\angle AOC.$$

What part of the Angle Addition Postulate did that person overlook?



$\overrightarrow{AL}$ bisects $\angle KAT$. Find the value of x.

29. $m\angle 3 = 6x$, $m\angle KAT = 90 - x$

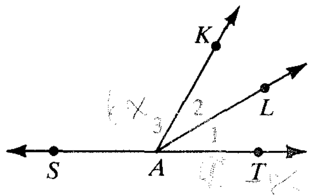
30. $m\angle 1 = 7x + 3$, $m\angle 2 = 6x + 7$

31. $m\angle 1 = 5x - 12$, $m\angle 2 = 3x + 6$

32. $m\angle 1 = x$, $m\angle 3 = 4x$

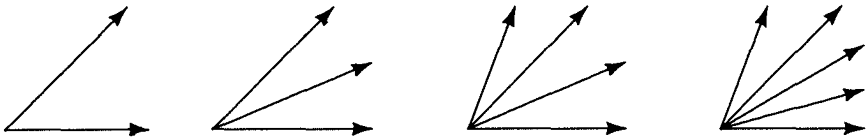
33. $m\angle 1 = 2x - 8$, $m\angle 3 = 116$

34. $m\angle 2 = x + 12$, $m\angle 3 = 6x - 20$



Exs. 29–34

C **35. a.** Complete.



2 rays	3 rays	4 rays	5 rays
1 angle	3 angles	_?_ angles	_?_ angles

b. Study the pattern in the four cases shown, and predict the number of angles formed by six noncollinear rays that have the same endpoint.

c. Which of the expressions below gives the number of angles formed by n noncollinear rays that have the same endpoint?

$n - 1$ $\qquad$ $2n - 3$ $\qquad$ $n^2 - 3$ $\qquad$ $\dfrac{n(n-1)}{2}$

36. $\overrightarrow{OC}$ bisects $\angle AOB$, $\overrightarrow{OD}$ bisects $\angle AOC$, $\overrightarrow{OE}$ bisects $\angle AOD$, $\overrightarrow{OF}$ bisects $\angle AOE$, and $\overrightarrow{OG}$ bisects $\angle FOC$.

a. If $m\angle BOF = 120$, then $m\angle DOE =$ _?_.

b. If $m\angle COG = 35$, then $m\angle EOG =$ _?_.

1-5 *Postulates and Theorems Relating Points, Lines, and Planes*

Recall that we have accepted, without proof, the following four basic assumptions.

The Ruler Postulate	The Segment Addition Postulate
The Protractor Postulate	The Angle Addition Postulate

These postulates deal with segments, lengths, angles, and measures. The following five basic assumptions deal with the way points, lines, and planes are related.

Postulate 5
A line contains at least two points; a plane contains at least three points not all in one line; space contains at least four points not all in one plane.

Postulate 6
Through any two points there is exactly one line.

Postulate 7
Through any three points there is at least one plane, and through any three noncollinear points there is exactly one plane.

Postulate 8
If two points are in a plane, then the line that contains the points is in that plane.

Postulate 9
If two planes intersect, then their intersection is a line.

Important statements that are *proved* are called **theorems.** In Classroom Exercise 1 you will see how Theorem 1-1 follows from the postulates. In Written Exercise 20 you will complete an argument that justifies Theorem 1-2. You will learn about writing proofs in the next chapter.

Theorem 1-1
If two lines intersect, then they intersect in exactly one point.

Theorem 1-2
Through a line and a point not in the line there is exactly one plane.

Theorem 1-3
If two lines intersect, then exactly one plane contains the lines.

The phrase ''exactly one'' appears several times in the postulates and theorems of this section. The phrase ''one and only one'' has the same meaning. For example, here is another correct form of Theorem 1-1:

If two lines intersect, then they intersect in one and only one point.

The theorem states that a point of intersection *exists* (there is *at least one* point of intersection) and the point of intersection is *unique* (*no more than one* such point exists).

Classroom Exercises

1. Theorem 1-1 states that two lines intersect in exactly one point. The diagram suggests what would happen if you tried to show two "lines" drawn through two points. State the postulate that makes this situation impossible.

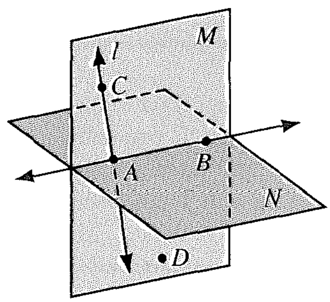
2. State Postulate 6 using the phrase *one and only one*.
3. Reword the following statement as two statements, one describing existence and the other describing uniqueness:

   A segment has exactly one midpoint.

Postulate 6 is sometimes stated as "Two points *determine* a line."

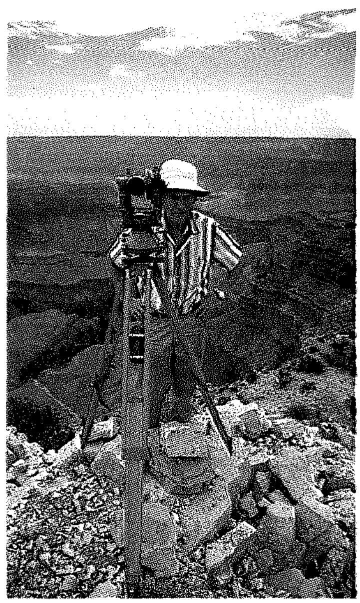
4. Restate Theorem 1-2 using the word *determine*.
5. Do two intersecting lines determine a plane?
6. Do three points determine a line?
7. Do three points determine a plane?

State a postulate, or part of a postulate, that justifies your answer to each exercise.



8. Name two points that determine line l.
9. Name three points that determine plane M.
10. Name the intersection of planes M and N.
11. Does $\overleftrightarrow{AD}$ lie in plane M?
12. Does plane N contain any points not on $\overleftrightarrow{AB}$?

Surveyors and photographers use a *tripod* for support.



13. Why does a three-legged support work better than one with four legs?
14. Explain why a four-legged table may rock even if the floor is level.

15. A carpenter checks to see if a board is warped by laying a straightedge across the board in several directions. State the postulate that is related to this procedure.
16. Think of the intersection of the ceiling and the front wall of your classroom as line l. Let the point in the center of the floor be point C.
    a. Is there a plane that contains line l and point C?
    b. State the theorem that applies.

Written Exercises

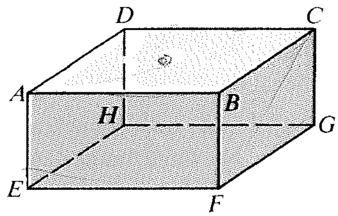
A **1.** State Theorem 1-2 using the phrase *one and only one.*

2. Reword Theorem 1-3 as two statements, one describing existence and the other describing uniqueness.

3. Planes M and N are known to intersect.
a. What kind of figure is the intersection of M and N?
b. State the postulate that supports your answer to part (a).

4. Points A and B are known to lie in a plane.
a. What can you say about $\overleftrightarrow{AB}$?
b. State the postulate that supports your answer to part (a).

In Exercises 5-11 you will have to visualize certain lines and planes not shown in the diagram of the box. When you name a plane, name it by using four points, no three of which are collinear.



Exs. 5-12

5. Write the postulate that assures you that $\overleftrightarrow{AC}$ exists.

6. Name a plane that contains $\overleftrightarrow{AC}$.

7. Name a plane that contains $\overleftrightarrow{AC}$ but that is not shown in the diagram.

8. Name the intersection of plane $DCFE$ and plane $ABCD$.

9. Name four lines shown in the diagram that don't intersect plane $EFGH$.

10. Name two lines that are not shown in the diagram and that don't intersect plane $EFGH$.

11. Name three planes that don't intersect $\overleftrightarrow{EF}$ and don't contain $\overleftrightarrow{EF}$.

12. If you measure $\angle EFG$ with a protractor you get more than 90°. But you know that $\angle EFG$ represents a right angle in a box. Using this as an example, complete the table.

	$\angle EFG$	$\angle AEF$	$\angle DCB$	$\angle FBC$
In the diagram	obtuse	?	?	?
In the box	right	?	?	?

State whether it is possible for the figure described to exist. Write *yes* or *no*.

B **13.** Two points both lie in each of two lines.

14. Three points all lie in each of two planes.

15. Three noncollinear points all lie in each of two planes.

16. Two points lie in a plane X, two other points lie in a different plane Y, and the four points are coplanar but not collinear.

17. Points R, S, and T are noncollinear points.

a. State the postulate that guarantees the existence of a plane X that contains R, S, and T.

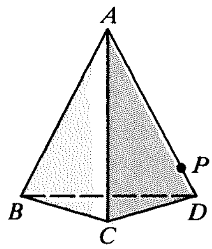
b. Draw a diagram showing plane X containing the noncollinear points R, S, and T.

c. Suppose that P is any point of $\overleftrightarrow{RS}$ other than R and S. Does point P lie in plane X? Explain.

d. State the postulate that guarantees that $\overleftrightarrow{TP}$ exists.

e. State the postulate that guarantees that $\overleftrightarrow{TP}$ is in Plane X.

18. Points A, B, C, and D are four noncoplanar points.



a. State the postulate that guarantees the existence of planes ABC, ABD, ACD, and BCD.

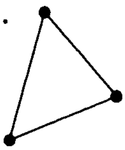
b. Explain how the Ruler Postulate guarantees the existence of a point P between A and D.

c. State the postulate that guarantees the existence of plane BCP.

d. Explain why there are an infinite number of planes through $\overline{BC}$.

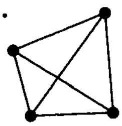
C **19.** State how many segments can be drawn between the points in each figure. No three points are collinear.

a.



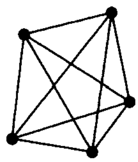
3 points
_?_ segments

b.



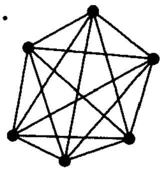
4 points
_?_ segments

c.



5 points
_?_ segments

d.

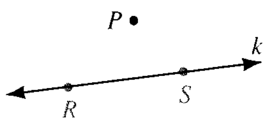


6 points
_?_ segments

e. Without making a drawing, predict how many segments can be drawn between seven points, no three of which are collinear.

f. How many segments can be drawn between n points, no three of which are collinear?

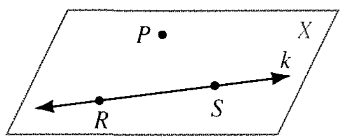
20. Parts (a) through (d) justify Theorem 1-2: Through a line and a point not in the line there is exactly one plane.



a. If P is a point not in line k, what postulate permits us to state that there are two points R and S in line k?

b. Then there is at least one plane X that contains points P, R, and S. Why?

c. What postulate guarantees that plane X contains line k? Now we know that there is a plane X that contains both point P and line k.



d. There can't be another plane that contains point P and line k, because then *two* planes would contain noncollinear points P, R, and S. What postulate does this contradict?

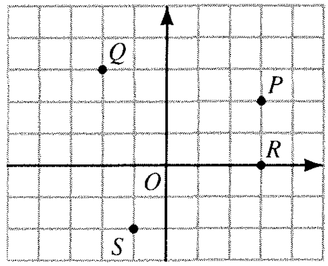
Application — *Locating Points*

Suppose you lived in an area with streets laid out on a grid. If you lived in a house located at point P in the diagram at the right below, you could tell someone where you lived by saying:

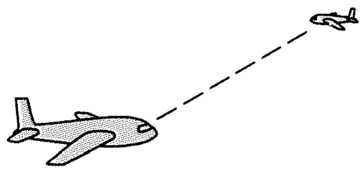
From the crossing at the center of town, go three blocks east and two blocks north.

A friend of yours living at Q might say she lives two blocks west and three blocks north of the town center.

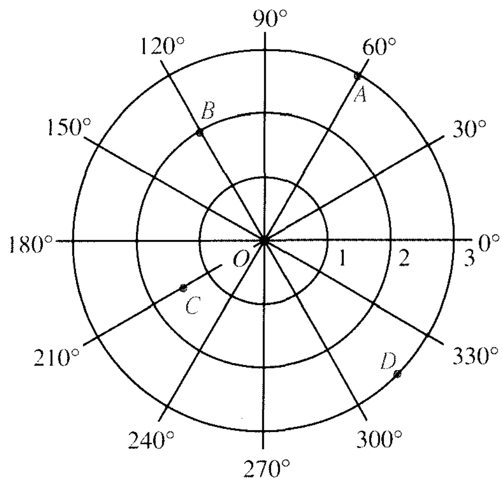
Mathematicians make such descriptions shorter by using a grid system and *coordinates*. They use (3, 2) for your house at point P, and (−2, 3) for your friend's house at Q. Point O at the center of town is (0, 0). Points R and S are (3, 0) and (−1, −2).



This grid system is not always the easiest way to describe a position. If you were a pilot and saw another airplane while flying, it would be difficult to give its position in this system. However, you might say the other plane is 4 km away at 11 o'clock, with 12 o'clock being straight ahead.

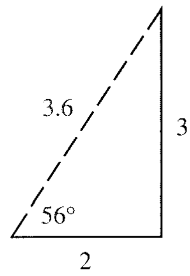


Mathematicians sometimes find it convenient to describe a point by a distance and an angle. Rotation in a clockwise direction is represented by a negative angle. Counterclockwise rotation is represented by a positive angle. A complete rotation, all the way around once, is 360° (or −360°). The labeled points in the diagram at the right are described as shown below.



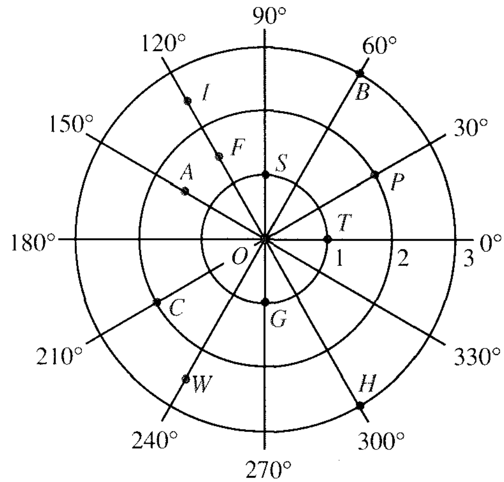
A	(3, 60°)
B	(2, 120°)
C	(1.5, 210°) or (1.5, −150°)
D	(3, 315°) or (3, −45°)

Sometimes you may want to change from one system to the other. For example, if you were at the town center and walked two blocks east and three blocks north, what would your position be in the distance-angle system? Use a centimeter ruler and draw the triangle suggested by your path. If you measure the triangle, you will get about (3.6, 56°).



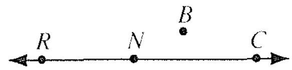
Exercises

1. Copy the grid system shown on the previous page onto a piece of graph paper. Then locate the following points.
   a. A point *T* five blocks due west of the center of town
   b. A point *U* five blocks east and two blocks south of the center of town
   c. A point *V* two blocks west and one block north of your house, which is located at point *P*



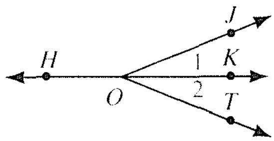
2. Give the letter that names each point.
   a. (2, 30°)
   b. (2.5, 120°)
   c. (1, −90°)
3. Give the distance and angle for each point.
   a. *C*
   b. *A*
   c. *T*
4. Give another way of naming each point.
   a. (1, −120°) **b.** (2, 300°) **c.** (2.5, −180°)
5. A point is given in the grid system. What would it be called in the distance-angle system? (*Hint*: See the discussion at the top of the page. Use a protractor and a centimeter ruler to help you answer the question.)
   a. (3, 4) **b.** (−2, 5)
   c. (4, 0) **d.** (8, −6)
6. A point is given in the distance-angle system. What would it be called, approximately, in the grid system? (*Hint*: Use a protractor and a centimeter ruler to draw the triangle suggested by the angle and distance. Measure the sides of the triangle.)
   a. (2, 50°) **b.** (1.5, −70°)
   c. (3, 90°) **d.** (1, 120°)

Self-Test 2

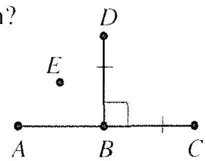


1. Write three names for the line pictured.
2. Name the ray that is opposite to $\overrightarrow{NC}$.
3. Is it correct to say that point B lies between points N and C?
4. When $RN = 7$, $NC = 3x + 5$, and $RC = 18$, what is the value of x?

Complete.



5. $m\angle 1 + m\angle 2 = m\angle$ _?_
6. If $\angle 1 \cong \angle 2$, then _?_ is the bisector of $\angle$ _?_.
7. $m\angle HOK =$ _?_, and $\angle HOK$ is called a(n) _?_ angle.
8. Which of the four things stated *can't* you conclude from the diagram?
   a. A, B, and C are collinear. **b.** $\angle DBC$ is a right angle.
   c. B is the midpoint of $\overline{AC}$. **d.** E is in the interior of $\angle DBA$.



Apply postulates and theorems to complete the statements.

9. Through any two points _?_.
10. If points A and B are in plane Z, _?_.
11. If two planes intersect, then _?_.
12. If there is a line j and a point P not in the line, then _?_.

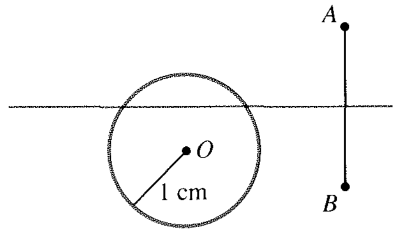
Chapter Summary

1. The concepts of *point*, *line*, and *plane* are basic to geometry. These undefined terms are used in the definitions of other terms.
2. $\overleftrightarrow{AB}$ represents a line, $\overline{AB}$ a segment, and $\overrightarrow{AB}$ a ray. AB represents the length of $\overline{AB}$; AB is a positive number.
3. Two rays with the same endpoint form an angle.
4. Congruent segments have equal lengths. Congruent angles have equal measures.
5. Angles are classified as acute, right, obtuse, or straight, according to their measures.
6. Diagrams enable you to reach certain conclusions. However, judgments about segment length and angle measure must not be made on the basis of appearances alone.
7. Statements that are accepted without proof are called postulates. Statements that are proved are called theorems.
8. Postulates and theorems in this chapter deal with distances, angle measures, points, lines, and planes.

Chapter Review

In Exercises 1–4 answer on the basis of what appears to be true.

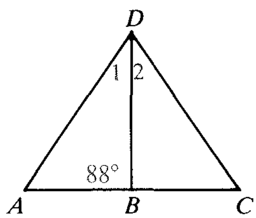
1. How many blue points are 1 cm from point O? 1–1
2. How many red points are 1 cm from O?
3. How many red points are 2 cm from O?
4. Each red point is said to be __?__ from points A and B.



Sketch and label the figures described.

5. Points A, B, C, and D are coplanar, but A, B, and C are the only three of those points that are collinear. 1–2
6. Line l intersects plane X in point P.
7. Plane M contains intersecting lines j and k.
8. Planes X and Y intersect in $\overleftrightarrow{AB}$.

9. Name a point on $\overrightarrow{ST}$ that is not on $\overline{ST}$. 1–3
10. Complete: RS = __?__ and ST = __?__
11. Complete: $\overline{RS}$ and $\overline{ST}$ are called __?__ segments.
12. If U is the midpoint of $\overline{TV}$, find the value of x.



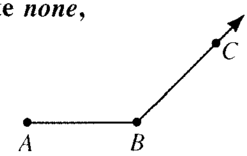
13. Name three angles that have vertex D. Which angles with vertex D are adjacent angles? 1–4
14. **a.** $m\angle CBD$ = __?__
    b. Name the postulate that justifies your answer in part (a).
15. What kind of angle is $\angle CBD$?
16. $\overrightarrow{DB}$ bisects $\angle ADC$, $m\angle 1 = 5x - 3$, and $m\angle 2 = x + 25$. Find the value of x.

Classify each statement as true or false.

17. It is possible to locate three points in such a position that an unlimited number of planes contain all three points. 1–5
18. It is possible for two intersecting lines to be noncoplanar.
19. Through any three points there is at least one line.
20. If points A and B lie in plane P, then so does any point of $\overrightarrow{AB}$.

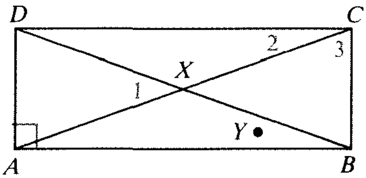
Chapter Test

State how many points meet the requirements. For each answer write *none*, *one*, or *an unlimited number*.



1. Equidistant from points A and B
2. On $\overrightarrow{BC}$ and equidistant from points A and B

Given the diagram, tell whether you can reach the conclusion shown.



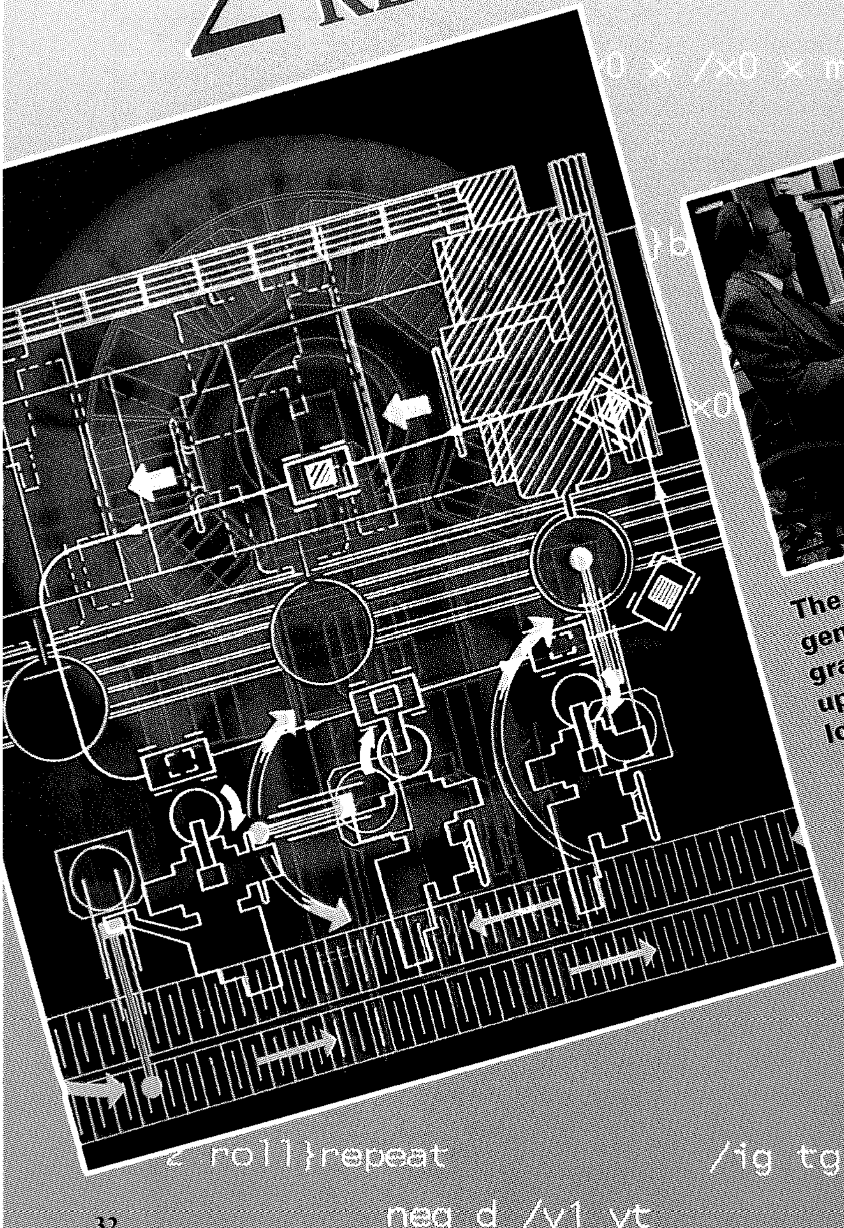
Exs. 3–15

3. $\angle AXC$ is a straight angle.
4. Point Y lies in the interior of $\angle 3$.
5. $\angle ADC$ is a right angle.
6. X is the midpoint of $\overline{AC}$.
7. Point Y lies between points A and B.
8. Name three collinear points.
9. Name the intersection of $\overrightarrow{CX}$ and $\overleftrightarrow{AB}$.
10. Which postulate justifies the statement $AX + XC = AC$?
11. If $\overline{AC}$ bisects $\overline{BD}$, name two congruent segments.
12. Name the vertex and sides of $\angle 1$.
13. Name a right angle.
14. If $m\angle 1 = 46$, find $m\angle DXC$ and $m\angle CXB$.
15. If $m\angle DAX = 70$, find the measure of $\angle XAB$.

Exercises 16–20 refer to a number line that is not pictured here. Point A has coordinate 2 and point B has coordinate 5.

16. What is the length of $\overline{AB}$?
17. What is the coordinate of the midpoint of $\overline{AB}$?
18. If A is the midpoint of $\overline{PB}$, what is the coordinate of P?
19. What is the coordinate of a point that is on $\overrightarrow{AB}$ and is 4 units from B?
20. What is the coordinate of a point that is 4 units from B, but is not on $\overrightarrow{AB}$?
21. Is it possible for a line and a point to be noncoplanar?
22. Is it possible for the intersection of two planes to consist of a segment?
23. Is a postulate an important proved statement, or is it a basic assumption?
24. Complete the statement of the postulate: If two points are in a plane, then _?_.

2 DEDUCTIVE REASONING



The computer program that generates a complex diagram such as this is made up of simple steps linked in logical sequence.

Using Deductive Reasoning

Objectives

1. Recognize the hypothesis and the conclusion of an if-then statement.
2. State the converse of an if-then statement.
3. Use a counterexample to disprove an if-then statement.
4. Understand the meaning of *if and only if*.
5. Use properties from algebra and properties of congruence in proofs.
6. Use the Midpoint Theorem and the Angle Bisector Theorem.
7. Know the kinds of reasons that can be used in proofs.

2-1 *If-Then Statements; Converses*

Your friend says, "If it rains after school, then I will give you a ride home."

A geometry student reads, "If B is between A and C, then AB + BC = AC."

These are examples of **if-then statements,** which are also called **conditional statements** or simply **conditionals.**

To represent an if-then statement symbolically, let p represent the **hypothesis,** shown in red, and let q represent the **conclusion,** shown in blue. Then we have the basic form of an if-then statement shown below:

If p, then q.
↑ ↑
p: **hypothesis** q: **conclusion**

The **converse** of a conditional is formed by interchanging the hypothesis and the conclusion.

Statement: If p, then q. Converse: If q, then p.

A statement and its converse say different things. In fact, some true statements have false converses.

Statement: If Ed lives in Texas, then he lives south of Canada.
False Converse: If Ed lives south of Canada, then he lives in Texas.

An if-then statement is false if an example can be found for which the hypothesis is true and the conclusion is false. Such an example is called a **counterexample.** It takes only one counterexample to disprove a statement. We know the converse above is false because we can find a counterexample: Ed could live in Kansas City, which *is* south of Canada and *is not* in Texas.

Some true statements have true converses.

Statement: If $4x = 20$, then $x = 5$.
True Converse: If $x = 5$, then $4x = 20$.

Conditional statements are not always written with the "if" clause first. Here are some examples. All these conditionals mean the same thing.

General Form	*Example*
If p, then q.	If $x^2 = 25$, then $x < 10$.
p implies q.	$x^2 = 25$ implies $x < 10$.
p only if q.	$x^2 = 25$ only if $x < 10$.
q if p.	$x < 10$ if $x^2 = 25$.

If a conditional and its converse are both true they can be combined into a single statement by using the words "if and only if." A statement that contains the words "if and only if" is called a **biconditional.** Its basic form is shown below.

p if and only if q.

Every definition can be written as a biconditional as the statements below illustrate.

Definition: Congruent segments are segments that have equal lengths.

Biconditional: Segments are congruent if and only if their lengths are equal.

Classroom Exercises

State the hypothesis and the conclusion of each conditional.

1. If $2x - 1 = 5$, then $x = 3$.

2. If she's smart, then I'm a genius.

3. $8y = 40$ implies $y = 5$.

4. $RS = \frac{1}{2}RT$ if S is the midpoint of $\overline{RT}$.

5. $\angle 1 \cong \angle 2$ if $m\angle 1 = m\angle 2$.

6. $\angle 1 \cong \angle 2$ only if $m\angle 1 = m\angle 2$.

7. Combine the conditionals in Exercises 5 and 6 into a single biconditional.

Provide a counterexample to show that each statement is false. You may use words or draw a diagram.

8. If $\overline{AB} \cong \overline{BC}$, then B is the midpoint of $\overline{AC}$.

9. If a line lies in a vertical plane, then the line is vertical.

10. If a number is divisible by 4, then it is divisible by 6.

11. If $x^2 = 49$, then $x = 7$.

State the converse of each conditional. Is the converse true or false?

12. If today is Friday, then tomorrow is Saturday.

13. If $x > 0$, then $x^2 > 0$.

14. If a number is divisible by 6, then it is divisible by 3.

15. If $6x = 18$, then $x = 3$.

16. Give an example of a false conditional whose converse is true.

Written Exercises

Write the hypothesis and the conclusion of each conditional.

A

1. If $3x - 7 = 32$, then $x = 13$.

2. I can't sleep if I'm not tired.

3. I'll try if you will.

4. If $m\angle 1 = 90$, then $\angle 1$ is a right angle.

5. $a + b = a$ implies $b = 0$.

6. $x = -5$ only if $x^2 = 25$.

Rewrite each pair of conditionals as a biconditional.

7. If B is between A and C, then $AB + BC = AC$.
If $AB + BC = AC$, then B is between A and C.

8. If $m\angle AOC = 180$, then $\angle AOC$ is a straight angle.
If $\angle AOC$ is a straight angle, then $m\angle AOC = 180$.

Write each biconditional as two conditionals that are converses of each other.

9. Points are collinear if and only if they all lie in one line.

10. Points lie in one plane if and only if they are coplanar.

Provide a counterexample to show that each statement is false. You may use words or a diagram.

11. If $ab < 0$, then $a < 0$.

12. If $n^2 = 5n$, then $n = 5$.

13. If point G is on $\overrightarrow{AB}$, then G is on $\overrightarrow{BA}$.

14. If $xy > 5y$, then $x > 5$.

15. If a four-sided figure has four right angles, then it has four congruent sides.

16. If a four-sided figure has four congruent sides, then it has four right angles.

Tell whether each statement is true or false. Then write the converse and tell whether it is true or false.

17. If $x = -6$, then $|x| = 6$.

18. If $x^2 = 4$, then $x = -2$.

19. If $b > 4$, then $5b > 20$.

20. If $m\angle T = 40$, then $\angle T$ is not obtuse.

21. If Pam lives in Chicago, then she lives in Illinois.

22. If $\angle A \cong \angle B$, then $m\angle A = m\angle B$.

B

23. $a^2 > 9$ if $a > 3$.

24. $x = 1$ only if $x^2 = x$.

25. $n > 5$ only if $n > 7$.

26. $ab = 0$ implies that $a = 0$ or $b = 0$.

27. If points D, E, and F are collinear, then $DE + EF = DF$.

28. P is the midpoint of $\overline{GH}$ implies that $GH = 2PG$.

29. Write a definition of congruent angles as a biconditional.

30. Write a definition of a right angle as a biconditional.

C

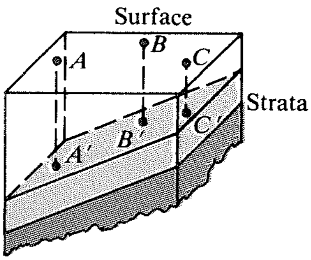
31. What can you conclude if the following sentences are all true?
(1) If p, then q. (2) p (3) If q, then not r. (4) s or r.

Career

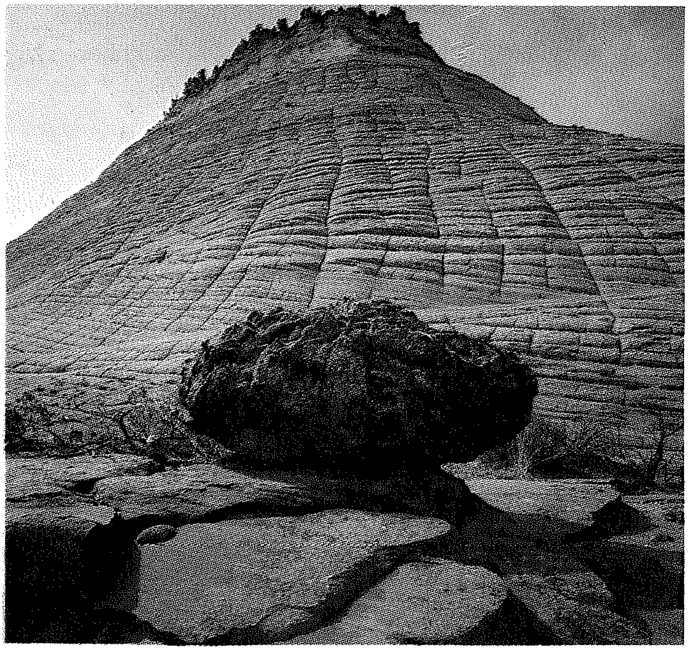
Geologist

Geologists study rock formations like those at Checkerboard Mountain in Zion National Park. Rock formations often occur in *strata*, or layers, beneath the surface of the Earth. Earthquakes occur at *faults*, breaks in the strata. In search of a fault, how would you determine the position of a stratum of rock buried deep beneath the surface of the Earth?

A geologist might start by picking three noncollinear points, A, B, and C, on the surface and drilling holes to find the depths of points A', B', and C' on the stratum. These three points determine the plane of the surface of the stratum.



Geologists may work for industry, searching for oil or minerals. They may work in research centers, developing ways to predict earthquakes.



Today, geologists are trying to locate sources of geothermal energy, energy generated by the Earth's internal heat. A career in geology usually requires knowledge of mathematics, physics, and chemistry, as well as a degree in geology.

Mixed Review Exercises

Complete. You may find that drawing a diagram will help you.

1. If M is the midpoint of $\overline{AB}$, then $\underline{\ ?\ } \cong \underline{\ ?\ }$.
2. If $\overrightarrow{BX}$ is the bisector of $\angle ABC$, then $\underline{\ ?\ } \cong \underline{\ ?\ }$.
3. If point B lies in the interior of $\angle AOC$, then $m\angle \underline{\ ?\ } + m\angle \underline{\ ?\ } = m\angle \underline{\ ?\ }$.
4. If $\angle POQ$ is a straight angle and R is any point not on $\overleftrightarrow{PQ}$, then $m\angle \underline{\ ?\ } + m\angle \underline{\ ?\ } = \underline{\ ?\ }$.

2-2 *Properties from Algebra*

Since the length of a segment is a real number and the measure of an angle is a real number, the facts about real numbers and equality that you learned in algebra can be used in your study of geometry. The properties of equality that will be used most often are listed below.

Properties of Equality

Addition Property	If $a = b$ and $c = d$, then $a + c = b + d$.
Subtraction Property	If $a = b$ and $c = d$, then $a - c = b - d$.
Multiplication Property	If $a = b$, then $ca = cb$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then either a or b may be substituted for the other in any equation (or inequality).
Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.

Recall that $DE = FG$ and $\overline{DE} \cong \overline{FG}$ can be used interchangeably, as can $m\angle D = m\angle E$ and $\angle D \cong \angle E$. Thus the following properties of congruence follow directly from the related properties of equality.

Properties of Congruence

Reflexive Property	$\overline{DE} \cong \overline{DE}$	$\angle D \cong \angle D$
Symmetric Property	If $\overline{DE} \cong \overline{FG}$, then $\overline{FG} \cong \overline{DE}$. If $\angle D \cong \angle E$, then $\angle E \cong \angle D$.	
Transitive Property	If $\overline{DE} \cong \overline{FG}$ and $\overline{FG} \cong \overline{JK}$, then $\overline{DE} \cong \overline{JK}$. If $\angle D \cong \angle E$ and $\angle E \cong \angle F$, then $\angle D \cong \angle F$.	

The properties of equality and other properties from algebra, such as the **Distributive Property,**

$$a(b + c) = ab + ac,$$

can be used to justify your steps when you solve an equation.

Example 1 Solve $3x = 6 - \frac{1}{2}x$ and justify each step.

Solution

Steps	*Reasons*
1. $3x = 6 - \frac{1}{2}x$	1. Given equation
2. $6x = 12 - x$	2. Multiplication Property of Equality
3. $7x = 12$	3. Addition Property of Equality
4. $x = \frac{12}{7}$	4. Division Property of Equality

Example 1 shows a proof of the statement "If $3x = 6 - \frac{1}{2}x$, then x *must* equal $\frac{12}{7}$." In other words, when given the information that $3x = 6 - \frac{1}{2}x$ we can use the properties of algebra to conclude, or *deduce*, that $x = \frac{12}{7}$.

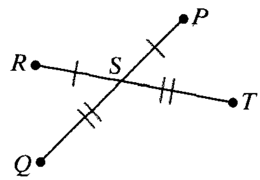
Many proofs in geometry follow this same pattern. We use certain given information along with the properties of algebra and accepted statements, such as the Segment Addition Postulate and Angle Addition Postulate, to show that other statements *must* be true. Often a geometric proof is written in two-column form, with statements on the left and a reason for each statement on the right.

In the following examples, congruent segments are marked alike and congruent angles are marked alike. For example, in the diagram below, the marks show that $\overline{RS} \cong \overline{PS}$ and $\overline{ST} \cong \overline{SQ}$. In the diagram for Example 3 the marks show that $\angle AOC \cong \angle BOD$.

Example 2

Given: $\overline{RT}$ and $\overline{PQ}$ intersecting at S so that $RS = PS$ and $ST = SQ$.

Prove: $RT = PQ$



Proof:

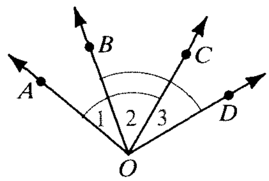
Statements	Reasons
1. $RS = PS$; $ST = SQ$	1. Given
2. $RS + ST = PS + SQ$	2. Addition Prop. of =
3. $RS + ST = RT$; $PS + SQ = PQ$	3. Segment Addition Postulate
4. $RT = PQ$	4. Substitution Prop.

In Steps 1 and 3 of Example 2, notice how statements can be written in pairs when justified by the same reason.

Example 3

Given: $m\angle AOC = m\angle BOD$

Prove: $m\angle 1 = m\angle 3$



Proof:

Statements	Reasons
1. $m\angle AOC = m\angle BOD$	1. Given
2. $m\angle AOC = m\angle 1 + m\angle 2$; $m\angle BOD = m\angle 2 + m\angle 3$	2. Angle Addition Postulate
3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	3. Substitution Prop.
4. $m\angle 2 = m\angle 2$	4. Reflexive Prop.
5. $m\angle 1 = m\angle 3$	5. Subtraction Prop. of =

Notice that the reason given for Step 4 is "Reflexive Property" rather than "Reflexive Property of Equality." Since the reflexive, symmetric, and transitive properties of equality are so closely related to the corresponding properties of congruence, we will simply use "Reflexive Property" to justify either

$$m\angle BOC = m\angle BOC \quad \text{or} \quad \angle BOC \cong \angle BOC.$$

Suppose, in a proof, you have made the statement that

$$m\angle R = m\angle S$$

and also the statement that

$$m\angle S = m\angle T.$$

You can then deduce that $m\angle R = m\angle T$ and use as your reason either "Transitive Property" or "Substitution Property." Similarly, if you know that

(1) $m\angle R = m\angle S$
(2) $m\angle S = m\angle T$
(3) $m\angle T = m\angle V$

you can go on to write (4) $m\angle R = m\angle V$

and use either "Transitive Property" or "Substitution Property" as your reason. Actually, you use the Transitive Property twice or else make a double substitution.

There are times when the Substitution Property is the simplest one to use. If you know that

(1) $m\angle 4 + m\angle 2 + m\angle 5 = 180$
(2) $m\angle 4 = m\angle 1$; $m\angle 5 = m\angle 3$

you can make a double substitution and get

(3) $m\angle 1 + m\angle 2 + m\angle 3 = 180.$

Note that you can't use the Transitive Property here.

Classroom Exercises

Justify each statement with a property from algebra or a property of congruence.

1. $\angle P \cong \angle P$

2. If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

3. If $RS = TW$, then $TW = RS$.

4. If $x + 5 = 16$, then $x = 11$.

5. If $5y = -20$, then $y = -4$.

6. If $\frac{z}{5} = 10$, then $z = 50$.

7. $2(a + b) = 2a + 2b$

8. If $2z - 5 = -3$, then $2z = 2$.

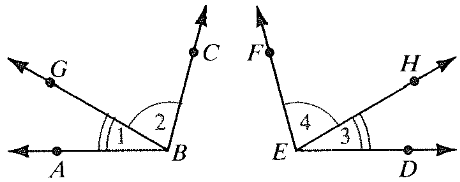
9. If $2x + y = 70$ and $y = 3x$, then $2x + 3x = 70$.

10. If $AB = CD$, $CD = EF$, and $EF = 23$, then $AB = 23$.

Complete each proof by supplying missing reasons and statements.

11. Given: $m\angle 1 = m\angle 3$;
$m\angle 2 = m\angle 4$

Prove: $m\angle ABC = m\angle DEF$

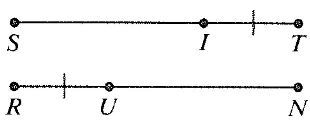


Proof:

Statements	Reasons
1. $m\angle 1 = m\angle 3$; $m\angle 2 = m\angle 4$	1. ?
2. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	2. ?
3. $m\angle 1 + m\angle 2 = m\angle ABC$; $m\angle 3 + m\angle 4 = m\angle DEF$	3. ?
4. $m\angle ABC = m\angle DEF$	4. ?

12. Given: $ST = RN$; $IT = RU$

Prove: $SI = UN$



Proof:

Statements	Reasons
1. $ST = RN$	1. ?
2. ? $= SI + IT$; ? $= RU + UN$	2. ?
3. $SI + IT = RU + UN$	3. ?
4. $IT = RU$	4. ?
5. ?	5. ?

Written Exercises

Justify each step.

A

1.
$$4x - 5 = -2$$
$$4x = 3$$
$$x = \frac{3}{4}$$

2.
$$\frac{3a}{2} = \frac{6}{5}$$
$$3a = \frac{12}{5}$$
$$a = \frac{4}{5}$$

3.
$$\frac{z + 7}{3} = -11$$
$$z + 7 = -33$$
$$z = -40$$

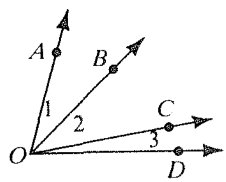
4.
$$15y + 7 = 12 - 20y$$
$$35y + 7 = 12$$
$$35y = 5$$
$$y = \frac{1}{7}$$

5.
$$\frac{2}{3}b = 8 - 2b$$
$$2b = 3(8 - 2b)$$
$$2b = 24 - 6b$$
$$8b = 24$$
$$b = 3$$

6.
$$x - 2 = \frac{2x + 8}{5}$$
$$5(x - 2) = 2x + 8$$
$$5x - 10 = 2x + 8$$
$$3x - 10 = 8$$
$$3x = 18$$
$$x = 6$$

Copy everything shown and supply missing statements and reasons.

7. Given: $\angle AOD$ as shown
Prove: $m\angle AOD = m\angle 1 + m\angle 2 + m\angle 3$



Proof:

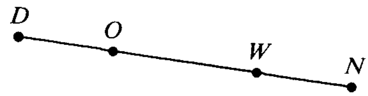
Statements	Reasons
1. $m\angle AOD = m\angle AOC + m\angle 3$	1. ?
2. $m\angle AOC = m\angle 1 + m\angle 2$	2. ?
3. ?	3. ?

8. Given: $FL = AT$
Prove: $FA = LT$

Proof:

Statements	Reasons
1. ?	1. Given
2. $LA = LA$	2. ?
3. $FL + LA = AT + LA$	3. ?
4. $FL + LA = FA$; $LA + AT = LT$	4. ?
5. ?	5. Substitution Prop.

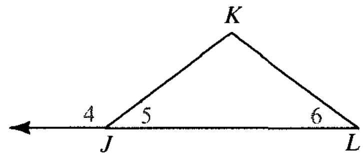
9. Given: $DW = ON$
Prove: $DO = WN$



Proof:

Statements	Reasons
1. $DW = ON$	1. ?
2. $DW = DO + OW$; $ON =$? $+$?	2. ?
3. ?	3. Substitution Prop.
4. $OW = OW$	4. ?
5. ?	5. ?

10. Given: $m\angle 4 + m\angle 6 = 180$
Prove: $m\angle 5 = m\angle 6$



Proof:

Statements	Reasons
1. $m\angle 4 + m\angle 6 = 180$	1. ?
2. $m\angle 4 + m\angle 5 = 180$	2. ?
3. $m\angle 4 + m\angle 5 = m\angle 4 + m\angle 6$	3. ?
4. $m\angle 4 = m\angle 4$	4. ?
5. ?	5. ?

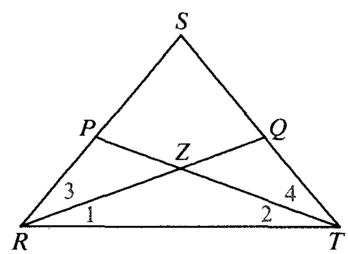
Copy everything shown and write a two-column proof.

B **11.** Given: $m\angle 1 = m\angle 2$; $m\angle 3 = m\angle 4$
Prove: $m\angle SRT = m\angle STR$

12. Given: $RP = TQ$; $PS = QS$
Prove: $RS = TS$

13. Given: $RQ = TP$; $ZQ = ZP$
Prove: $RZ = TZ$

14. Given: $m\angle SRT = m\angle STR$; $m\angle 3 = m\angle 4$
Prove: $m\angle 1 = m\angle 2$



Exs. 11–14

C **15.** Consider the following statements:

Reflexive Property: Robot A is as rusty as itself.

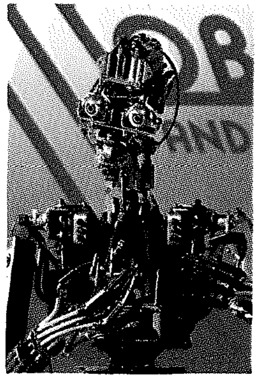
Symmetric Property: If Robot A is as rusty as Robot B, then Robot B is as rusty as Robot A.

Transitive Property: If Robot A is as rusty as Robot B and Robot B is as rusty as Robot C, then Robot A is as rusty as Robot C.

A *relation* such as "is as rusty as" that is reflexive, symmetric, and transitive is an *equivalence relation*. Which of the following are equivalence relations?

a. is rustier than **b.** has the same length as

c. is opposite (for rays) **d.** is coplanar with (for lines)



2-3 *Proving Theorems*

Chapter 1 included three *theorems*, statements that are proved. The theorems were deduced from *postulates*, statements that are accepted without proof. We will prove additional theorems throughout the book. When writing proofs, we will treat properties from algebra as postulates.

Suppose you are told that Y is the midpoint of $\overline{XZ}$ and that $XZ = 12$. You probably realize that $XY = 6$. Your conclusion about one particular situation suggests the general statement shown below as Theorem 2-1. The theorem uses the definition of a midpoint to prove additional properties of a midpoint that are not explicitly included in the definition. In this case, the theorem states something obvious. Later theorems may not be so obvious. In fact, some of them may surprise you.

Theorem 2-1 *Midpoint Theorem*

If M is the midpoint of $\overline{AB}$, then $AM = \frac{1}{2}AB$ and $MB = \frac{1}{2}AB$.

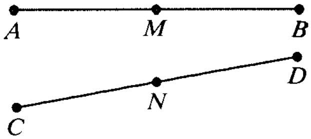
Given: M is the midpoint of $\overline{AB}$.

Prove: $AM = \frac{1}{2}AB$; $MB = \frac{1}{2}AB$

Proof:

Statements	Reasons
1. M is the midpoint of $\overline{AB}$.	1. Given
2. $\overline{AM} \cong \overline{MB}$, or $AM = MB$	2. Definition of midpoint
3. $AM + MB = AB$	3. Segment Addition Postulate
4. $AM + AM = AB$, or $2AM = AB$	4. Substitution Prop. (Steps 2 and 3)
5. $AM = \frac{1}{2}AB$	5. Division Prop. of =
6. $MB = \frac{1}{2}AB$	6. Substitution Prop. (Steps 2 and 5)

Example 1 Given: M is the midpoint of $\overline{AB}$;
N is the midpoint of $\overline{CD}$;
$AB = CD$
What can you deduce?



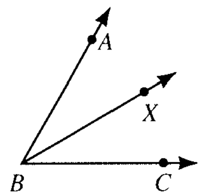
Solution Because M and N are midpoints, you know that $AM = MB$ and $CN = ND$.
From the Midpoint Theorem, you know that $AM = \frac{1}{2}AB$ and $CN = \frac{1}{2}CD$.
Since $AB = CD$, you know that $\frac{1}{2}AB = \frac{1}{2}CD$.
By substitution, you get $AM = CN$.
Thus you can deduce that AM, MB, CN, and ND are all equal.

The next theorem is similar to the Midpoint Theorem. It proves properties of the angle bisector that are not given in the definition. The proof is left as Classroom Exercise 10.

Theorem 2-2 *Angle Bisector Theorem*

If $\overrightarrow{BX}$ is the bisector of $\angle ABC$, then

$$m\angle ABX = \tfrac{1}{2}m\angle ABC \text{ and } m\angle XBC = \tfrac{1}{2}m\angle ABC.$$

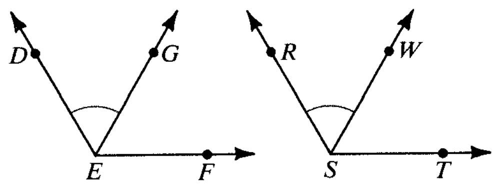


Given: $\overrightarrow{BX}$ is the bisector of $\angle ABC$.
Prove: $m\angle ABX = \frac{1}{2}m\angle ABC$; $m\angle XBC = \frac{1}{2}m\angle ABC$

In addition to postulates and definitions, theorems may be used to justify steps in a proof. Notice the use of the Angle Bisector Theorem in Example 2.

Example 2

Given: $\overrightarrow{EG}$ is the bisector of $\angle DEF$;
$\overrightarrow{SW}$ is the bisector of $\angle RST$;
$m\angle DEG = m\angle RSW$
Prove: $m\angle DEF = m\angle RST$



Proof:

Statements	Reasons
1. $\overrightarrow{EG}$ is the bisector of $\angle DEF$; $\overrightarrow{SW}$ is the bisector of $\angle RST$.	1. Given
2. $m\angle DEG = \frac{1}{2}m\angle DEF$; $m\angle RSW = \frac{1}{2}m\angle RST$	2. Angle Bisector Theorem
3. $m\angle DEG = m\angle RSW$	3. Given
4. $\frac{1}{2}m\angle DEF = \frac{1}{2}m\angle RST$	4. Substitution Prop. (Steps 2 and 3)
5. $m\angle DEF = m\angle RST$	5. Multiplication Prop. of =

The two-column proofs you have seen in this section and the previous one are examples of **deductive reasoning.** We have proved statements by reasoning from postulates, definitions, theorems, and given information. The kinds of reasons you can use to justify statements in a proof are listed below.

Reasons Used in Proofs

Given information

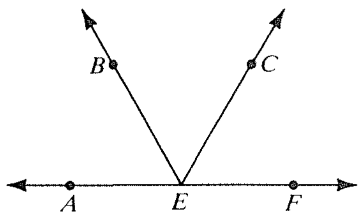
Definitions

Postulates (These include properties from algebra.)

Theorems that have already been proved

Classroom Exercises

What postulate, definition, or theorem justifies the statement about the diagram?



Exs. 1–9

1. $m\angle AEB + m\angle BEC = m\angle AEC$

2. $AE + EF = AF$

3. $m\angle AEB + m\angle BEF = 180$

4. If E is the midpoint of $\overline{AF}$, then $\overline{AE} \cong \overline{EF}$.

5. If E is the midpoint of $\overline{AF}$, then $AE = \frac{1}{2}AF$.

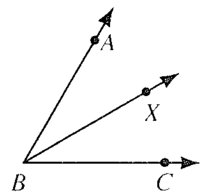
6. If E is the midpoint of $\overline{AF}$, then $\overrightarrow{EC}$ bisects $\overline{AF}$.

7. If $\overrightarrow{EB}$ bisects $\overline{AF}$, then E is the midpoint of $\overline{AF}$.

8. If $\overrightarrow{EB}$ is the bisector of $\angle AEC$, then $m\angle AEB = \frac{1}{2}m\angle AEC$.

9. If $\angle BEC \cong \angle CEF$, then $\overrightarrow{EC}$ is the bisector of $\angle BEF$.

10. Complete the proof of Theorem 2-2.
Given: $\overrightarrow{BX}$ is the bisector of $\angle ABC$.
Prove: $m\angle ABX = \frac{1}{2}m\angle ABC$; $m\angle XBC = \frac{1}{2}m\angle ABC$



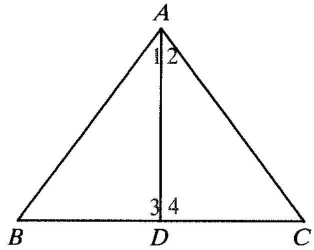
Proof:

Statements	Reasons
1. $\overrightarrow{BX}$ is the bisector of $\angle ABC$.	1. ?
2. $\angle ABX \cong$?, or $m\angle ABX =$?	2. ?
3. $m\angle ABX + m\angle XBC = m\angle ABC$	3. ?
4. $m\angle ABX + m\angle ABX = m\angle ABC$, or $2m\angle ABX = m\angle ABC$	4. ?
5. $m\angle ABX = \frac{1}{2}m\angle ABC$	5. ?
6. $m\angle XBC = \frac{1}{2}m\angle ABC$	6. Substitution Prop. (Steps ? and ?)

Written Exercises

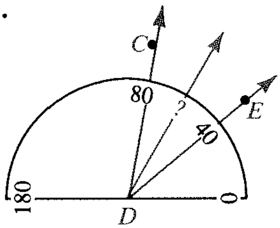
Name the definition, postulate, or theorem that justifies the statement about the diagram.

A
1. If D is the midpoint of $\overline{BC}$, then $\overline{BD} \cong \overline{DC}$.
2. If $\angle 1 \cong \angle 2$, then $\overrightarrow{AD}$ is the bisector of $\angle BAC$.
3. If $\overrightarrow{AD}$ bisects $\angle BAC$, then $\angle 1 \cong \angle 2$.
4. $m\angle 3 + m\angle 4 = 180$
5. If $\overline{BD} \cong \overline{DC}$, then D is the midpoint of $\overline{BC}$.
6. If D is the midpoint of $\overline{BC}$, then $BD = \frac{1}{2}BC$.
7. $m\angle 1 + m\angle 2 = m\angle BAC$
8. $BD + DC = BC$



Exs. 1–8

Write the number that is paired with the bisector of $\angle CDE$.

9.


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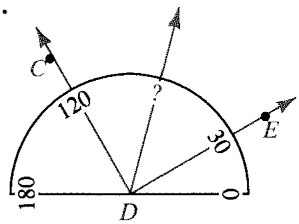
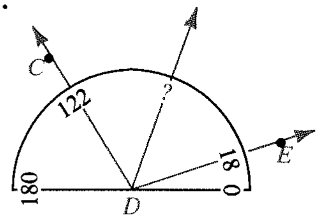
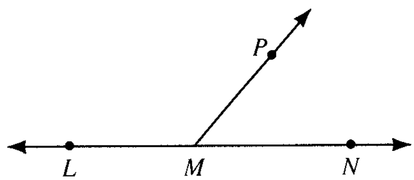


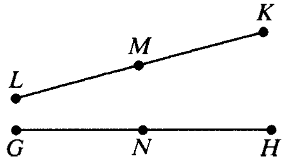
11.


12. **a.** Draw a diagram similar to the one shown.
    b. Use a protractor to draw the bisectors of $\angle LMP$ and $\angle PMN$.
    c. What is the measure of the angle formed by these bisectors?
    d. Explain how you could have known the answer to part (c) without measuring.

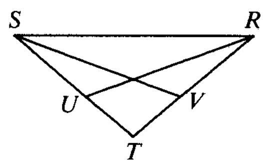


B
13. The coordinates of points L and X are 16 and 40, respectively. N is the midpoint of $\overline{LX}$, and Y is the midpoint of $\overline{LN}$. Sketch a diagram and find:
    a. LN **b.** the coordinate of N **c.** LY **d.** the coordinate of Y
14. $\overrightarrow{SW}$ bisects $\angle RST$ and $m\angle RST = 72$. $\overrightarrow{SZ}$ bisects $\angle RSW$, and $\overrightarrow{SR}$ bisects $\angle NSW$. Sketch a diagram and find $m\angle RSZ$ and $m\angle NSZ$.

15. **a.** Suppose M and N are the midpoints of $\overline{LK}$ and $\overline{GH}$, respectively. What segments are congruent?
    b. What additional information about the figure would enable you to deduce that $LM = NH$?



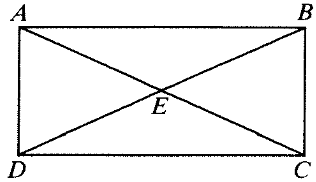
16. **a.** Suppose $\overrightarrow{SV}$ bisects $\angle RST$ and $\overrightarrow{RU}$ bisects $\angle SRT$. What angles are congruent?
    b. What additional information would enable you to deduce that $m\angle VSU = m\angle URV$?



What can you deduce from the given information?

17. Given: $AE = DE$;
$CE = BE$

18. Given: $\overline{AC}$ bisects $\overline{DB}$;
$\overline{DB}$ bisects $\overline{AC}$;
$CE = BE$



19. Copy and complete the following proof of the statement: If points A and B have coordinates a and b, with $b > a$, and the midpoint M of $\overline{AB}$ has coordinate x, then $x = \frac{a + b}{2}$.

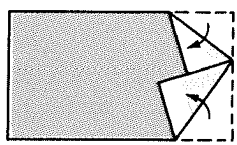
Given: Points A and B have coordinates a and b;
$b > a$; midpoint M of $\overline{AB}$ has coordinate x.

Prove: $x = \frac{a + b}{2}$

Proof:

Statements	Reasons
1. A, M, and B have coordinates a, x, and b respectively; $b > a$	1. ?
2. $AM = x - a$; $MB = b - x$	2. ?
3. M is the midpoint of $\overline{AB}$.	3. ?
4. $\overline{AM} \cong \overline{MB}$, or $AM = MB$	4. ?
5. $x - a = b - x$	5. ?
6. $2x =$?	6. ?
7. $x = \frac{a + b}{2}$	7. ?

C **20.** Fold down a corner of a rectangular sheet of paper. Then fold the next corner so that the edges touch as in the figure. Measure the angle formed by the fold lines. Repeat with another sheet of paper, folding the corner at a different angle. Explain why the angles formed are congruent.



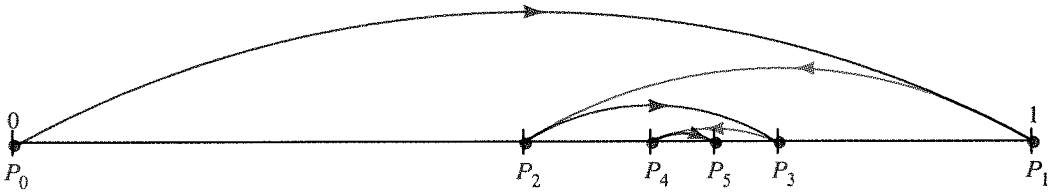
21. M is the midpoint of $\overline{AB}$, Q is the midpoint of $\overline{AM}$, and T is the midpoint of $\overline{QM}$. If the coordinates of A and B are a and b, find the coordinates of Q and T in terms of a and b.

22. Point T is the midpoint of $\overline{RS}$, W is the midpoint of $\overline{RT}$, and Z is the midpoint of $\overline{WS}$. If the length of $\overline{TZ}$ is x, find the following lengths in terms of x. (*Hint*: Sketch a diagram and let $y = WT$.)

a. RW **b.** ZS **c.** RS **d.** WZ

♦ Computer Key-In

A bee starts at point P_0, flies to point P_1, and lands. The bee then returns half of the way to P_0, landing at P_2. From P_2, the bee returns half of the way to P_1, landing at P_3, and so forth. Can you predict the bee's location after 10 trips?



Assuming that P_0 and P_1 have coordinates 0 and 1, respectively, the BASIC program below will compute and print the bee's location at the end of trips 2 through 10. P_n represents the position of the bee after n trips. Since P_n is the midpoint of the bee's previous two positions, P_{n-1} and P_{n-2}, line 50 calculates $P(N)$ by using the statement proved in Exercise 19, page 47.

```
10  DIM P(50)
20  LET P(0) = 0
30  LET P(1) = 1
40  FOR N = 2 TO 10
50  LET P(N) = (1/2) * (P(N - 2) + P(N - 1))
60  PRINT N, P(N)
70  NEXT N
80  END
```

Exercises

1. Enter the program on your computer and RUN it. Do you notice any patterns or trends in the coordinates? Change line 40 so that the computer will print the coordinates up to P_{40}. What simple fraction is approximated by P_{40}?

2. In line 50, $P(n)$ could instead be computed from the *series*

$$1 - \tfrac{1}{2} + \tfrac{1}{4} - \tfrac{1}{8} + \cdots + (-\tfrac{1}{2})^{n-1}$$

where each term of the series reflects the bee's return half of the way from P_{n-1} to P_{n-2}. Replace line 50 with the line below and RUN the new program.

```
50  LET P(N) = P(N - 1) + (-1/2) ↑ (N - 1)
```

Check that both programs produce the same results. (Some slight variations will be expected, due to rounding off.)

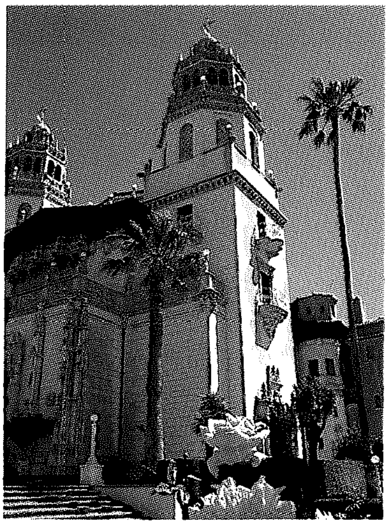
3. Suppose that on each trip the bee returned one third of the way to the previous point instead of half of the way. How would the series in Exercise 2 be modified? How would line 50 of Exercise 2 be modified? RUN a modified program for 30 trips and determine what point the bee seems to be approaching.

Self-Test 1

Use the conditional: If $\overline{AB}$ and $\overline{CD}$ intersect, then $\overrightarrow{AB}$ and $\overrightarrow{CD}$ intersect.

1. Write the hypothesis and the conclusion of the conditional.
2. Write the converse of the conditional. Is the converse true or false?
3. Rewrite the following pair of conditionals as a biconditional:
   $\overline{AB} \cong \overline{CD}$ if $AB = CD$; $\overline{AB} \cong \overline{CD}$ only if $AB = CD$.
4. Provide a counterexample to disprove the statement:
   If $m\angle A$ is less than 100, then $\angle A$ is an acute angle.
5. Given: $m\angle A + m\angle B = 180$; $m\angle C = m\angle B$
   What property of equality justifies the statement $m\angle A + m\angle C = 180$?
6. Point M is the midpoint of $\overline{RT}$. $RM = x$ and $RT = 4x - 6$. Find the value of x.
7. The measure of $\angle ABC$ is 108. $\overrightarrow{BD}$ is the bisector of $\angle ABC$, and $\overrightarrow{BE}$ is the bisector of $\angle ABD$. Find the measure of $\angle EBC$.
8. You can use given information and theorems as reasons in proofs. Name two other kinds of reasons you can use.

Biographical Note *Julia Morgan*



Julia Morgan (1872–1959), the first successful woman architect in the United States, was born in San Francisco. Though best known for her design of San Simeon, the castle-like former home of William Randolph Hearst pictured at the left, she designed numerous public buildings and private homes. Even today, to own "a Julia Morgan house" carries considerable prestige.

To become an architect, Morgan needed great determination as well as a brilliant mind. Since the University of California did not have an architecture curriculum at that time, she prepared for graduate work in Paris by studying civil engineering. In Paris the École des Beaux-Arts, which had just begun to admit foreigners, was particularly reluctant to admit a foreign woman. She persisted, however, and became the school's first woman graduate.

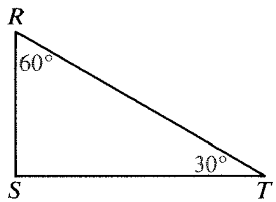
Theorems about Angles and Perpendicular Lines

Objectives

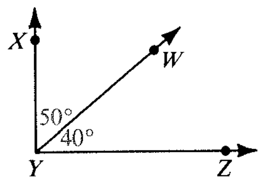
1. Apply the definitions of complementary and supplementary angles.
2. State and use the theorem about vertical angles.
3. Apply the definition and theorems about perpendicular lines.
4. State and apply the theorems about angles supplementary to, or complementary to, congruent angles.
5. Plan proofs and then write them in two-column form.

2-4 *Special Pairs of Angles*

Complementary angles (comp. ∠s) are two angles whose measures have the sum 90. Each angle is called a *complement* of the other.

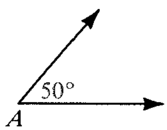


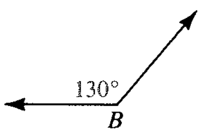
∠*R* and ∠*T* are complementary.



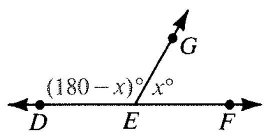
∠*XYW* is a complement of ∠*WYZ*.

Supplementary angles (supp. ∠s) are two angles whose measures have the sum 180. Each angle is called a *supplement* of the other.





∠*A* and ∠*B* are supplementary.



∠*DEG* is a supplement of ∠*GEF*.

Example 1 A supplement of an angle is three times as large as a complement of the angle. Find the measure of the angle.

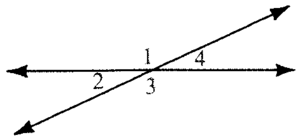
Solution Let x = the measure of the angle.
Then $180 - x$ = the measure of its supplement,
and $90 - x$ = the measure of its complement.

$$180 - x = 3(90 - x)$$
$$180 - x = 270 - 3x$$
$$2x = 90$$
$$x = 45$$

The measure of the angle is 45.

Vertical angles (vert. $\angle s$) are two angles such that the sides of one angle are opposite rays to the sides of the other angle. When two lines intersect, they form two pairs of vertical angles.

$\angle 1$ and $\angle 3$ are vert. $\angle s$. $\angle 2$ and $\angle 4$ are vert. $\angle s$.

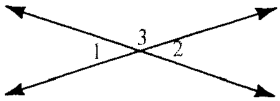


Theorem 2-3

Vertical angles are congruent.

Given: $\angle 1$ and $\angle 2$ are vertical angles.

Prove: $\angle 1 \cong 2$

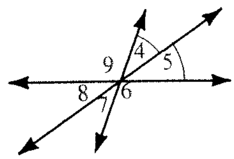


Proof:

Statements	Reasons
1. $m\angle 1 + m\angle 3 = 180$; $m\angle 2 + m\angle 3 = 180$	1. Angle Addition Postulate
2. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	2. Substitution Prop.
3. $m\angle 3 = m\angle 3$	3. Reflexive Prop.
4. $m\angle 1 = m\angle 2$, or $\angle 1 \cong \angle 2$	4. Subtraction Prop. of =

Example 2 In the diagram, $\angle 4 \cong \angle 5$.
Name two other angles congruent to $\angle 5$.

Solution $\angle 8 \cong \angle 5$ since vertical angles are congruent.
Since $\angle 7 \cong \angle 4$ and $\angle 4 \cong \angle 5$, $\angle 7 \cong \angle 5$ by the Transitive Property.



Classroom Exercises

Find the measures of a complement and a supplement of $\angle A$.

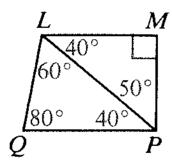
1. $m\angle A = 10$ **2.** $m\angle A = 75$ **3.** $m\angle A = 89$ **4.** $m\angle A = y$

5. Name two right angles.

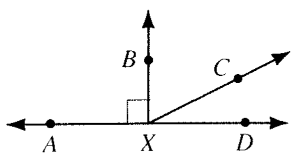
6. Name two adjacent complementary angles.

7. Name two complementary angles that are not adjacent.

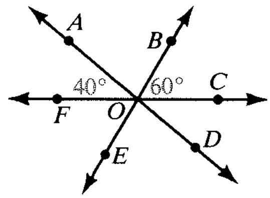
8. a. Name a supplement of $\angle MLQ$.
b. Name another pair of supplementary angles.



9. In the diagram, $m\angle AXB = 90$. Name:
a. two congruent supplementary angles
b. two supplementary angles that are not congruent
c. two complementary angles
d. a straight angle

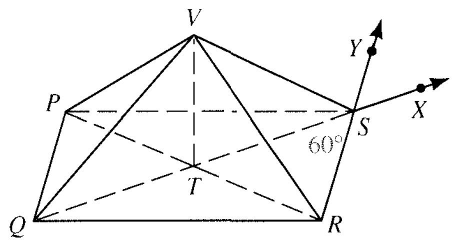


Complete.

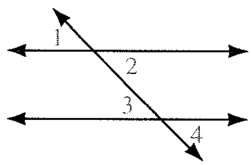


10. $\angle AOB \cong$?
11. $\angle AOE \cong$?
12. $\angle FOB \cong$?
13. $\angle COA \cong$?
14. $m\angle FOE =$?
15. $m\angle COD =$?
16. $m\angle DOB =$?
17. $m\angle AOB =$?
18. $m\angle COE =$?
19. $m\angle FOB =$?

20. The four angles of figure $PQRS$ are right angles. $\angle VTR$ is a right angle. $m\angle QSR = 60$. Find the measures.
 a. $m\angle VTP$ b. $m\angle XSY$
 c. $m\angle RSX$ c. $m\angle PSY$



21. Given: $\angle 2 \cong \angle 3$
 a. What can you deduce?
 b. Explain how you would prove your conclusion.



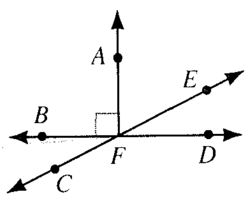
Written Exercises

Find the measures of a complement and a supplement of $\angle K$.

A 1. $m\angle K = 20$ 2. $m\angle K = 72\frac{1}{2}$ 3. $m\angle K = x$ 4. $m\angle K = 2y$

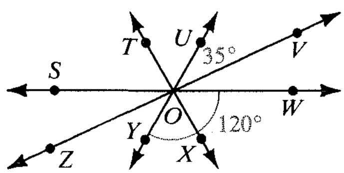
5. Two complementary angles are congruent. Find their measures.
6. Two supplementary angles are congruent. Find their measures.

In the diagram, $\angle AFB$ is a right angle. Name the figures described.



7. Another right angle
8. Two complementary angles
9. Two congruent supplementary angles
10. Two noncongruent supplementary angles
11. Two acute vertical angles
12. Two obtuse vertical angles

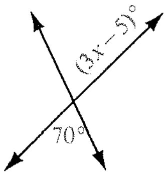
In the diagram, $\overrightarrow{OT}$ bisects $\angle SOU$, $m\angle UOV = 35$, and $m\angle YOW = 120$. Find the measure of each angle.



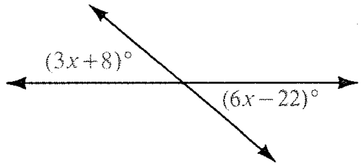
13. $m\angle ZOY$
14. $m\angle ZOW$
15. $m\angle VOW$
16. $m\angle SOU$
17. $m\angle TOU$
18. $m\angle ZOT$

Find the value of x.

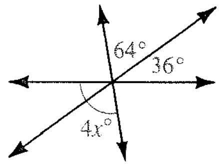
19.



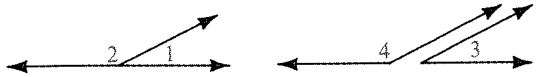
20.



21.



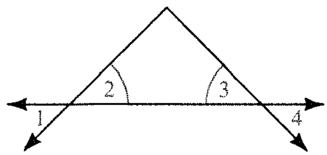
22. $\angle 1$ and $\angle 2$ are supplements.
$\angle 3$ and $\angle 4$ are supplements.



a. If $m\angle 1 = m\angle 3 = 27$, find $m\angle 2$ and $m\angle 4$.
b. If $m\angle 1 = m\angle 3 = x$, find $m\angle 2$ and $m\angle 4$ in terms of x.
c. If two angles are congruent, must their supplements be congruent?

23. Copy everything shown. Complete the proof.

Given: $\angle 2 \cong \angle 3$
Prove: $\angle 1 \cong \angle 4$



Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. ___?___
2. $\angle 2 \cong \angle 3$	2. ___?___
3. $\angle 3 \cong \angle 4$	3. ___?___
4. ___?___	4. Transitive Property (used twice)

If $\angle A$ and $\angle B$ are supplementary, find the value of x, $m\angle A$, and $m\angle B$.

B **24.** $m\angle A = 2x$, $m\angle B = x - 15$

25. $m\angle A = x + 16$, $m\angle B = 2x - 16$

If $\angle C$ and $\angle D$ are complementary, find the value of y, $m\angle C$, and $m\angle D$.

26. $m\angle C = 3y + 5$, $m\angle D = 2y$

27. $m\angle C = y - 8$, $m\angle D = 3y + 2$

Use the given information to write an equation and solve the problem.

28. Find the measure of an angle that is twice as large as its supplement.

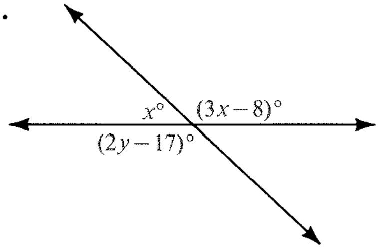
29. Find the measure of an angle that is half as large as its complement.

30. The measure of a supplement of an angle is 12 more than twice the measure of the angle. Find the measures of the angle and its supplement.

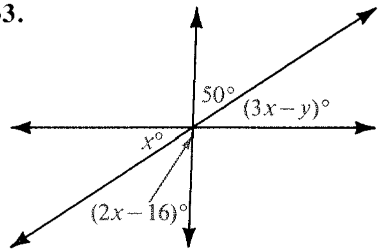
31. A supplement of an angle is six times as large as a complement of the angle. Find the measures of the angle, its supplement, and its complement.

Find the values of x and y for each diagram.

32.



33.



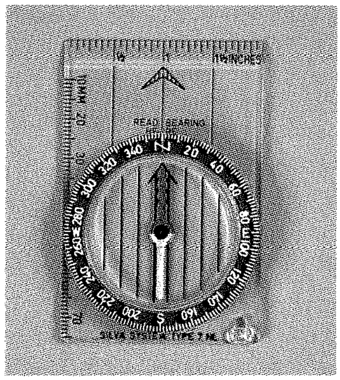
C **34.** Can the measure of a complement of an angle ever equal exactly half the measure of a supplement of the angle? Explain.

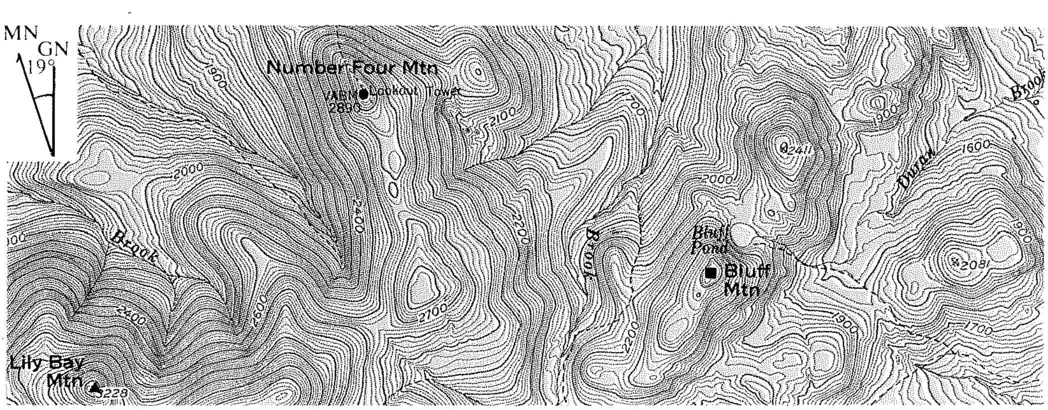
35. You are told that the measure of an acute angle is equal to the difference between the measure of a supplement of the angle and twice the measure of a complement of the angle. What can you deduce about the angle? Explain.

Application — *Orienteering*

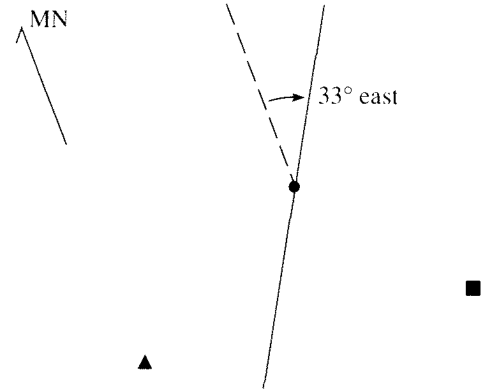
The sport of orienteering involves finding your way from control point to control point in a wilderness area, using a map and protractor-type compass. Similar methods can be used by hikers, hunters, boaters, and backpackers.

One thing you want to be able to do is locate your position on the map. This can be done by taking sightings of specific objects. For example, suppose you can see a lookout tower (on Number Four Mountain at • on the map shown below).



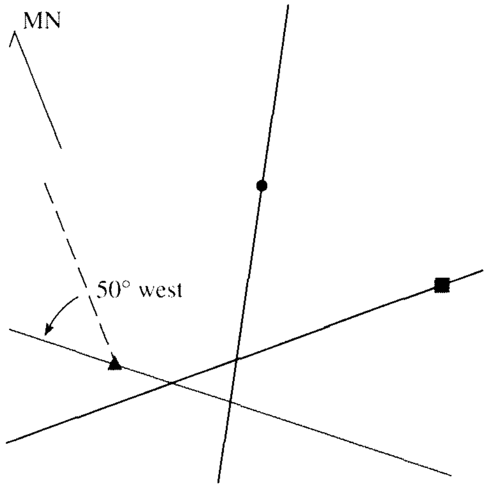


You sight across your compass and discover the tower is 33° east of magnetic north (MN). On your map you draw a line through the tower at a 33° angle to magnetic north. Be sure to use magnetic north rather than true north, for they may differ by as much as 20°. Hiking maps and nautical charts usually give both. All compass readings here are given in terms of magnetic north.



You are somewhere on the line you have drawn. If there is a feature near you (a trail, stream or pond), then your position is where the line crosses the feature on the map. Otherwise, you will need to take a second sighting, on the peak of Lily Bay Mountain (at ▲ on the map). It is 50° west of north. Draw a line on your map through the peak at a 50° angle with magnetic north. You are close to the point where the lines cross.

Since a third landmark is visible, the summit of Bluff Mountain (■ on the map), you can check your position with a third sighting. The three lines might cross at a single point. However, there is usually some error in sighting and drawing the angles, so instead of meeting exactly at a point, the three lines drawn often form a triangle. If the triangle is small, it gives you a good idea of your true position.



Exercises

1. Another orienteering party sights on Lily Bay Mountain and the lookout tower and finds the following angles: mountain, 58° west of north; tower, 40° east of north. Are they north or south of you?

2. If you head due east from Lily Bay Mountain (90° east of magnetic north), will you pass Bluff Mountain on your right or on your left?

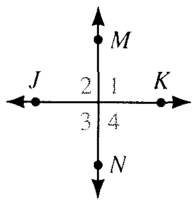
3. Lillian and Ray both sight Lily Bay Mountain at 70° west of north, but Lillian sees the lookout tower at 40° east of north, while Ray sees it at 20° east of north. Which person is closer to Bluff Mountain?

4. Sailors use this method of finding their position when they are navigating near shore, sighting on lighthouses, smokestacks, and other landmarks shown on their charts. They call the small triangle formed by the three sighting lines a "cocked hat," and usually mark their position at the corner closest to the nearest hazard. Why is this a sensible rule?

2-5 *Perpendicular Lines*

In the town shown, roads that run east-west are called streets, while those that run north-south are called avenues. Each of the streets is *perpendicular* to each of the avenues.

Perpendicular lines are two lines that intersect to form right angles (90° angles). Because lines that form one right angle always form four right angles (see Exercise 26, page 21), you can conclude that two lines are perpendicular, by definition, once you know that any one of the angles they form is a right angle. The definition of perpendicular lines can be used in the two ways shown below.



1. If $\overleftrightarrow{JK}$ is perpendicular to $\overleftrightarrow{MN}$ (written $\overleftrightarrow{JK} \perp \overleftrightarrow{MN}$), then each of the numbered angles is a right angle (a 90° angle).
2. If any one of the numbered angles is a right angle (a 90° angle), then $\overleftrightarrow{JK} \perp \overleftrightarrow{MN}$.

The word *perpendicular* is also used for intersecting rays and segments. For example, if $\overleftrightarrow{JK} \perp \overleftrightarrow{MN}$ in the diagram, then $\overline{JK} \perp \overline{MN}$ and the sides of $\angle 2$ are perpendicular.

The definition of perpendicular lines is closely related to the following theorems. Notice that Theorem 2-4 and Theorem 2-5 are *converses* of each other. For the proofs of the theorems, see the exercises.

Theorem 2-4

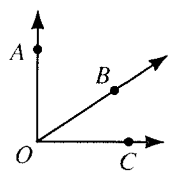
If two lines are perpendicular, then they form congruent adjacent angles.

Theorem 2-5

If two lines form congruent adjacent angles, then the lines are perpendicular.

Theorem 2-6

If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.



Given: $\overrightarrow{OA} \perp \overrightarrow{OC}$

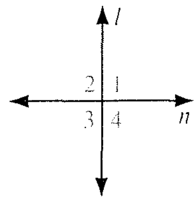
Prove: $\angle AOB$ and $\angle BOC$ are comp. $\angle$s.

Classroom Exercises

1. Complete the proof of Theorem 2-4: If two lines are perpendicular, then they form congruent adjacent angles.

Given: $l \perp n$

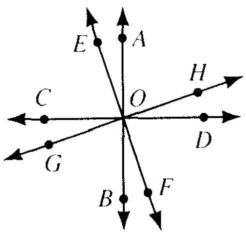
Prove: $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are congruent angles.



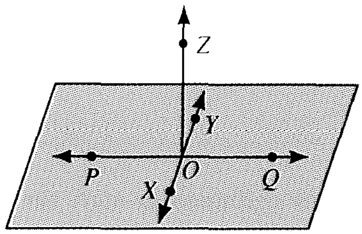
Proof:

Statements	Reasons
1. $l \perp n$	1. ___?___
2. $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ are 90° ∠s.	2. Definition of ___?___
3. $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ are ≅ ∠s.	3. Definition of ___?___

2. In the diagram, $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and $\overleftrightarrow{EF} \perp \overleftrightarrow{GH}$. Name eight right angles.

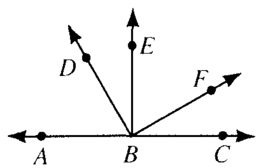


3. In the diagram, $\overrightarrow{OZ} \perp \overleftrightarrow{PQ}$, $\overrightarrow{OZ} \perp \overleftrightarrow{XY}$, and $\overleftrightarrow{PQ} \perp \overleftrightarrow{XY}$. Name eight right angles.



In the diagram, $\overrightarrow{BE} \perp \overleftrightarrow{AC}$ and $\overrightarrow{BD} \perp \overrightarrow{BF}$. Find the measures of the following angles.

	$m\angle CBF$	$m\angle EBF$	$m\angle DBE$	$m\angle DBA$	$m\angle DBC$
4.	40	?	?	?	?
5.	x	?	?	?	?



Name the definition or state the theorem that justifies the statement about the diagram.

6. If $\angle 6$ is a right angle, then $\overleftrightarrow{RS} \perp \overleftrightarrow{TV}$.

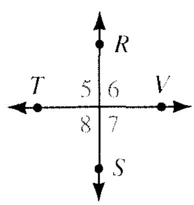
7. If $\overleftrightarrow{RS} \perp \overleftrightarrow{TV}$, then $\angle 5$, $\angle 6$, $\angle 7$, and $\angle 8$ are right angles.

8. If $\overleftrightarrow{RS} \perp \overleftrightarrow{TV}$, then $\angle 8 \cong \angle 7$.

9. If $\overleftrightarrow{RS} \perp \overleftrightarrow{TV}$, then $m\angle 6 = 90$.

10. If $\angle 5 \cong \angle 6$, then $\overleftrightarrow{RS} \perp \overleftrightarrow{TV}$.

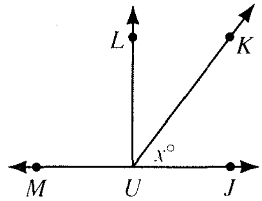
11. If $m\angle 5 = 90$, then $\overleftrightarrow{RS} \perp \overleftrightarrow{TV}$.



Written Exercises

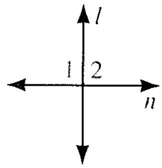
A **1.** In the diagram, $\overrightarrow{UL} \perp \overleftrightarrow{MJ}$ and $m\angle JUK = x$. Express in terms of x the measures of the angles named.

a. $\angle LUK$ **b.** $\angle MUK$



2. Copy and complete the proof of Theorem 2-5: If two lines form congruent adjacent angles, then the lines are perpendicular.

Given: $\angle 1 \cong \angle 2$
Prove: $l \perp n$



Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$, or $m\angle 1 = m\angle 2$	1. ?
2. $m\angle 1 + m\angle 2 = 180$	2. ?
3. $m\angle 2 + m\angle 2 = 180$, or $2m\angle 2 = 180$	3. ?
4. $m\angle 2 = 90$	4. ?
5. ?	5. Def. of $\perp$ lines

Name the definition or state the theorem that justifies the statement about the diagram.

3. If $\angle EBC$ is a right angle, then $\overrightarrow{BE} \perp \overleftrightarrow{AC}$.

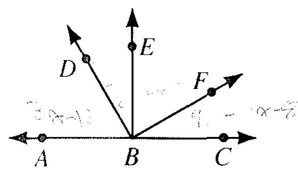
4. If $\overleftrightarrow{AC} \perp \overrightarrow{BE}$, then $\angle ABE$ is a right angle.

5. If $\overrightarrow{BE} \perp \overleftrightarrow{AC}$, then $\angle ABD$ and $\angle DBE$ are complementary.

6. If $\angle ABD$ and $\angle DBE$ are complementary angles, then $m\angle ABD + m\angle DBE = 90$.

7. If $\overrightarrow{BE} \perp \overleftrightarrow{AC}$, then $m\angle ABE = 90$.

8. If $\angle ABE \cong \angle EBC$, then $\overleftrightarrow{AC} \perp \overrightarrow{BE}$.



Exs. 3–12

In the diagram, $\overrightarrow{BE} \perp \overleftrightarrow{AC}$ and $\overrightarrow{BD} \perp \overrightarrow{BF}$. Find the value of x.

9. $m\angle ABD = 2x - 15$, $m\angle DBE = x$

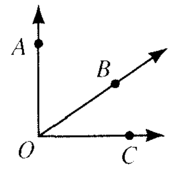
10. $m\angle DBE = 3x$, $m\angle EBF = 4x - 1$

11. $m\angle ABD = 3x - 12$, $m\angle DBE = 2x + 2$, $m\angle EBF = 2x + 8$

12. $m\angle ABD = 6x$, $m\angle DBE = 3x + 9$, $m\angle EBF = 4x + 18$, $m\angle FBC = 4x$

13. Copy and complete the proof of Theorem 2-6: If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.

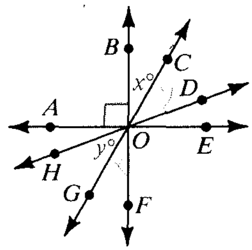
Given: $\overrightarrow{OA} \perp \overrightarrow{OC}$
Prove: $\angle AOB$ and $\angle BOC$ are comp. $\angle$s.



Proof:

Statements	Reasons
1. $\overrightarrow{OA} \perp \overrightarrow{OC}$	1. __?__
2. $m\angle AOC = 90$	2. Def. of $\perp$ lines
3. $m\angle AOB + m\angle BOC = m\angle AOC$	3. __?__
4. __?__	4. Substitution Prop.
5. __?__	5. Def. of comp. $\angle$s

In the figure $\overleftrightarrow{BF} \perp \overleftrightarrow{AE}$, $m\angle BOC = x$, and $m\angle GOH = y$. Express the measure of the angle in terms of x, y, or both.



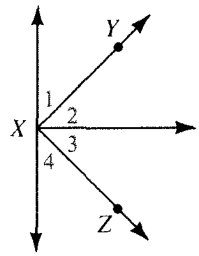
B **14.** $\angle COA$

15. $\angle COH$

16. $\angle HOF$

17. $\angle DOE$

Can you conclude from the information given for each exercise that $\overrightarrow{XY} \perp \overrightarrow{XZ}$?



18. $m\angle 1 = 46$ and $m\angle 4 = 44$

19. $\angle 1$ and $\angle 3$ are complementary.

20. $\angle 2 \cong \angle 3$

21. $m\angle 1 = m\angle 4$

22. $\angle 1$ and $\angle 3$ are congruent and complementary.

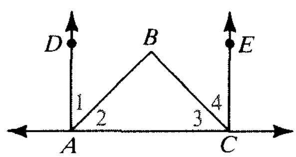
23. $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$

24. $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

25. $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$

What can you conclude from the information given?

26. Given: $\overrightarrow{AB}$ bisects $\angle DAC$;
$\overrightarrow{CB}$ bisects $\angle ECA$;
$m\angle 2 = 45$;
$m\angle 3 = 45$

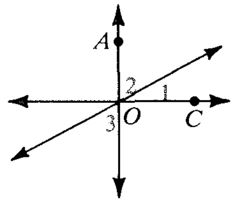


27. Given: $\overrightarrow{AD} \perp \overleftrightarrow{AC}$; $\overrightarrow{CE} \perp \overleftrightarrow{AC}$; $m\angle 1 = m\angle 4$

28. Copy everything shown and write a two-column proof.

Given: $\overleftrightarrow{AO} \perp \overleftrightarrow{CO}$

Prove: $\angle 1$ and $\angle 3$ are comp. $\angle$s.



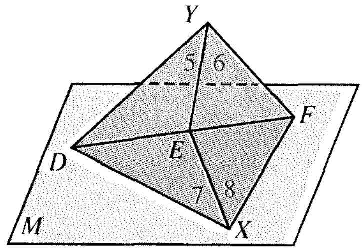
C **29.** First find two lines (other than $\overleftrightarrow{YD}$ and $\overleftrightarrow{YF}$) that are perpendicular. Then write a two-column proof that the lines are perpendicular.

Given: $\overleftrightarrow{YD} \perp \overleftrightarrow{YF}$;

$m\angle 7 = m\angle 5$;

$m\angle 8 = m\angle 6$

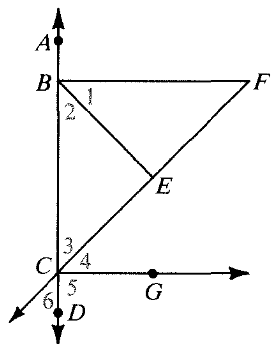
Prove: $\underline{\ ?\ } \perp \underline{\ ?\ }$



Mixed Review Exercises

Write something you can conclude from the given information.

1. Given: $m\angle 1 = m\angle 4$; $m\angle 2 = m\angle 3$
2. Given: $AB = CD$
3. Given: $m\angle 6 = m\angle 4$
4. Given: $\overline{FB} \perp \overline{AD}$; $\overrightarrow{BE}$ bisects $\angle FBC$.
5. Given: $BE = EF$; E is the midpoint of $\overline{FC}$.
6. Given: $\angle 1$ and $\angle 2$ are complements.
7. Given: $\angle 4$ and $\angle 6$ are complements.



2-6 *Planning a Proof*

As you have seen in the last few sections, a proof of a theorem consists of five parts:

1. *Statement* of the theorem
2. A *diagram* that illustrates the given information
3. A list, in terms of the figure, of what is *given*
4. A list, in terms of the figure, of what you are to *prove*
5. A series of *statements and reasons* that lead from the given information to the statement that is to be proved

In many of the proofs in this book, the diagram and the statements of what is given and what is to be proved will be supplied for you. Sometimes you will be asked to provide them.

When you draw a diagram, try to make it reasonably accurate, avoiding special cases that might mislead. For example, when a theorem refers to an angle, don't draw a *right* angle.

Before you write the steps in a two-column proof you will need to plan your proof. Sometimes you will read the statement of a theorem and see immediately how to prove it. Other times you may need to try several approaches before you find a plan that works.

If you don't see a method of proof immediately, try reasoning back from what you would like to prove. Think: "This conclusion will be true if _?_ is true. This, in turn, will be true if _?_ is true . . . ." Sometimes this procedure leads back to a given statement. If so, you have found a method of proof.

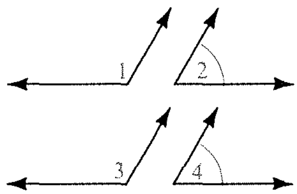
Studying the proofs of previous theorems may suggest methods to try. For example, the proof of the theorem that vertical angles are congruent suggests the proof of the following theorem.

Theorem 2-7

If two angles are supplements of congruent angles (or of the same angle), then the two angles are congruent.

Given: $\angle 1$ and $\angle 2$ are supplementary;
$\angle 3$ and $\angle 4$ are supplementary;
$\angle 2 \cong \angle 4$

Prove: $\angle 1 \cong \angle 3$



Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary; $\angle 3$ and $\angle 4$ are supplementary.	1. Given
2. $m\angle 1 + m\angle 2 = 180$; $m\angle 3 + m\angle 4 = 180$	2. Def. of supp. $\angle$s
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3. Substitution Prop.
4. $\angle 2 \cong \angle 4$, or $m\angle 2 = m\angle 4$	4. Given
5. $m\angle 1 = m\angle 3$, or $\angle 1 \cong \angle 3$	5. Subtraction Prop. of =

The proof of the following theorem is left as Exercise 18.

Theorem 2-8

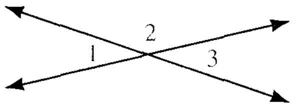
If two angles are complements of congruent angles (or of the same angle), then the two angles are congruent.

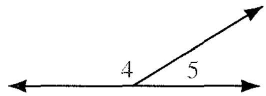
There is often more than one way to prove a particular statement, and the amount of detail one includes in a proof may differ from person to person. You should show enough steps so the reader can follow your argument and see why the theorem you are proving is true. As you gain more experience in writing proofs, you and your teacher may agree on what steps may be combined or omitted.

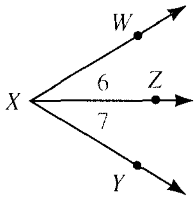
Classroom Exercises

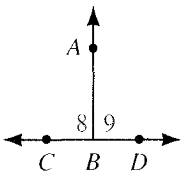
a. In each exercise use the information given to conclude that two angles are congruent.
b. Name or state the definition or theorem that justifies your conclusion.

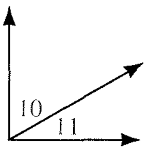
1. $\angle 6$ is comp. to $\angle 10$;
   $\angle 7$ is comp. to $\angle 10$.
2. $m\angle 5 = 31$; $m\angle 7 = 31$
3. $\overline{AB} \perp \overline{CD}$
4. $\overrightarrow{XZ}$ bisects $\angle WXY$.
5. $\angle 4$ is supp. to $\angle 6$;
   $\angle 2$ is supp. to $\angle 7$;
   $\angle 6 \cong \angle 7$
6. Given only the diagrams, and no additional information





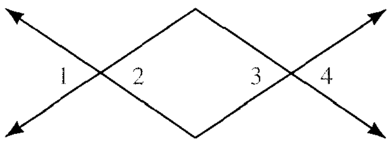




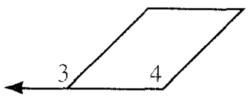


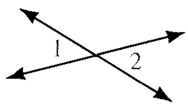
Describe your plan for proving the following. You don't need to give all the details.

7. Given: $\angle 2 \cong \angle 3$
   Prove: $\angle 1 \cong \angle 4$

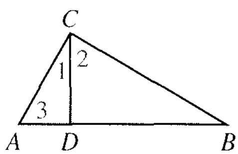


8. Given: $\angle 3$ is supp. to $\angle 1$;
   $\angle 4$ is supp. to $\angle 2$.
   Prove: $\angle 3 \cong \angle 4$

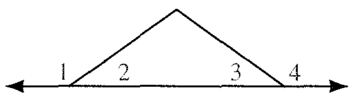




9. Given: $\overline{AC} \perp \overline{BC}$;
   $\angle 3$ is comp. to $\angle 1$.
   Prove: $\angle 3 \cong \angle 2$

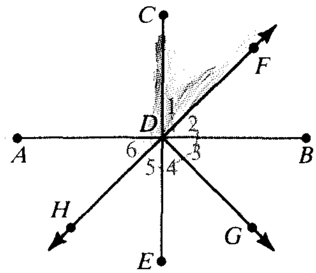


10. Given: $m\angle 1 = m\angle 4$
    Prove: $m\angle 2 = m\angle 3$



Written Exercises

Write the name or statement of the definition, postulate, property, or theorem that justifies the statement about the diagram.



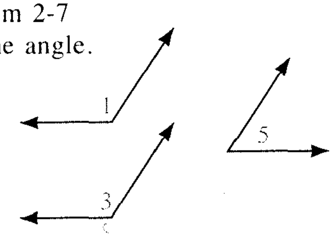
Exs. 1–14

A

1. $AD + DB = AB$
2. $m\angle 1 + m\angle 2 = m\angle CDB$
3. $\angle 2 \cong \angle 6$
4. If D is the midpoint of $\overline{AB}$, then $AD = \frac{1}{2}AB$.
5. If $\overrightarrow{DF}$ bisects $\angle CDB$, then $\angle 1 \cong \angle 2$.
6. $m\angle ADF + m\angle FDB = 180$
7. If $\overline{CD} \perp \overline{AB}$, then $m\angle CDB = 90$.
8. If $\angle 4 \cong \angle 3$, then $\overrightarrow{DG}$ bisects $\angle BDE$.
9. If $m\angle 3 + m\angle 4 = 90$, then $\angle 3$ and $\angle 4$ are complements.
10. If $\angle ADF$ and $\angle 4$ are supplements, then $m\angle ADF + m\angle 4 = 180$.
11. If $\overline{AB} \perp \overline{CE}$, then $\angle ADC \cong \angle ADE$.
12. If $\angle 4$ is complementary to $\angle 5$ and $\angle 6$ is complementary to $\angle 5$, then $\angle 4 \cong \angle 6$.
13. If $\angle FDG$ is a right angle, then $\overrightarrow{DF} \perp \overrightarrow{DG}$.
14. If $\angle FDG \cong \angle GDH$, then $\overrightarrow{DG} \perp \overleftrightarrow{HF}$.
15. Copy everything shown and complete the proof of Theorem 2-7 for the case where two angles are supplements of the same angle.

    Given: $\angle 1$ and $\angle 5$ are supplementary;
    $\angle 3$ and $\angle 5$ are supplementary.

    Prove: $\angle 1 \cong \angle 3$

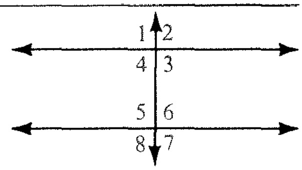


Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 5$ are supplementary; $\angle 3$ and _?_.	1. _?_
2. $m\angle 1 + m\angle 5 = 180$; $m\angle 3 + m\angle 5 = 180$	2. _?_
3. $m\angle 1 + m\angle 5 = m\angle 3 + m\angle 5$	3. _?_
4. $m\angle 5 = m\angle 5$	4. Reflexive Prop.
5. $m\angle 1 = m\angle 3$, or $\angle 1 \cong \angle 3$	5. _?_

16. **a.** Are there any angles in the diagram that must be congruent to $\angle 4$? Explain.

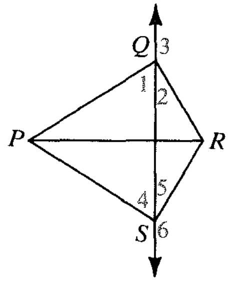
    b. If $\angle 4$ and $\angle 5$ are supplementary, name all angles shown that must be congruent to $\angle 4$.



17. a. Copy everything shown and complete the proof.

Given: $\overline{PQ} \perp \overline{QR}$;
$\overline{PS} \perp \overline{SR}$;
$\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 5$



Proof:

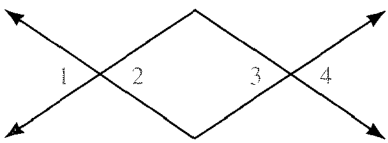
Statements	Reasons
1. $\overline{PQ} \perp \overline{QR}$; $\overline{PS} \perp \overline{SR}$	1. ___?___
2. $\angle 2$ is comp. to $\angle 1$; $\angle 5$ is comp. to $\angle 4$.	2. ___?___
3. $\angle 1 \cong \angle 4$	3. ___?___
4. $\angle 2 \cong \angle 5$	4. ___?___

b. After proving that $\angle 2 \cong \angle 5$ in part (a), tell how you could go on to prove that $\angle 3 \cong \angle 6$.

B **18.** Prove Theorem 2-8: If two angles are complements of congruent angles, then the two angles are congruent. *Note*: You will need to draw your own diagram and state what is given and what you are to prove in terms of your diagram. (*Hint*: See the proof of Theorem 2-7 on page 61.)

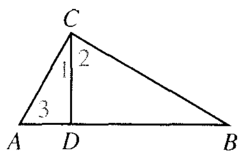
Copy everything shown and write a two-column proof.

19. Given: $\angle 2 \cong \angle 3$
Prove: $\angle 1 \cong \angle 4$

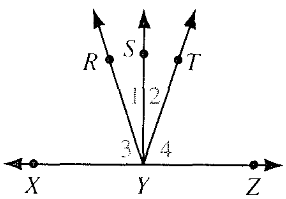


20. Given: $\angle 3$ is supp. to $\angle 1$;
$\angle 4$ is supp. to $\angle 2$.
Prove: $\angle 3 \cong \angle 4$

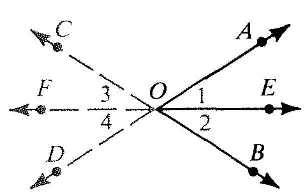
21. Given: $\overline{AC} \perp \overline{BC}$;
$\angle 3$ is comp. to $\angle 1$.
Prove: $\angle 3 \cong \angle 2$



22. Given: $m\angle 1 = m\angle 2$;
$m\angle 3 = m\angle 4$
Prove: $\overrightarrow{YS} \perp \overleftrightarrow{XZ}$

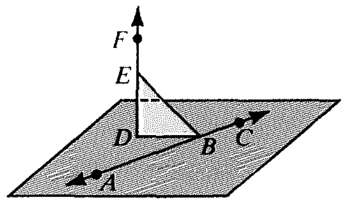


23. Draw any $\angle AOB$ and its bisector $\overrightarrow{OE}$. Now draw the rays opposite to $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OE}$. What can you conclude about the part of the diagram shown in red? Prove your conclusion.



C **24.** Make a diagram showing $\angle PQR$ bisected by $\overrightarrow{QX}$. Choose a point Y on the ray opposite to $\overrightarrow{QX}$.
Prove: $\angle PQY \cong \angle RQY$

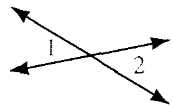
25. Given: $m\angle DBA = 45$;
$m\angle DEB = 45$
Prove: $\angle DBC \cong \angle FEB$



Self-Test 2

1. It is known that $\angle HOK$ has a supplement, but can't have a complement. Name one possible measure for $\angle HOK$.

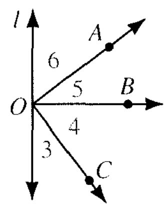
2. $m\angle 1 = 3x - 5$ and $m\angle 2 = x + 25$
a. $x =$ _?_ **b.** $m\angle 1 =$ _?_ (numerical value)



For Exercises 3 and 4 you are given that $\overrightarrow{OB} \perp l$ and $\overrightarrow{OA} \perp \overrightarrow{OC}$.

3. If $m\angle 3 = 37$, complete:
$m\angle 4 =$ _?_ $m\angle 5 =$ _?_ $m\angle 6 =$ _?_

4. If $m\angle 3 = t$, express the measures of the other numbered angles in terms of t.

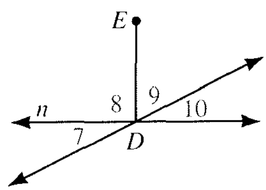


In the diagram, $\overline{DE} \perp n$. State the theorem or name the definition that justifies the statement about the diagram.

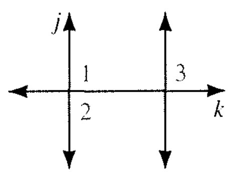
5. $\angle 8$ is a 90° angle.

6. $\angle 7 \cong \angle 10$

7. $\angle 9$ and $\angle 10$ are complementary.



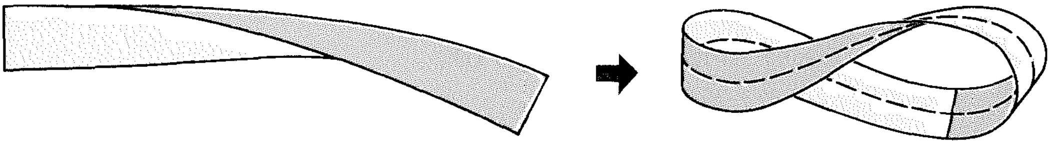
8. Give a plan for the following proof.
Given: $\angle 1$ is supp. to $\angle 3$;
$\angle 2$ is supp. to $\angle 3$.
Prove: $j \perp k$



9. Write a proof for Exercise 8 in two-column form.

Extra Möbius Bands

Take a long narrow strip of paper. Give the strip a half-twist. Tape the ends together. The result is a *Möbius band.*



Exercises

1. Make a Möbius band. Color one side of the Möbius band. How much of the band is left uncolored? The original strip of paper had two sides. How many sides does a Möbius band have?
2. Cut the Möbius band lengthwise down the middle. (Start at a point midway between the edges and cut around the band.) What is the result? Cut the band a second time down the middle. Write a sentence or two describing what happens.
3. Give a full twist to a long, narrow strip of paper. Tape the ends together. How many sides does this band have? Cut this band lengthwise down the middle. Write a brief description of what is formed.
4. Make a Möbius band. Let the band be 3 cm wide. Make a lengthwise cut, staying 1 cm from the right-hand edge. Describe the result.
5. Take two long narrow strips of paper. Fasten them together so they are perpendicular and form a plus sign. Twist one strip so it is a Möbius band and fasten its ends together. Don't twist the other strip at all, just fasten its ends. Cut the Möbius band down the middle lengthwise. Then cut the other band down the middle. Describe the final result.

Chapter Summary

1. *If p, then q* is a conditional statement. p is the hypothesis and q is the conclusion. *If q, then p* is the converse. The statement *p if and only if q* is a biconditional that means both the conditional and its converse are true.
2. Properties of algebra (see page 37) can be used to reach conclusions in geometry. Properties of congruence are related to some of the properties of equality.
3. Deductive reasoning is a process of proving conclusions. Given information, definitions, postulates, and previously proved theorems are the four kinds of reasons that can be used to justify statements in a proof.

4. When $m\angle A + m\angle B = 90$, $\angle A$ and $\angle B$ are complementary. When $m\angle C + m\angle D = 180$, $\angle C$ and $\angle D$ are supplementary. Complements (or supplements) of the same angle or of congruent angles are congruent.
5. Vertical angles are congruent.
6. Perpendicular lines are two lines that form right angles (90° angles). If two lines are perpendicular, then they form congruent adjacent angles. If two lines form congruent adjacent angles, then the lines are perpendicular.
7. If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.
8. The proof of a theorem consists of five parts, which are listed on page 60.

Chapter Review

Use the conditional: If $m\angle 1 = 120$, then $\angle 1$ is obtuse.

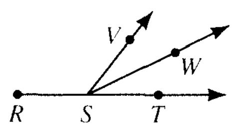
1. Write the hypothesis and the conclusion of the conditional. 2–1
2. Write the converse of the conditional.
3. Provide a counterexample to disprove the converse.
4. Write a definition of a straight angle as a biconditional.

Justify each statement with a property from algebra or a property of congruence.

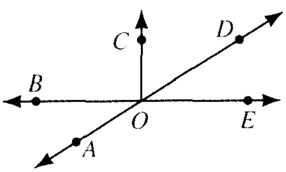
5. If $m\angle A + m\angle B + m\angle C = 180$ and $m\angle C = 50$, then $m\angle A + m\angle B + 50 = 180$. 2–2
6. If $m\angle A + m\angle B + 50 = 180$, then $m\angle A + m\angle B = 130$.
7. If $6x = 18$, then $x = 3$.
8. If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Name the definition, postulate, or theorem that justifies the statement.

9. If $\overline{RS} \cong \overline{ST}$, then S is the midpoint of $\overline{RT}$. 2–3
10. If $\overrightarrow{SW}$ bisects $\angle VST$, then $\angle VSW \cong \angle WST$.
11. If $\overrightarrow{SW}$ bisects $\angle VST$, then $m\angle WST = \frac{1}{2}m\angle VST$.



12. If $\angle BOC$ is a right angle and $m\angle COD = 58$, then $m\angle DOE = $ _?_, $m\angle BOA = $ _?_, and $m\angle AOC = $ _?_. 2–4
13. Name a supplement of $\angle AOE$.



14. A supplement of a given angle is four times as large as a complement of the angle. Find the measure of the given angle.

Name the definition or state the theorem that justifies the statement about the diagram.

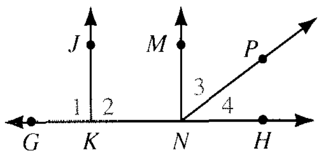
2–5

15. If $\overrightarrow{KJ} \perp \overleftrightarrow{GH}$, then $\angle 1$ is a right angle.

16. If $\angle 2$ is a 90° angle, then $\overrightarrow{KJ} \perp \overleftrightarrow{GH}$.

17. If $\overrightarrow{NM} \perp \overleftrightarrow{GH}$, then $\angle MNK \cong \angle MNH$.

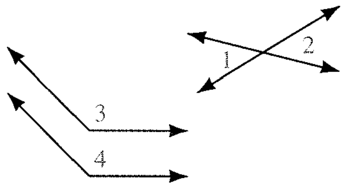
18. If $\overrightarrow{NM} \perp \overleftrightarrow{GH}$, then $\angle 3$ and $\angle 4$ are complementary.



2–6

19. Write a plan for a proof.
Given: $\angle 3$ is a supplement of $\angle 1$;
$\angle 4$ is a supplement of $\angle 2$.
Prove: $\angle 3 \cong \angle 4$

20. Write a proof in two-column form for Exercise 19.



Chapter Test

1. Use the conditional: Two angles are congruent if they are vertical angles.
a. Write the hypothesis. **b.** Write the converse.

2. Provide a counterexample to disprove the statement:
If $x^2 > 4$, then $x > 2$.

3. Write the biconditional as two conditionals that are converses of each other:
Angles are congruent if and only if their measures are equal.

4. Supply reasons to justify the steps:

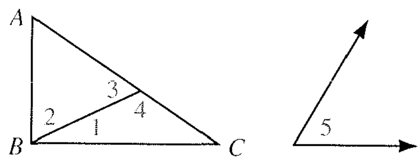
Steps	*Reasons*
1. $y = 12$	1. Given
2. $5x = 2x + y$	2. Given
3. $5x = 2x + 12$	3. __?__
4. $3x = 12$	4. __?__
5. $x = 4$	5. __?__

5. $\overrightarrow{OB}$ is the bisector of $\angle AOC$ and $\overrightarrow{OC}$ is the bisector of $\angle BOD$. $m\angle AOC = 60$. Find $m\angle COD$.

6. S is the midpoint of $\overline{RT}$ and W is the midpoint of $\overline{ST}$. If $RT = 32$, find ST, WT, and RW.

7. In the diagram, $\overline{AB} \perp \overline{BC}$. Name:
a. two supplementary angles
b. two complementary angles

8. Given: $\angle 5$ is supplementary to $\angle 4$.
a. What can you conclude about $\angle 5$ and $\angle 3$?
b. State the theorem that justifies your conclusion.

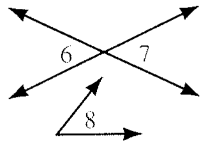


Exs. 7–9

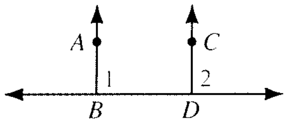
9. Suppose $m\angle 3 = 3x + 5$ and $m\angle 4 = 6x + 13$. Find the value of x.

10. State the theorem that justifies the statement $\angle 6 \cong \angle 7$.

11. Suppose you have already stated that $\angle 6 \cong \angle 7$ and $\angle 7 \cong \angle 8$. What property of congruence justifies the conclusion that $\angle 6 \cong \angle 8$?



12. Write a proof in two-column form.
Given: $\overrightarrow{DC} \perp \overleftrightarrow{BD}$; $\angle 1 \cong \angle 2$
Prove: $\overrightarrow{BA} \perp \overleftrightarrow{BD}$



Algebra Review: *Systems of Equations*

Solve each system of equations by the substitution method.

Example 1 (1) $y = 5 - 2x$
(2) $5x - 6y = 21$

Solution Substitute $5 - 2x$ for y in (2): $5x - 6(5 - 2x) = 21$
$17x - 30 = 21$; $x = 3$

Substitute 3 for x in (1): $y = 5 - 2(3) = -1$

The solution is $x = 3$, $y = -1$.

1. $y = 3x$
$5x + y = 24$

2. $y = 2x + 5$
$3x - y = 4$

3. $x = 8 + 3y$
$2x - 5y = 8$

4. $3x + 2y = 71$
$y = 4 + 2x$

5. $4x - 5y = 92$
$x = 7y$

6. $y = 3x + 8$
$x = y$

7. $8x + 3y = 26$
$2x = y - 4$

8. $x - 7y = 13$
$3x - 5y = 23$

9. $3x + y = 19$
$2x - 5y = -10$

Solve each system by the method of addition or subtraction.

Example 2 (1) $3x - y = 13$
(2) $4x + y = 22$

Solution Add (1) and (2):
$7x = 35$; $x = 5$

Substitute 5 for x in (2):
$4(5) + y = 22$; $y = 2$

The solution is $x = 5$, $y = 2$.

Example 3 (1) $6x + 15y = 90$
(2) $6x - 14y = 32$

Solution Subtract (2) from (1):
$29y = 58$; $y = 2$

Substitute 2 for y in (1):
$6x + 15(2) = 90$; $x = 10$

The solution is $x = 10$, $y = 2$.

10. $5x - y = 20$
$3x + y = 12$

11. $x + 3y = 7$
$x + 2y = 4$

12. $3x - 2y = 11$
$3x - y = 7$

13. $7x + y = 29$
$5x + y = 21$

14. $8x - y = 17$
$6x + y = 11$

15. $9x - 2y = 50$
$6x - 2y = 32$

16. $7y = 2x + 35$
$3y = 2x + 15$

17. $2y = 3x - 1$
$2y = x + 21$

18. $19 = 5x + 2y$
$1 = 3x - 4y$

Preparing for College Entrance Exams

Strategy for Success

When you are taking a college entrance exam, be sure to read the directions, the questions, and the answer choices very carefully. In the test booklet, you may want to underline important words such as *not, exactly, false, never,* and *except,* and to cross out answer choices that are clearly incorrect.

Indicate the best answer by writing the appropriate letter.

1. On a number line, point M has coordinate -3 and point R has coordinate 6. Point Z is on $\overrightarrow{RM}$ and $RZ = 4$. Find the coordinate of Z.
(A) -7 **(B)** 1 **(C)** 2 **(D)** 10 **(E)** cannot be determined

2. $\angle 1$ and $\angle 2$ are complementary. $m\angle 1 = 5x + 15$ and $m\angle 2 = 10x$. The measure of $\angle 1$ is:
(A) 5 **(B)** 11 **(C)** 40 **(D)** 70 **(E)** 30

3. Vertical angles are never:
(A) complementary **(B)** supplementary **(C)** right angles
(D) adjacent **(E)** congruent

4. A reason that cannot be used to justify a statement in a proof is:
(A) a postulate **(B)** a definition **(C)** given information
(D) yesterday's theorem **(E)** tomorrow's theorem

5. Which of the following must be true?
(I) If two lines form congruent adjacent angles, then the lines are perpendicular.
(II) If two lines are perpendicular, then they form congruent adjacent angles.
(III) If the exterior sides of two adjacent obtuse angles are perpendicular, then the angles are complementary.
(A) I only **(B)** II only **(C)** III only
(D) I and II only **(E)** I, II, and III

6. $\angle 1$ and $\angle 2$ are congruent angles. $m\angle 1 = 10x - 20$ and $m\angle 2 = 8x + 2$. $\angle 1$ is a(n) __?__ angle.
(A) acute **(B)** right **(C)** obtuse **(D)** straight
(E) answer cannot be determined

7. If you know that $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then what reason can you give for the statement that $m\angle A = m\angle C$?
(I) Reflexive Property (II) Transitive Property (III) Substitution Property
(A) I only **(B)** II only **(C)** III only
(D) either I or II **(E)** either II or III

8. Which of the following is *not* the converse of the statement: If b, then c.
(A) If c, then b. **(B)** b if c. **(C)** c if and only if b.
(D) c only if b. **(E)** c implies b.